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NATIONAL AND REGIONAL AIR POLLUTION SYSTEMS ANALYSIS PROGRAM

TRW Systems Group

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National and Regional Air Pollution
System Analysis Program
Phase II

June 1970

Prepared for

U.S. Department of Health, Education and Welfare
Public Health Service
Environmental Health Service
National Air Pollution Control Administration
Washington, D.C.

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1.0 PROJECT SUMMARY

1.1 BACKGROUND -- PHASE I

In May of 1968, TRW Systems Group submitted a proposal to the National Air Pollution Control Administration entitled "National and Regional Air Pollution Systems Analysis Program." The objectives of this study were to design, assemble, and deliver analytical tools to be used in regional and national air pollution planning and management. Programs were to be developed which would give decision-makers a rapid means for determining the most effective tactics and strategies for both regional and national air pollution control efforts.

Our view at that time was that regional and national air pollution analyses represented parts of a single complex problem, not because the results of one effort determined the nature of the other, but because of the similiarity of the required analytical processes and data. Therefore, a combined study program covering both regional and national activities was proposed. The analytical tools could be developed at a minimum cost due to the overlapping of data and modeling concepts in the combined program.

At the conclusion of Phase I, "First Year's Activities," TRW demonstrated the operation of a Regional Air Pollution Analysis Program, using the St. Louis Air Quality Control Region as the source of data. St. Louis provided an excellent demonstration example, since a great deal of data had been accumulated in this area and a diffusion model had recently been calibrated by NAPCA. The results of the demonstration were published in a final report which provided the reader with results of computer calculations as well as the technical basis for the computer programs and the actual program coding.

The second major output of the first year's activities is the National Air Pollution Analysis. This output presented the initial concepts of a National Air Pollution Analysis (NAPA) model designed to: (1) identify and help implement federal financial programs for the alleviation of air pollution problems, (2) provide information required for basic resource allocation decisions, and (3) assess the direct and indirect economic effects of air pollution control. The report pointed out that after several months of NAPA activities it had become apparent that a lack of data and a still-evolving definition of NAPA's role made it infeasible to pursue the development of a NAPA computer model. Although the report did not specify an actual computer model, it discussed the structure around which policy issues are generated and resolved at the national level. The report summarized an analysis of legislative and technological forecasting, the two techniques used to generate data on policy issues. The preliminary specifications of several mathematical models which could be used in resolving the issues were also developed. Although none of the issues that were raised were resolved, the completed work served to focus on and provide a foundation for activities in subsequent NAPA efforts.

The National Air Pollution Analysis activity also included a detailed Regional Economic Model which was to be the precursor of a National Economic Model. The Regional Economic Model developed in the first phase was intended to provide the following types of information for public-policy analysis:

(a) Regional economic declines expected to result from enforcement of varying emission control standards upon high-emission industries.

(b) Regional economic expansions expected to result from reduction of industrial damage and growth in air pollution equipment industries.

(c) Regional output, investment, employment, income, and consumption under varying emission control standards.

(d) Fiscal effects of regional implementation of air quality control programs, including tax-base impacts of economic change and the effects of tax credits upon economic change and the rate of achievement of emission standards in terms of the implementation plan.

(e) Statistical indicators, including industry-specific ratios of air pollution control costs to total investment and to value added in production.

In the first year (Phase I), a Regional Economic Model was developed for the St. Louis Region. This model describes the growth patterns of key economic sectors (both high- and low-emission industries), with estimates of regional product, employment, capital stock and investment change, value added by industry, tax receipts, and regional unemployment. These estimates are sensitive to a variety of air pollution controls applicable by the Regional Air Quality Control Agency and associated States or by the National Air Pollution Control Administration in conjunction with Federal legislation. The economic effects of five hypothetical air quality control strategies were simulated and interpreted, using the model for the St. Louis Region.

1.2 CURRENT ACTIVITIES -- PHASE II

In May of 1969, TRW Systems Group proposed a second phase of activities under the National and Regional Air Pollution Systems Analysis Program. The major efforts of this continued activity were:

(a) Converting the Regional Air Pollution Analysis Program, developed as a Phase I demonstration, into an operational program to be installed on government computing facilities.

(b) Performing experiments and tests to help validate the use of this type of computer program.

(c) Modifying the various models contained in the Phase I programs to include additional detail and expanded coverage.

(d) Testing national air pollution control strategies on several air quality control regions.

(e) Providing continuing NAPCA liaison in connection with further model developments, program evaluations, implementation planning, and air pollution emission standards.

TRW's efforts were aimed at combining NAPCA's on-going activities into a regional air pollution analysis program demonstrating the utility of a coordinated and integrated systems approach.

During the course of these Phase II activities, the study plan was modified somewhat in preparation for the application of implementation plans in several air quality control regions. These plans would require extensive use of the Regional Air Pollution Analysis Program, together with a technical analysis of both the inputs and the outputs from these programs. In an attempt to streamline the application of the programs to implementation planning, a separate set of programs were documented and installed on Department of Health, Education and Welfare computing facilities. These programs -- the Air Quality Display Model and the Implementation Planning Program -- were considered to be offshoots of the basic

Regional Air Pollution Analysis. The Implementation Planning Program was applied to the Air Quality Control Regions of St. Louis, Cincinnati, Washington, D. C., and Philadelphia. In each case, application of the program resulted in a great deal of output material which is being used in the development of State air quality implementation plans.

Continuing activities on the National Air Pollution Analysis project included evaluations of national economic factors that are influenced by air pollution control. During the first phase, a Regional Economic Model was developed for the St. Louis Air Quality Control Region. The second phase of activities in this area resulted in an expansion of this model to include the largest Standard Metropolitan Statistical Areas. These areas encompassed approximately 75 million people and more than 50% of national industrial production.

The current year (Phase II) has witnessed an extension of the model to 31 large metropolitan areas. These large urban areas have a varied industrial structure highly representative of the national industrial composition and comprise a significant segment of national output. The structure and outputs of this model are very similar to those of the St. Louis model.

The completion of the work on this model provides an opportunity to extend further work in two directions. First, the economic models could be extended to embrace all air quality control regions, at the same time providing a broader range of economic effects than has been possible so far.

Second, preliminary work on the estimation of national effects of air pollution control strategies can be implemented, building on the current year's work. Analysis of national effects is seen as a possible prelude to the development of a national economic model that could provide estimates of economic effects in the same detail as the regional models in their last phase of construction.

1.3 FUTURE ACTIVITIES -- PHASE III

The main theme of our approach to Phase III of this project is to develop trends or patterns of information based on our detailed implementation planning analysis of several air quality control regions, and to extend these patterns in order to derive broad-based conclusions on a national basis. At the present time, four air quality control regions (Cincinnati, St. Louis, Philadelphia, and Washington) are under study. The total population represented by the Standard Metropolitan Statistical Areas of these regions is approximately 12 million people. In addition, these regions represent a sizable cross-section in the mix of industrial, commercial, and residential land-use characteristics. It is expected that, on the basis of a combination of these features, it will be possible to derive general patterns of air pollution control costs, fossil fuel demands, land-use characteristics, control device demands, and so forth. In addition, implementation planning is also in progress for such other regions as Denver, Chicago, and New York. Data resulting from these implementation plans will become available in the very near future and will constitute another broad source of information from which generalized patterns can be derived. To avoid the pitfalls associated with the wide fluctuations in source characteristics or commercial land use characteristics, the patterns describing each of these regions will be arrived at by obtaining aggregated values for the 10 or so largest two-digit Standard Industrial Classification (SIC) codes and area sources representative of commercial, agricultural, and residential zones. For each of these categories, our study is expected to determine the regional response to selected emission standards or combinations of emission standards in terms of air pollution control strategies. These responses will then be indicative of air pollution control strategies or emission standards throughout the nation. Similarities will be established between various regions of the nation simply by scaling the magnitude of area source characteristics and the major industrial categories selected for those regions in which detailed implementation planning studies have been undertaken. The results of this work will provide a broad and generalized answer to the questions posed regarding air pollution control policy issues for the national aggregation of regions, including those regions which have not undergone detailed analysis. In addition, this type of study will provide air pollution analysis coverage beyond the limits of presently designated air quality control regions. The patterns developed for the detailed studies will be applicable to similar industrial, commercial, residential, and agricultural regions located outside the boundaries of air quality control regions presently included in the basic regional air pollution analysis.

2.0 REGIONAL AIR POLLUTION ANALYSIS PROGRAM

To assist the Federal Government and Air Quality Control Regions in developing and evaluating meaningful air pollution control strategies, the Regional Air Pollution Analysis (RAPA) Program* has been developed. The regional air quality implementation process is based on the application of a set of stationary source emission controls in the form of emission standards. The many combinations of these standards result in a variety of alternative control strategies. Through application of computer simulation techniques, the program is used in selecting appropriate emission standards, evaluating the resulting air quality, and estimating the costs associated with the various alternative control strategies.

The applicability of the program is bounded by certain operational restrictions. The control regulations, or emission standards, evaluated by the program are restricted to stationary sources of pollution. The program accounts for the effects of Federal control regulations on mobile emissions by including both existing and future mobile source contributions to the total area source inventory of pollutant emissions.

The computer simulation initially took into consideration only two pollutants, sulfur oxides and suspended particulate matter. Criteria and control technique information have now been published for several additional pollutants, however, and the program is being modified accordingly.

Although certain data on air quality, meteorological conditions, and other parameters are expressed in short time spans (days or months), the computer simulation is concerned primarily with an annual time scale. The statistically derived short-term parameters are included for future application in more critical analyses. Use of short-term data in the program is restricted because of the use of both a long-term atmospheric diffusion model and emission standards that require the application of continuous and permanent control measures.

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*The Implementation Planning Program, a limited version of RAPA, was developed for use by local air quality control region officials. A program manual entitled "Air Quality Implementation Planning Program" was submitted to NAPCA in March 1970. A separate activity under the PH 22-68-60 contract has been initiated to improve this program and its documentation. This effort is being directed by P. Bierbaun and G. Mersch of the Division of Abatement. A new manual and improved program are expected to be available in the fall of 1970.

The selection and evaluation of emission standards in a control strategy is restricted to the political jurisdictions within an air quality control region. Pollutants from man-made sources outside this region and natural ground-level background concentration are assumed to be uniformly distributed and unaffected by meteorological conditions or emission standards. When a sufficient number of regions use this type of computer simulation technique, it will be possible to analyze the effects of control strategies in one region on the air quality in a neighboring region.

Finally, the accuracy with which the computer simulation models predict air quality conditions and control costs is dependent on the quality of input information. As in most computer simulations of the real world, better input information results in better output results.

2.1 PROGRAM INPUTS

A wide variety of input information is required by the RAPA Program, including meteorological data, air quality data, a source emission inventory, data on air pollution control technology, regional economic data, and descriptions of the emission standards to be used in candidate control strategies.

The meteorological data required by the program describe those meteorological elements which have an important effect on the atmospheric transport and dispersion of air pollutants, including wind direction, wind speed, stability class, mixing depth, ambient temperature, and ambient pressure. Wind direction determines whether or not the pollutant is transported from a pollutant emission source to a particular receptor location, and wind speed determines the amount of air available for dilution of the pollutant. Atmospheric stability, a measure of turbulence, may be in the form of an arbitrary measurement based on time of year, amount of solar radiation, and wind speed, or it may be a conversion from data on vertical temperature differences. Mixing depth determines the amount of vertical mixing possible in the atmosphere. Normally, air pollutants can only mix through a relatively shallow atmospheric layer. The mixing depth may be as high as 5 or 6 thousand feet in the afternoon and only a few hundred feet at night. Data inputs for these four most important meteorological elements are available from the National Weather Records Center (NWRC), operated by the Environmental Sciences Services Administration in Asheville, North Carolina. Here all meteorological data collected by Federal agencies are stored, tabulated, and summarized for the user's convenience.

To be compatible with the requirements of the implementation plan, an air quality monitoring network must provide data for comparison with air quality standards, analysis of emission control effects, evaluation of short-term emergency conditions, and verification of the atmospheric diffusion modeling. Specifically, data are required on sampling station location, annual arithmetic mean concentration, and standard geometric deviation. The amount and quality of such data needed for the verification process cannot be stated in an absolute sense. Naturally, as more data becomes available and as the quality of the data improves, more confidence can be applied to the atmospheric diffusion modeling.

An emission inventory is used to identify the existence of air pollution sources and to define the location, type, magnitude, frequency, duration, and relative contribution of the emissions from these sources.

The ideal stationary source emission inventory would include emission estimates for every source in an air quality control region, as determined from stack gas sampling. Since this is not possible for all sources and pollutants, various techniques have been developed to estimate emissions from sources where the above data are not available.

Several types of air pollution sources are encountered in any given metropolitan area. Generally, these sources can be divided into four major categories and several subcategories. The breakdown used by the National Air Pollution Control Administration in presenting emission inventory data for the computer program is as follows:

FUEL COMBUSTION (Stationary Sources)

- Residential
- Commercial-Institutional-Governmental
- Industrial
- Steam-Electric Power Plants

INDUSTRIAL PROCESS LOSSES

SOLID WASTE DISPOSAL

- Incineration
- Open Burning

TRANSPORTATION (Mobile Sources)

- Motor Vehicles
- Aircraft
- Railroads
- Ships

Information on these categories of pollution sources is further broken down according to two types: (1) Area sources, and (2) Point Sources. Area sources include small residential, commercial, governmental, institutional, and industrial fuel combustion operations, on-site solid waste disposal, and mobile sources. Point sources include major fuel users, incinerators, open burning dumps, and industrial process sources.

The costs and efficiencies associated with air pollution control devices influence the engineering and economic consequences of control strategies. Inputs to the program must account for (1) regional information such as labor costs, sales tax, and interest rates; (2) pollutant source information from the emission inventory; and (3) control device information such as installation costs, manufacturers' price, expected life, rated efficiency, etc. The Program contains a great deal of this information for the devices described in the control techniques documents* published by the Department of Health, Education and Welfare. In addition, the program allows the user to supplement this information with new control devices or more refined data on devices already included in the Program.

Legal regulations prohibiting or restricting emissions into the atmosphere have long been a major tool used by communities against air pollution. Many of the earlier control regulations were designed to effect a reduction in local nuisance problems. More recently, control regulations have been adopted with a view toward achieving a general improvement in the quality of a community's air resources.

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*Control Techniques for Particulates (and Sulfur Oxides) Air Pollutants, U. S. Department of Health, Education and Welfare; Public Health Service; Environmental Health Service, NAPCA, Washington, D. C., January 1969.

Emission standards, from which emission regulations are developed, generally limit the amount of a pollutant which may be emitted into the atmosphere from a source. Figure 2-1 illustrates a typical emission standard format limiting allowable emissions to some function of plant capacity such as the quantity of input raw materials. The allowable emission may be either directly stated (e.g., a process/weight-type standard) or implied (e.g., a specific type of fossil fuel). It is usually necessary to adopt several standards in order to control all the pollution sources in the area. For example, particulate pollution control may require separate emission standards covering industrial processes, fuel combustion, and solid waste disposal.

The specific emission standards to be used as candidates for a region's control strategy must be determined on the basis of many factors, including such aspects as types of industry and local availability of different qualities of fuel for combustion sources. The RAPA Program has been designed to accept an infinite variety of emission standards, the only requirement being that the standards can be described in mathematical terms.

Based on direct control costs, borne by the industries affected by a control strategy, the program estimates the total economic impact on the region. This impact is measured in terms of gross regional product, industrial output, changes in profit, etc. The input information required for this type of economic analysis includes industrial data on employment, value-added, exports and imports, regional data on local taxes, government expenditures, regional income, labor force, and many other economic factors. Both time series and cross-sectional data have been used in the estimations.

2.2 PROGRAM STRUCTURE

The RAPA Program is divided into several separate elements: an Atmospheric Diffusion Model, Control Cost Model, Control Strategy Model, Regional Economic Model, and an Information System which stores and maintains air pollution control data and provides this data to each of the

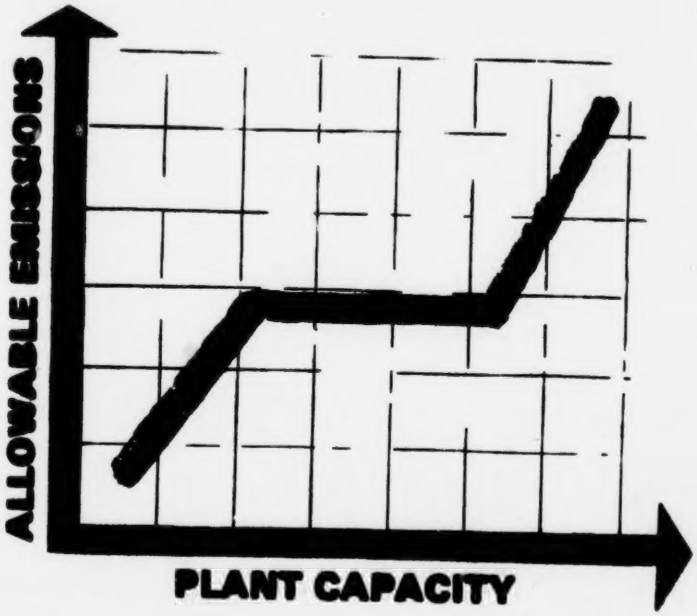


Figure 2-1. Typical Emission Standard Format

models. Figure 2-2 illustrates the major models in the program and their interaction with human decisions which permit the user to determine the amount of control based on the atmospheric diffusion, air quality standards, available control technology, and the cost of air pollution control.

The purpose of the several models which constitute the program is to provide an integrated and comprehensive analytical tool to assess the air pollution problem in an air quality control region and to assist local officials in the development of control strategies designed to meet the regional air quality goals.

2.3 ATMOSPHERIC DIFFUSION MODEL

The first step in examining the existing or projected regional air quality conditions is an analysis of the movement of pollutants. The atmospheric transport of pollutants from any given source takes place through convection (due to the net air motion) and diffusion (due to random atmospheric turbulence). The pollutant plume moves away from the source in the downwind direction and expands laterally. The Atmospheric Diffusion Model provides mathematical transfer functions between individual source emission within the region and regional measures of air quality. In addition, the transfer functions are also used to estimate the effects of various source emission standards contained in overall control strategies on regional air pollution.

Since the objective of the Atmospheric Diffusion Model is both to produce a frequency distribution of pollutant concentrations and to determine the individual influence of each source at each receptor point, a combination of two modeling techniques is used. A deterministic technique transforms source emission data into average ground-level long-term pollutant concentrations, and a statistical technique estimates a corresponding frequency distribution for ground-level concentrations at different averaging times. It should be noted that the deterministic model alone cannot be used to develop a concentration frequency distribution, since the upper end of the distribution (i.e., those concentrations exceeded, say, one percent of the time, or the expected annual maximum) occurs under short-term meteorological conditions.

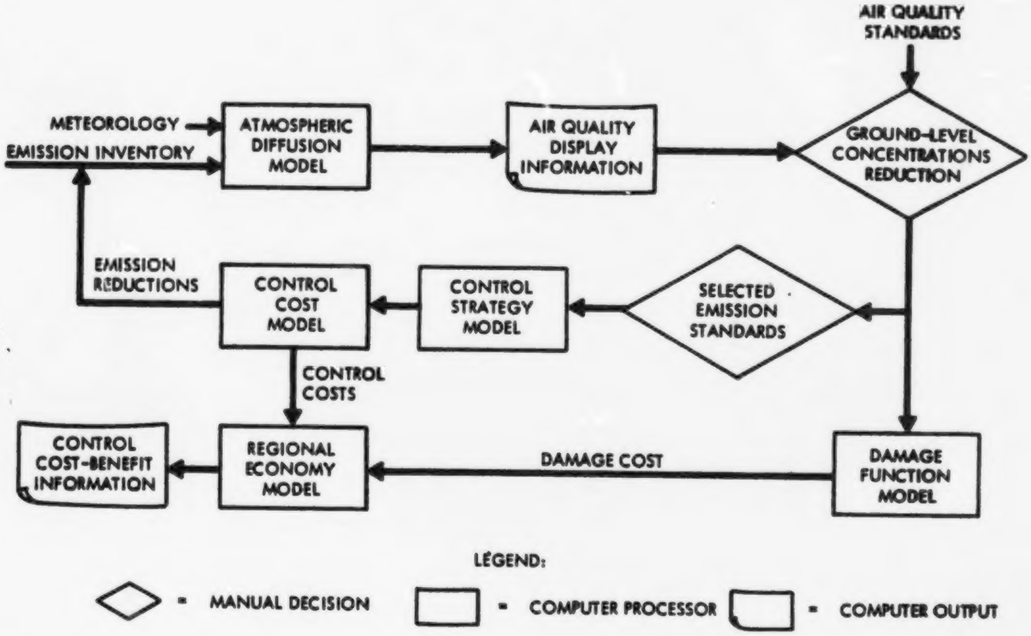

AIR QUALITY STANDARDS
METEOROLOGY
EMISSION INVENTORY
ATMOSPHERIC DIFFUSION MODEL
AIR QUALITY DISPLAY INFORMATION
GROUND-LEVEL CONCENTRATIONS REDUCTION
EMISSION REDUCTIONS
CONTROL COST MODEL
CONTROL STRATEGY MODEL
SELECTED EMISSION STANDARDS
CONTROL COSTS
CONTROL COST-BENEFIT INFORMATION
REGIONAL ECONOMY MODEL
DAMAGE COST
DAMAGE FUNCTION MODEL
LEGEND:
= MANUAL DECISION
= COMPUTER PROCESSOR
= COMPUTER OUTPUT

Figure 2-2. Regional Air Pollution Analysis Program Structure

The deterministic model computes the arithmetic annual average ground-level concentration resulting from specified area and point source emissions. Area sources are aggregations of small sources (e.g., a residential area) while point sources are major emitters of effluent gas, usually from a plant stack. For each receptor location, the model sums the effects of all sources over a complete range of meteorological conditions.

An added feature to the operation of the deterministic model is the use of a "correlation" option, a method of adjusting and verifying the model to make the computed ground-level concentration agree with available measured ground-level concentrations. The method of adjustment uses least-square regression lines of measured concentrations on theoretical concentrations for each pollutant.

The statistical model (Figure 2-3) uses correlated arithmetic averages of the deterministic model and input values reflecting the geometric standard deviation for each pollutant to estimate desired frequency distributions (i.e., maximum values and frequencies of occurrence with which specified values are exceeded). The key assumptions involved in the use of the statistical segment are that (1) ground-level air pollutant concentrations are approximately lognormally distributed for all pollutants and for all averaging times, and (2) the geometric standard deviations are known for each pollutant and are independent of the concentration magnitude but dependent upon its location within the region.

2.4 CONTROL COST MODEL

The purpose of computing control cost and control effectiveness values is to provide an indication of the lowest-cost set of air pollution control measures which will satisfy required control regulations. The direct costs of these control measures, when summed over their application to all air pollution sources, provide estimates of regional air pollution control costs. By calculating the direct costs of air pollution control and the efficiencies of the selected control measures, this model provides the program with estimates of point source control cost-effectiveness measures for evaluating candidate air pollution control strategies.

Each air pollution source (e.g., an industrial plant, power-generating station, or apartment house) has one or more control-measure options for

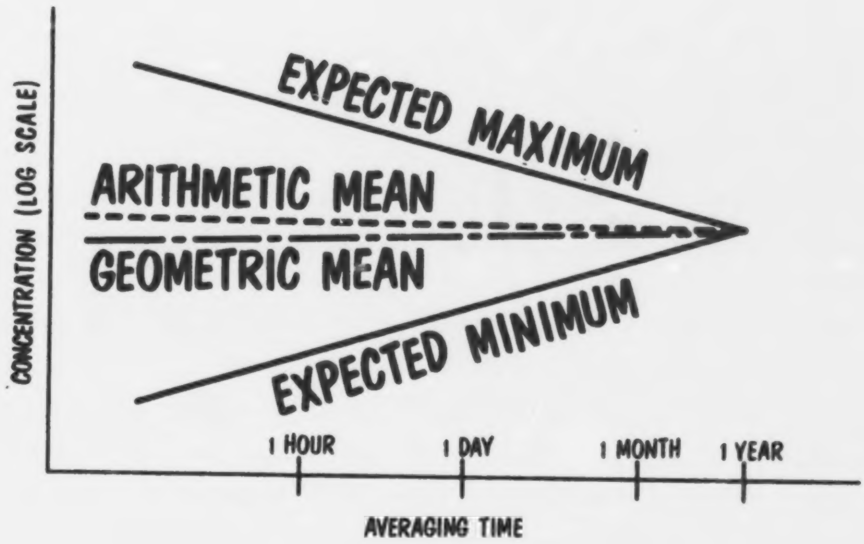

EXPECTED MAXIMUM
ARITHMETIC MEAN
GEOMETRIC MEAN
EXPECTED MINIMUM
CONCENTRATION (LOG SCALE)
1 HOUR
1 DAY
1 MONTH
1 YEAR
AVERAGING TIME

Figure 2-3. Statistical Distribution of Ground Level Concentrations

reducing its pollutant emissions. Each control measure has various cost and control efficiencies, depending on the source to which it is applied. The Control Cost Model sorts out applicable source/control measure combinations and provides the user with cost-effectiveness estimates for each specific point source or source category.

Figure 2-4 illustrates the cost of control applied to the chemical industry (Standard Industrial Classification 28) in the St. Louis and Cincinnati regions. Total annual costs are shown, including manufacturer's price, installation costs, annual capital charge, and operation and maintenance costs. In computing these major expense items, the model takes into account such factors as equipment depreciation schedules, interest rates on capital, and cost or credit for pollutant disposal. The model also considers variations in costs from region to region to account for equipment transportation costs, service utility connections, labor costs, etc.

The outputs of the Control Cost Model may be varied according to several user options. Results can be grouped to display the set of alternative control measures available to any single air pollution emission source, together with estimates of the annual cost and efficiency of each control measure. By further aggregation of the results, a similar display can be constructed for groups of sources (e.g., Standard Industrial Classification codes) or for all combustion sources. Figure 2-5 illustrates the costs of maximum control technology to a group of major industry categories in the St. Louis Air Quality Control Region. In this case, maximum control represents the most effective device or non-device control measure available to each plant.

2.5 CONTROL STRATEGY MODEL

A control strategy is essentially a plan for reducing pollutant emissions from specific sources within a specified period of time. The emission limitations of a control strategy generally include emission standards applicable to air pollution sources, regulations pertaining to fuel use, rules for the location of new industrial plants and other sources of air pollution, restrictions on open burning, plans for disposal of solid wastes, etc.

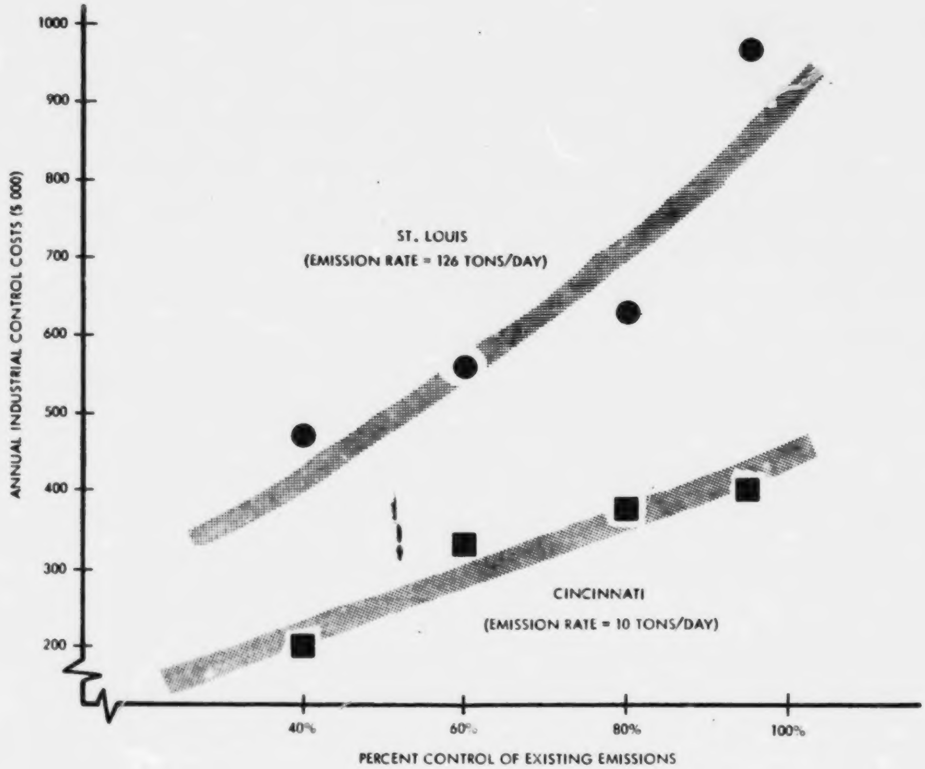


Figure 2-4. Particulate Matter Control Cost for the Chemical and Allied Products Industry (SIC 28)

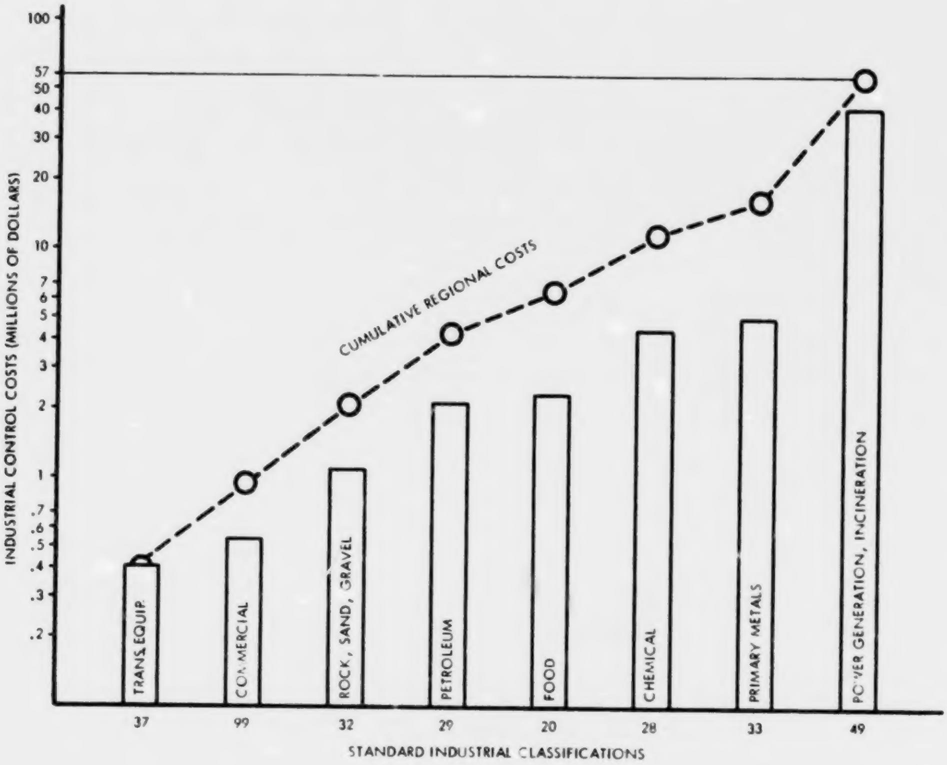


Figure 2-5. Cost of Maximum Sulfur Oxide and Particulate Matter Control Technology

Control strategies are preventive or controlling schemes directed at particular pollution sources (e.g., combustion sources, industrial sources, Federal facilities) or select types of regulations (e.g., process weight or fossil fuel restrictions). The program can process a control strategy whereby maximum control technology is applied to fuel combustion sources, process weight restrictions are applied to industrial sources, and potential emission standards are applied to solid waste disposal sources.

The purpose of the Control Strategy Model is to provide a reasonably accurate and quantitative procedure for testing the impact of alternative control strategies in a computer-processed mathematical model and assessing the degree to which air quality goals are met under each strategy. The program can thus be used as a "gaming" tool, with the user specifying a control strategy, observing the resulting change in regional air quality, and, on the basis of the results, modifying the specifications for another "run."

In the context of the RAPA Program, a control strategy is considered to be an overall plan which specifies the types of air pollution sources to be controlled and methods for their control. The objective of a strategy is to reduce both long- and short-term air pollution. As long-term standards are made more and more stringent, the frequency of short-term emergency conditions will be correspondingly reduced. The Control Strategy Model, in conjunction with the statistical segment of the Atmospheric Diffusion Model, allows the user to observe the interaction between chronic and acute regional air pollution problems.

The program postulates the type of control measures which would be used by an air pollution source facility to meet the proposed source emission requirements. Inherent in the program is the assumption that individuals or corporations responsible for the air pollution sources will use control techniques which result in the lowest cost to the source facility. Simulation of a strategy with assumed lowest-cost control measures applied to the sources of pollution assists the user in evaluating implementation plans with regard to reasonable ground-level pollutant concentrations and direct control costs. The Control Strategy Model provides sufficient information to evaluate the regional impact of selected strategies from the standpoint of: (1) the types of sources affected by the strategy; (2) the degree to which these sources are affected in terms of control costs; (3) the resulting change in pollutant emissions; and (4) regionally aggregated values of control costs by source category, total reduction in pollutant emissions, changes to regional fuel-use patterns, and resulting air quality levels.

2.6 REGIONAL ECONOMIC MODEL

The purpose of the Regional Economic Model is to provide the program with a means of assessing economic impacts in a region that contains industries required to invest in air pollution control measures.

In deciding upon production plans, industries traditionally have not had to consider the external social costs generated by their activities. Were they required to do so, they would notice an increase in the costs of production, perhaps to a prohibitive degree. In particular, industries emitting harmful pollutants into the air during the production process have, by not bearing the cost of the resulting social damage, enjoyed the advantages of lower production costs. With the additional concern for air quality, industries will be encouraged to control the amount of pollutants that they discharge into the air. To industry this means that, for the same amount of output produced prior to consideration of air pollution controls, production costs will be increased as a result of the required investments and operating expenses for air pollution control. Thus, certain economic sectors that in the past have enjoyed the advantages of low-cost production may have to face some degree of economic decline due to the effects of air pollution control strategies.

The Regional Economic Model draws upon four basic types of information in its economic estimation process:

- Average annual cost for air pollution control equipment for each control strategy, including all initial and on-going costs, annual costs of fuel changes and associated equipment changeover costs, less by-product revenues.
- National industry output data and projections (by Standard Industrial Classification code).
- Initial-period regional activity level and capital stock data.
- Regional air quality control strategy variables, such as tax credits.

Given these sets of data, the model produces estimates of regional economic variables, including:

- Gross regional product.
- Employment change by industry.
- Investment change and capital stock by industry.
- Industry value-added.
- Regional tax receipts.
- Regional unemployment.

As a result of a calibration process, estimates can be made of the economic impact resulting from the additional industry investments in air pollution control measures. Figure 2-6 summarizes the overall economic impact of a maximum control technology strategy. This strategy assumes that every plant invests in the control measures which result in the greatest pollutant emissions reduction.

2.7 PROGRAM OUTPUT

The RAPA Program provides the user with a wide variety of output information designed to assist him in developing and evaluating air pollution control strategies. Final decisions may thus be based on a combination of the experience and intuition of the user and the data-processing capabilities of the computer simulation.

The simplest output is in the form of reports displaying the contents of the data files. These reports are capable of summarizing the vast amount of data stored in the emission inventory file, control cost file, and regional data file. Figure 2-7 summarizes pollutant emission rates in the Philadelphia Air Quality Control Region. This type of output organizes the vast and detailed information on a region in a format that can be readily assimilated for evaluation and decision processes.

The Atmospheric Diffusion Model provides the user with contour maps, tabular data on average ground-level concentrations, and statistical data on expected maximum concentrations. The tabular data is in the form of a grid system, an arrangement which makes it easy to accurately locate specific receptors in a region. Figure 2-8 illustrates the ground-level concentration, in the form of a contour map, prior to application of a

```
               ST  LOUIS  -  ECONOMETRIC MODEL  (1966)

                          REGIONAL   DATA

        MAXIMUM CONTROL TECHNOLOGY

REGIONAL EXPENDITURE ($MILLION)             (Y*)* =  6964.734 ,   -0.613 %

CONSUMPTION ($MILLION)                         C* =  4413.297 ,   -0.401 %

REGIONAL INCOME ($MILLION)                  (Y)* =  7243.336 ,   -0.774 %

EMPLOYMENT BY NON-MANUFACTURING
                  INDUSTRIES (1000)       N-BAR* =  647.3262 ,   -0.477 %

TOTAL EMPLOYMENT (1000)                        N* =  933.8894 ,   -0.394 %

TOTAL LABOR FORCE (1000)                       L* =  964.7620 ,   -0.358 %

UNEMPLOYMENT (1000)                            U* =  34.3381  ,   12.050 %

LOCAL UNEMPLOYMENT RATE (PERCENT)      SMALL U* =  3.5465   ,   12.050 %

ELECTRIC POWER DEMAND FOR
       RESIDENTIAL USE  (1000 KWH)            QC* =  2642.255 ,   -0.634 %

ELECTRIC POWER DEMAND FOR
       NON-MANUFACTURING INDUSTRIES
                      (1000 KWH)          Q-BAR* =  1935.559 ,    -0.68 %
```

Figure 2-6. St. Louis Regional Economic Summary with Maximum Control Technology Investments

POLLUTANT EMISSIONS BY SOURCE CATEGORY (TONS/YEAR)
PHILADELPHIA
06/05/70

POLLUTANT	POLITICAL JURISDICTION	FUEL COMBUSTION	INDUSTRIAL PROCESS	WASTE DISPOSAL	TOTAL
SO2	1	201673	30839	0	232512
SO2	2	157683	18320	0	176003
SO2	3	236655	26751	471	263877
SO2	4	197580	10836	473	208889
	TOTAL	793591	86746	944	881281
PARTICULATE	1	25396	17443	364	43203
PARTICULATE	2	12741	2618	472	15831
PARTICULATE	3	25264	18813	2589	46666
PARTICULATE	4	17309	2948	1275	21532
	TOTAL	80710	41822	4700	127232

Figure 2-7. Summary of Regional Pollutant Emission Rates

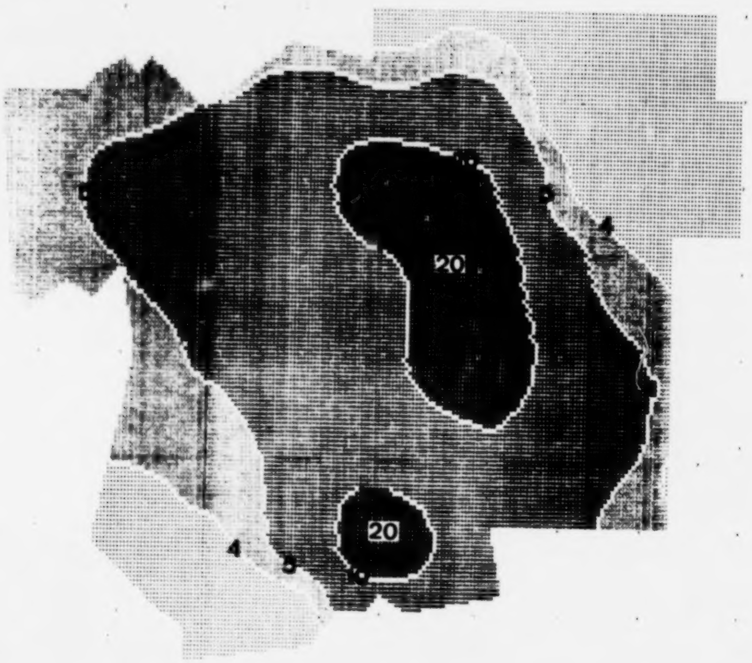


Figure 2-8. Particulate Matter Concentrations Before Control
(Micrograms per Cubic Meter)

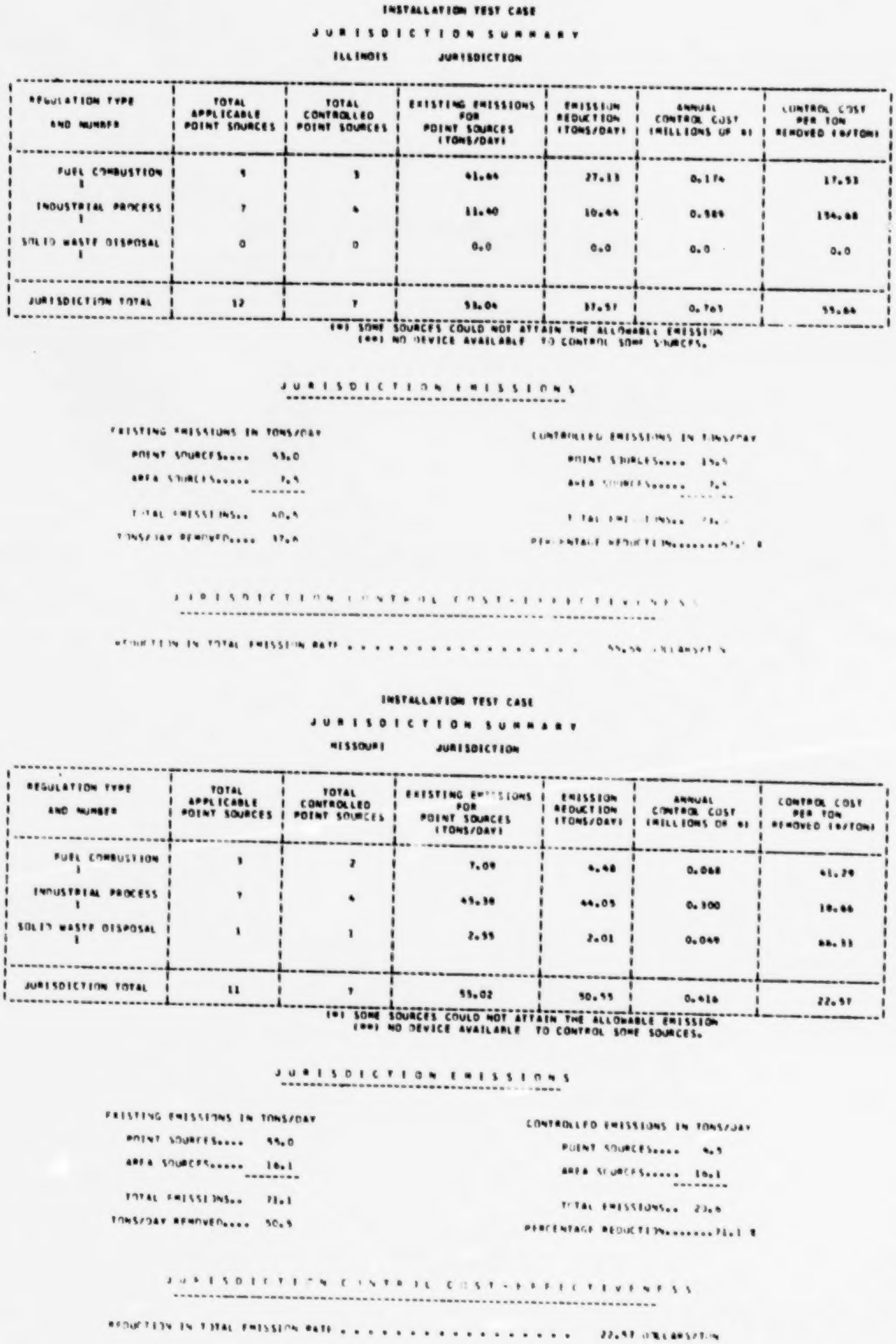
control strategy. This map represents a sampling of sources (23 points and 30 areas) in the St. Louis Air Quality Control Region.

The Control Cost Model provides results showing the types of control measures that are applicable to each type of pollutant source. Along with the control measure identification, the model calculates annual costs and expected emission reduction efficiency as applied to a specific source.

The Control Strategy Model contains the emission standards which, in most cases, require the application of the control measures identified by the Control Cost Model. The outputs illustrate the actual control measures selected for each source, together with the annual costs and data on the degree to which emissions have been controlled. The Model also displays the conversion from emission reductions to reductions in annual and expected maximum ground-level concentrations for selected emission standards. A regional summary, Figure 2-9, culminates the program's output and includes a description, by political jurisdiction, of regional costs, regional emission rates, number of sources affected by the control strategy, and figures of merit for each strategy (e.g., cost per ton of pollutant removed, cost per microgram/cubic meter of reduced ground-level concentrations). In addition, a ground-level concentration contour map is provided which highlights the effect of the control strategy. The contour map in Figure 2-10 as well as the tabulated summaries illustrate a comparison in ground level concentration before (Figure 2-8) and after the application of a control strategy to a small sample of stationary sources in the St. Louis region.

The ability of the program to determine the consequences of specific emission regulations on the basis of computer simulations provides the user with a means for objectively comparing pollution control strategies before they are put into effect. The program assists the local air pollution control official in selecting, from the large number of control strategy alternatives available to a region, that strategy which will be most effective in solving the region's air pollution problems.

Ideally, the regional air pollution control official is expected to strike a balance between regional expenditures on air pollution control and regional savings resulting from increasing air quality levels. Information on air quality savings (or, conversely, the costs of air pollution



```
                              INSTALLATION TEST CASE
                          J U R I S D I C T I O N   S U M M A R Y
                              ILLINOIS      JURISDICTION

-------------------------------------------------------------------------------------------------------------------------
! REGULATION TYPE      !     TOTAL       !     TOTAL       ! EXISTING EMISSIONS !  EMISSION   !     ANNUAL       !  CONTROL COST  !
!                      !   APPLICABLE    !   CONTROLLED    !        FOR         !  REDUCTION  !  CONTROL COST    !    PER TON     !
!  AND NUMBER          ! POINT SOURCES   ! POINT SOURCES   !   POINT SOURCES    ! (TONS/DAY)  ! (MILLIONS OF $)  ! REMOVED ($/TON)!
!                      !                 !                 !    (TONS/DAY)      !             !                  !                !
-------------------------------------------------------------------------------------------------------------------------
!     FUEL COMBUSTION  !        5        !        3        !       41.64        !    27.13    !      0.174       !      17.53     !
!            1         !                 !                 !                    !             !                  !                !
!  INDUSTRIAL PROCESS  !        7        !        4        !       11.40        !    10.44    !      0.589       !     154.68     !
!            1         !                 !                 !                    !             !                  !                !
! SOLID WASTE DISPOSAL !        0        !        0        !        0.0         !     0.0     !      0.0         !       0.0      !
!            1         !                 !                 !                    !             !                  !                !
-------------------------------------------------------------------------------------------------------------------------
! JURISDICTION TOTAL   !       12        !        7        !       53.04        !    37.57    !      0.763       !      55.64     !
-------------------------------------------------------------------------------------------------------------------------
                        (*) SOME SOURCES COULD NOT ATTAIN THE ALLOWABLE EMISSION
                         (**) NO DEVICE AVAILABLE  TO CONTROL SOME SOURCES.

                        J U R I S D I C T I O N   E M I S S I O N S
                        -------------------------------------------

    EXISTING EMISSIONS IN TONS/DAY                        CONTROLLED EMISSIONS IN TONS/DAY
      POINT SOURCES....   53.0                              POINT SOURCES....  15.5
      AREA SOURCES.....    7.5                              AREA SOURCES.....   7.5
                        --------                                              --------
      TOTAL EMISSIONS..   60.5                              TOTAL EMISSIONS..  23.0
    TONS/DAY REMOVED....  37.6                           PERCENTAGE REDUCTION........[illegible] %

             J U R I S D I C T I O N   C O N T R O L   C O S T - E F F E C T I V E N E S S
             -----------------------------------------------------------------------------

    REDUCTION IN TOTAL EMISSION RATE . . . . . . . . . . . . . . . . .    55.64 DOLLARS/TON


                              INSTALLATION TEST CASE
                          J U R I S D I C T I O N   S U M M A R Y
                              MISSOURI      JURISDICTION

-------------------------------------------------------------------------------------------------------------------------
! REGULATION TYPE      !     TOTAL       !     TOTAL       ! EXISTING EMISSIONS !  EMISSION   !     ANNUAL       !  CONTROL COST  !
!                      !   APPLICABLE    !   CONTROLLED    !        FOR         !  REDUCTION  !  CONTROL COST    !    PER TON     !
!  AND NUMBER          ! POINT SOURCES   ! POINT SOURCES   !   POINT SOURCES    ! (TONS/DAY)  ! (MILLIONS OF $)  ! REMOVED ($/TON)!
!                      !                 !                 !    (TONS/DAY)      !             !                  !                !
-------------------------------------------------------------------------------------------------------------------------
!     FUEL COMBUSTION  !        3        !        2        !        7.09        !     4.48    !      0.068       !      41.29     !
!            1         !                 !                 !                    !             !                  !                !
!  INDUSTRIAL PROCESS  !        7        !        4        !       45.38        !    44.05    !      0.300       !      18.66     !
!            1         !                 !                 !                    !             !                  !                !
! SOLID WASTE DISPOSAL !        1        !        1        !        2.55        !     2.01    !      0.049       !      66.33     !
!            1         !                 !                 !                    !             !                  !                !
-------------------------------------------------------------------------------------------------------------------------
! JURISDICTION TOTAL   !       11        !        7        !       55.02        !    50.55    !      0.416       !      22.57     !
-------------------------------------------------------------------------------------------------------------------------
                        (*) SOME SOURCES COULD NOT ATTAIN THE ALLOWABLE EMISSION
                         (**) NO DEVICE AVAILABLE  TO CONTROL SOME SOURCES.

                        J U R I S D I C T I O N   E M I S S I O N S
                        -------------------------------------------

    EXISTING EMISSIONS IN TONS/DAY                        CONTROLLED EMISSIONS IN TONS/DAY
      POINT SOURCES....   55.0                              POINT SOURCES....   4.5
      AREA SOURCES.....   16.1                              AREA SOURCES.....  16.1
                        --------                                              --------
      TOTAL EMISSIONS..   71.1                              TOTAL EMISSIONS..  20.6
    TONS/DAY REMOVED....  50.5                           PERCENTAGE REDUCTION.......71.1 %

             J U R I S D I C T I O N   C O N T R O L   C O S T - E F F E C T I V E N E S S
             -----------------------------------------------------------------------------

    REDUCTION IN TOTAL EMISSION RATE . . . . . . . . . . . . . . . . .    22.57 DOLLARS/TON
```

Figure 2-9. Regional Control Cost-Effectiveness Summary

```
                              INSTALLATION TEST CASE                                 03/19/70
                       C O N T R O L   S T R A T E G Y   S U M M A R Y


                           R E G I O N A L   E M I S S I O N S
                        ----------------------------------------

EXISTING EMISSIONS IN TONS/DAY                          CONTROLLED EMISSIONS IN TONS/DAY

   POINT SOURCES....  108.1                                 POINT SOURCES....  19.9

   AREA SOURCES.....   23.6                                 AREA SOURCES.....  23.6
                     --------                                                 --------
   TOTAL EMISSIONS..  131.7                                 TOTAL EMISSIONS..  43.5

 TONS/DAY REMOVED....  88.1                             PERCENTAGE REDUCTION.......66.9 %


              R E G I O N A L   C O N T R O L   C O S T - E F F E C T I V E N E S S
             ----------------------------------------------------------------------

REDUCTION IN TOTAL EMISSION RATE . . . . . . . . . . . . . . . . . .    36.67 DOLLARS/TON

REDUCTION IN GROUND LEVEL CONCENTRATION . . .  364742.81 DOLLARS/MICROGRAM/CUBIC METER
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Figure 2-9. Regional Control Cost-Effectiveness Summary (Continued)

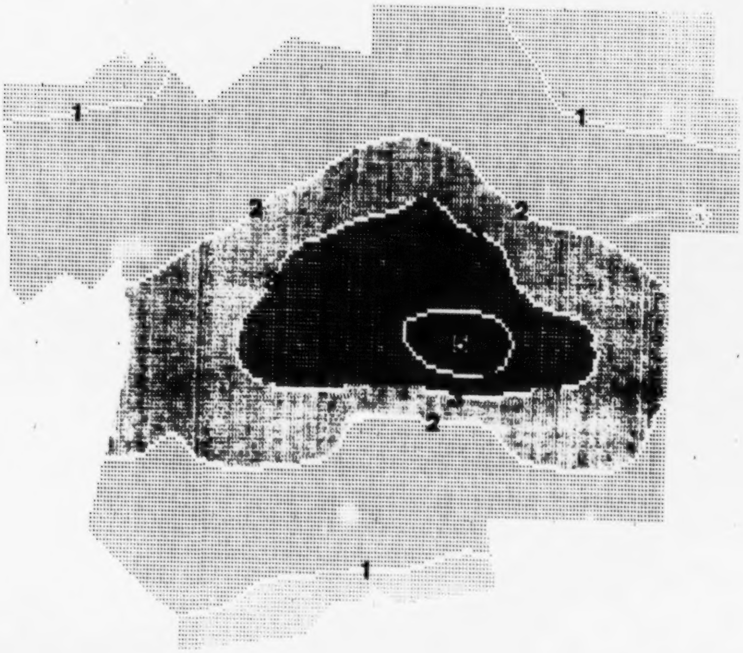

Figure 2-10. Particulate Matter Concentrations After Control (Micrograms per Cubic Meter)

damage) is much scarcer than cost data on air pollution control discussed previously. However, for illustrative purposes the format of a typical cost-benefit relationship is presented in Figure 2-11. The cost function states the added industrial and power-generating costs associated with investments in sulfur-oxide and particulate matter control technology. This strategy has been applied, via the program, to 75% of the major point sources of pollution in the St. Louis Region (the line between zero added investment and the $55 million maximum added control costs implies a continuous function through intermediate degrees of control). The benefit function states the added per capita cost resulting from the soiling damages caused by particulate matter pollution. This soiling damage function was previously estimated for the Washington, D. C. area in 1968 by R. Wilson and D. Minnotte of the National Air Pollution Control Administration.

It is clear from this type of a diagram that the savings from reducing particulate matter soiling damage alone almost offset the combined costs of maximum sulfur oxide and particulate matter control. If the added savings inherent in improved human and animal health, increased crop yields, enhanced visibility, and overall aesthetic values are added to the savings from decreased soiling damage, then regional air quality savings far outstrip industrial air pollution control costs.

The "optimum" management strategy is represented by a point which balances the marginal utility of additional air pollution control expenditures with additional benefits of air quality. For example, after a point has been reached where total control cost equals total air quality savings, an additional dollar of control investments may return more than one dollar in air quality benefits.

Unfortunately, quantitative assessments of the benefit-cost functions are still in the future. The control costs and regional economic estimates of these industrial investments are but a first step in the development of these functions. As soon as additional air pollution damage data are evaluated, estimates of benefits will be made in conjunction with the estimates of control costs, resulting in more quantitative appraisals of atmospheric resource management strategies.

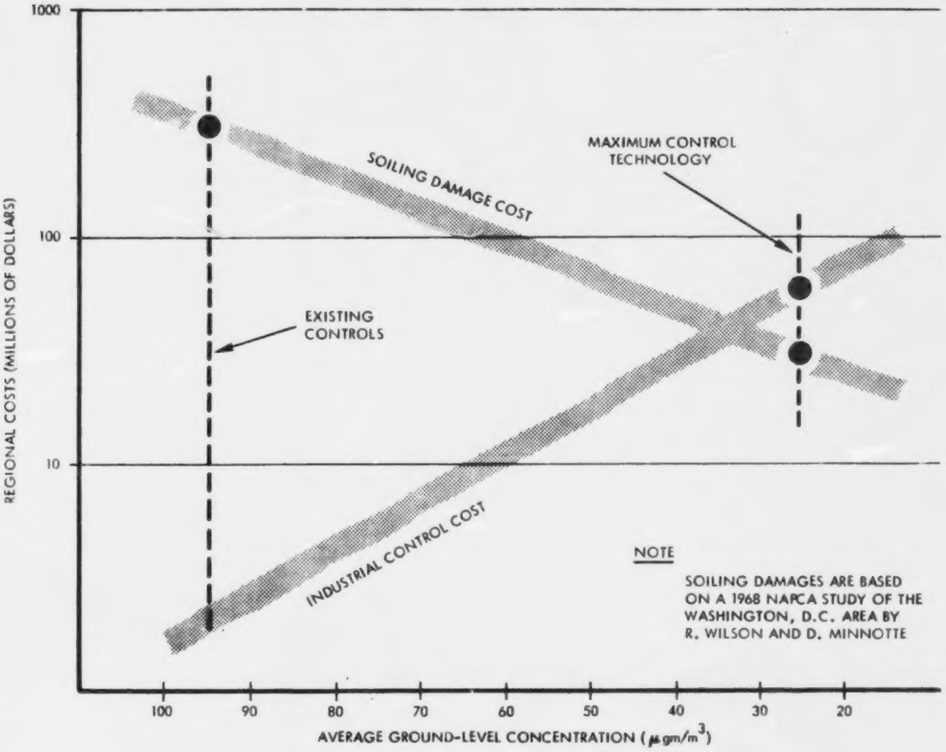

Figure 2-11. Air Pollution Control Cost versus Air Quality Benefits

3.0 AIR QUALITY DISPLAY MODEL

3.1 INTRODUCTION

The Air Quality Display Model manual describes the use of a computer program designed to estimate the spatial distribution of sulfur dioxide and particulate concentrations. Application of the AQDM to analysis of a specific region will require a general-purpose computer, certain input data, the instructions contained in the AQDM manual, and the program itself (in the form of a deck of cards or a magnetic tape).

The AQDM program is written in FORTRAN IV (level G) and is designed for installation on an IBM 360/40 (or equivalent) computer. The program and manual are available from the National Air Pollution Control Administration; the user supplies the computer, installs the program, and prepares the input data according to the procedures described. He then runs the program and interprets the printed output. A brief outline of the structure and usage of the AQDM is given in the following paragraphs.

Through a mathematical simulation of the atmospheric diffusion process [Martin and Tikvart, 1968], the program determines the estimated arithmetic average pollutant concentrations at ground level over an annual period. The resulting spatial distribution of average concentration values is obtained in both tabular and punched-card form. The punched-card output is designed to be used either with a program for the x-y plotter [CALCOMP, 1968], which produces contour-line maps (isopleths) of the various concentration levels in the region, or with the SYMAP [1968] program, which produces the same isopleths but in the form of shaded-area diagrams. These isopleth maps give a complete regional picture of air quality which could be used for submission of levels of air quality along with air quality standards [Guidelines, 1969, Sec. 2.22, page 19].

To provide a direct comparison with air quality criteria, AQDM includes a statistical model [Larsen, 1969] which is used to transform average concentration data from a limited number of receptor points into expected geometric-mean and maximum-concentration values for several different averaging times.

For each pollutant, the user may select a basic set of up to 237 receptor locations within the region for which estimated average concentration values are to be computed. From this basic set, 12 locations may be selected to provide additional output consisting of (1) expected geometric mean and (2) expected maximum concentrations for up to three different averaging times.

For each pollutant type, the AQDM also provides a table which gives the average concentration contribution from each source to any five receptors selected from the basic set of 237. The contribution is indicated both as an average concentration and as a percentage of the total average concentration at the specified receptor. If no selection is made by the user, data for the five receptors reporting the highest total average concentrations will automatically be entered.

The program can also be used in an analysis of control-device effectiveness. By scaling down the emission rates in the emission inventory in accordance with the efficiency of applicable devices, the impact of controls on air quality may be estimated. However, a more efficient and less costly means of calculating the effects of a control strategy will be forthcoming in the Implementation Planning Model to be released by NAPCA in the Fall of 1970.

3.2 MODEL STRUCTURE

The pollutant concentration output from the AQDM is derived from (1) an atmospheric diffusion model, which transforms regional source emission and meteorological data for a given annual period into estimated arithmetic average pollutant concentration values, and (2) a statistical model, which determines the geometric mean and expected maximum concentration values. Calculation of these parameters by the latter model is based on (a) arithmetic concentration values from the diffusion model, (b) input values of the standard geometric deviation* of the pollutant concentration, and (c) selected averaging times. The basic input data required by these models is discussed in the following sections.

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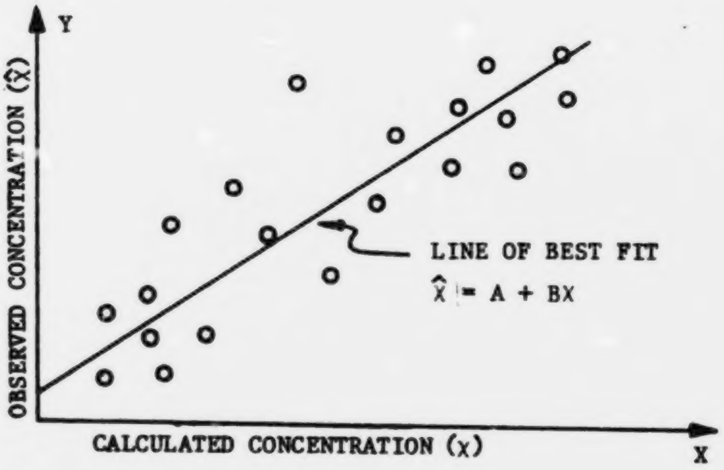
*Sometimes referred to as the geometric standard deviation.

Before using the AQDM to estimate regional air quality, the atmospheric diffusion model should be calibrated with existing air quality data (various calibration options are provided in the AQDM). In addition to the source emission and meteorological data, the following information is required for calibration:

(a) Existing Air Quality Data. For each pollutant, from 3 to 100 values of observed average concentrations may be specified, together with the respective receptor locations. The data must cover the same annual period represented by the emission inventory and meteorological data.

(b) Background Concentration. Average background concentration values for each pollutant type may be specified. These values represent that part of the regional pollutant concentration not attributable to the sources within the region. Background concentration data may be omitted if accurate values are not available, since a correlation option (detailed below) is provided which will automatically account for the background levels.

The calibration procedure (repeated for each pollutant) begins with the use of the diffusion model to calculate concentration values at each of the measuring sites. The calculated values at these points will include any input background values. Note that no receptor locations need be specified, since the program will automatically use the input measuring stations as receptor points. A least-square regression line of observed (i.e., measured) concentration values on the calculated values is then obtained. This procedure is used to determine the straight line which best fits the plot of calculated vs. observed concentrations. The scatter diagram below illustrates the procedure.



Each point plotted represents both the calculated and observed concentration values at a given receptor. For each pollutant, then, there is a regression line:

$$\hat{\chi}_p = A_p + B_p \chi_p$$

where

$\hat{\chi}_p$ = observed concentration of pollutant p,

χ_p = calculated concentration of pollutant p,

A_p = y-axis (observed concentration) intercept of best-fit line,

B_p = slope of best-fit line.

Since the observed concentration values include the background concentration, the line of best fit will have the same slope, B_p, whether or not background values are input. Inclusion of background values (which are added to the calculated values) simply shifts the line of best fit to the right. Thus the y-axis intercept, A_p, will automatically account for missing input background concentration values.

If the regression line adequately describes the relationship between the observed and calculated concentration values, it may then be applied to the region in general (within the range of calculated values used to determine the regression line). The model output would thus be adjusted at each receptor point according to the regression line equation.

To determine if the regression lines are adequate, the coefficient of correlation, a measure of data-scatter about the regression line, is calculated and compared with the maximum theoretical value that could arise due to chance.* The maximum theoretical value is determined for a 5% confidence level (that is, there are fewer than 5 chances in 100 that a coefficient of correlation as high as this value would arise due to random sampling variation). If the coefficient of correlation is greater than the theoretical value, the regression line is used to adjust the calculated values. If, on the other hand, the coefficient of correlation is less than the theoretical value, the regression line will not be used. A poor correlation warns the user that the input data (e.g., emission inventory, meteorological data, and observed concentration data) should be revised and, if an improved correlation does not result, the model assumptions should be reviewed by a meteorologist familiar with such modeling techniques.

If only a limited number of observed concentration values are available (fewer than 20 for most regions), or if their locations cover only a small portion of the region, calibration of the model may not be meaningful. However, it would still be of interest to check the model output against the known data points. In this way at least a partial check of the model's adequacy can be made. So that such checks may be performed and the regression results analyzed prior to region-wide application, the program has the following options:

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*For a general discussion of these concepts, see Fisher [1954].

OPTION	PROGRAM DESCRIPTION
0	The user must input the regression constants A_p and B_p. The program will apply the resulting regression line equations to the calculated concentration values prior to output.
1	The program will determine the regression lines and, if correlation is good, will adjust calculated concentration values accordingly. If the correlation is poor, the calculated concentrations will be output without adjustment and a message printed to indicate that poor correlation was obtained. In either case, the regression constants and correlation coefficients are output.
2	The program will determine the regression lines and correlation coefficients and stop.

Since model correlation is usually a trial-and-error process involving refinement of the input data, Option 2 is most economical. If accurate background levels are known, they should be used; otherwise, it is best to let the correlation procedure account for background values. Once good correlation has been achieved, Option 0 can be used. (Note, however, that if input background values are used in determining the coefficients A_p and B_p in Option 2, then the same background input values must be used in Option 0.)

3.3 ATMOSPHERIC DIFFUSION MODEL

The Martin-Tikvart [1968] diffusion model is based on the Gaussian-diffusion equation which describes the spreading, or diffusing, of a plume as it is transported downwind from an elevated, continuously emitting point source. The model is utilized here to compute annual arithmetic average ground-level pollutant concentrations resulting from specified point and area sources. The model calculates the effects of each source on each receptor for the observed combinations of wind direction, wind speed, and stability class. The relative frequency of occurrence for each combination is then included as a factor, and the resulting data are summed for each receptor over all combinations and all sources. The plume is assumed to be normally distributed about its center line (i.e., a Gaussian distribution) in the vertical direction and distributed as a linear function of crosswind distance in the crosswind direction. The pollutant distribution in the vertical direction is controlled by the magnitude of the vertical standard deviation of pollutant concentration. The program uses the vertical standard deviation values given by Pasquill [1961] and Gifford [1961]. These values are functions of the source-receptor downwind distance and atmospheric stability category. The stability categories (1 through 5, in order of increasing stability) are defined on the basis of the objective criteria specified by Turner [1964]. These categories are used to define both the standard deviation of the plume concentration distribution and the mixing depth of the atmosphere.

Input data to the diffusion model includes the following:

(a) Receptor Data. Up to 225 receptor locations corresponding to the intersections in a grid pattern having equidistant vertical and horizontal spacing may be specified, together with a maximum of 12 individually located receptors. The grid pattern is defined by the location of its lower left-hand corner, the number of rows and columns of receptors, and the distance between adjacent rows and columns. A convenient system with which to specify the receptor locations is the Universal Transverse Mercator system described in Section 3.0.

(b) Correlation Data. Depending on the correlation option (as discussed in the introduction to this section), the correlation coefficients and/or pollutant background levels may be input for one or both pollutant types.

(c) Source Data. Both area and point sources may be specified. For each point source the location, average annual emission rate (tons/day), and exhaust stack parameters must be specified. The stack parameters (physical stack height, stack exit diameter, effluent exit velocity, and effluent temperature) are used to obtain the effective height of release (i.e., the height at which the plume becomes horizontal) for use in the diffusion calculations. If the stack parameters are not known, an effective stack height value may be input. For each area source (assumed square in shape) the area, centroid location, average annual emission rate, and effective height of release must be input.

(d) Stability Wind Rose Data. This data gives the relative frequency of occurrence for each wind direction, wind speed class, and stability category combination as observed for the region and time period of interest. Although the wind direction is a continuous variable, for purposes of computation it is specified in sectors corresponding to 16-point compass headings, with sectors 1 through 16 corresponding to winds blowing from the directions N, NNE, NE,....., NW, and NNW, respectively. Since there are 5 stability classes, 16 wind directions, and 6 wind speed classes, the complete set of stability wind rose data consists of 480 frequency values. When summed over all categories, the relative frequency is 1.0. The diffusion model utilizes a single set of stability wind rose data for the entire region. For some regions this data may already be available in the required format. To check on data availability and/or for assistance in processing local meteorological data, consult the nearest NAPCA Regional Office.

(e) Mixing Depth. The mixing depth is defined as the depth above the surface through which the pollutants are mixed. Since this depth varies greatly from season to season, day to day, and diurnally, it is practical to account for only major changes. The program uses a single input value--the annual average afternoon mixing depth--and modifies it according to the stability class. If the mixing-depth value is not available, it may be estimated from Holzworth's [1964] estimates of the monthly mean afternoon mixing depths in the contiguous United States.

The diffusion model described above should only be applied when competent meteorological advice is available to interpret the limitations in the model. Important cautionary notes include:

(a) The basis of the model calculations, the point source diffusion formula, was developed to represent the behavior of plumes from actual point sources. The field data available for confirmation of the plume behavior were obtained from open, flat terrain and for travel distances of no more than a few kilometers in most cases. Thus, the use of the point-source model for urban scale projections has three specific weaknesses: (1) area sources are only imperfectly modeled by "effective point sources"; (2) plume behavior during horizontal transport of more than a few miles is not well known; and (3) plume behavior in regions of varying thermal and surface-roughness characteristics has not been systematically observed.

(b) The climatological data used in the model calculations is obtained from airport weather observing stations in nearly all cases. Frequently, the character of the lower atmosphere varies significantly between an out-of-town observing site and the urban central region.

(c) The use of surface meteorological data only in the estimation of mixing-layer depths has not been very successful. Better modeling should result when climatological data based upon vertical wind and temperature soundings is used.

(d) The emission inventories presently available for use in the model calculations are subject to error, and they cannot fully describe the variability within the urban region.

(e) The present model uses annual average emission inventory data. However, significant diurnal and seasonal variabilities in emission rates normally occur. Such variabilities are often (partially) correlated with the stability wind rose data. A future version of the model should include co-variable meteorological and emissions data in the development of annual average concentration estimates.

The listing of limitations cited here is not intended to imply that the present model be kept from use. On the contrary, improved models can only be developed when significant experience with the present formulation has been achieved.

3.4 STATISTICAL MODEL

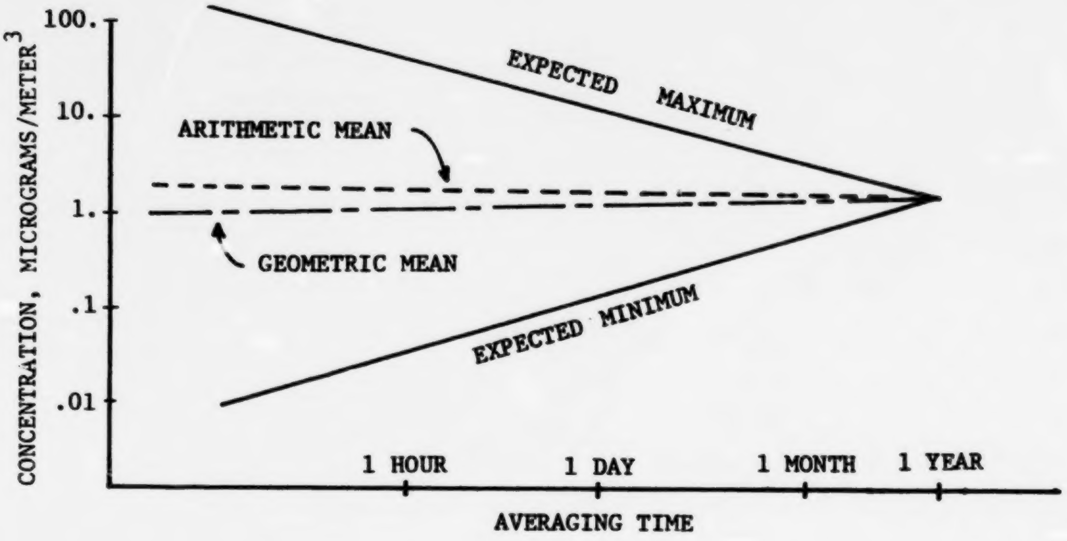
The statistical model [Larsen, 1969] is used to define the distribution of pollutant concentrations at each selected receptor as a function of averaging time, arithmetic average concentration, and standard geometric deviation at the given receptor. This statistical output includes such concentration distribution characteristics as the geometric mean concentration, expected maximum concentration, and standard geometric deviation (all of which are output for each of the specified averaging times).

The model is constructed on the basis of the following empirical characteristics of pollutant concentration distributions:

(a) Concentrations are lognormally distributed for all pollutants in all cities for all averaging times.

(b) The median (geometric mean) concentration is proportional to averaging time raised to an exponent.

From these characteristics, relationships between the arithmetic-mean, geometric-mean, and expected maximum concentrations are obtained. These relationships are illustrated in the sketch on the following page. A key result of the model is that the arithmetic mean is not a function of averaging time, and the arithmetic mean line in the figure is thus horizontal.



The standard geometric deviation, which describes the slope of the concentration-vs-frequency line for a fixed averaging time, does not appear here but is required in order to relate the arithmetic mean to the geometric mean and expected maximum concentration.

It should be noted that concentrations due to large single sources are not usually distributed lognormally. Thus, the model projections must be viewed conservatively in localities where a few single sources account for the major fraction of the observed concentrations. In particular, the frequency of occurrence of high concentrations may be underestimated by the model when a large single source exists in the neighborhood of a receptor location.

In addition to the arithmetic average concentration value at each receptor selected for output, the following input data to the statistical model is required:

(a) Output Receptor Selection. For each pollutant, a maximum of 12 output receptor locations may be selected from the basic set (for which arithmetic average concentrations have been computed).

(b) Standard Geometric Deviation. For each of the selected output receptors, a standard geometric deviation value for a 24-hour averaging time must be input for each pollutant (the program automatically converts this value to the desired averaging time). For output receptors located at measuring sites, the standard geometric deviation values will be obtained from the measuring station data. For output receptors not located at measuring sites, the standard geometric deviations must be interpolated from nearby measuring station values.

(c) Averaging Time. Three different averaging times for each pollutant type may be specified. A set of data will be output for each averaging time selected. These averaging times may be any desired values, but are most effectively determined after a study of the air quality statements in the Criteria Documents [1969].

4.0 RAPA DEMONSTRATION

A major activity during Phase II was the use of the RAPA program to develop control strategies for the National Capitol Air Quality Control Region and the Philadelphia Air Quality Control Region. Several other activities, such as the Damage Function Model Demonstration and the RAPA Experiments (described in Sections 4.5 and 4.6) were sharply curtailed to allow greater depth in the regional analysis. The results of these analyses are summarized in Sections 4.3.1 and 4.3.2 below. Results for the Cincinnati Air Quality Control Region which were prepared under a separate contract have also been included as a demonstration of the application of RAPA.

4.1 CONTROL STRATEGY ANALYSIS PROCEDURE

The heart of any plan designed to achieve specified air quality objectives is the set of emission control regulations proposed. These regulations specify the pollutant reductions required of existing facilities and limit the emissions from any new sources. Compliance is judged and enforcement actions are based upon the performance of emission sources in a region with respect to emission standards or criteria.

An Emission Control Strategy is made up of a compatible set of emission standards. Each standard applies to emissions from a specific type of operation (i.e., fuel combustion, industrial process, incineration), and the summation of the several standards included within each strategy defines allowable emission levels from each source in the region. In an air quality control region made up of several political jurisdictions (states, counties or cities), an emission control strategy may include several sets of emission standards, each set applying to a specific political jurisdiction within the region. Separate emission control strategies are defined for each pollutant.

The procedure for developing a control strategy consists of a number of steps, from the determination of existing air quality to the simulation of the final emission standards. For the regions described here, the analysis activity included the following items:

(a) Calibrate the Atmospheric Diffusion Model - This involves collection of all data needed by the diffusion model, plus measured annual average ground level concentrations for SO_x and particulates. A number of calibration runs are normally required, each with a more carefully edited set of measurements, until satisfactory calibration is achieved.

(b) Select the Devices to be Applied to Each Source Type and Run the Cost Model - Normally, the source/device applicability assumptions will not vary from region to region. However, the possibility of new source types and local control problems makes review of these assumptions important.

(c) Run the Atmospheric Diffusion Model - The receptor grid must be selected, and, if the region is large (say, over 500 point and area sources), the receptors must be grouped to allow the use of several short runs rather than a single long run to compute the regional ground-level concentrations. The shorter runs are much easier to schedule in the computer laboratory, and the penalty for an error (machine or input) in any given run is much less. This step begins after step (a) and proceeds in parallel with step (b).

(d) Run the Control Strategy Model, Job Step 1 (CONREG) - All emission standards to be considered are simulated in this job step, and a disk file is created which forms the basic input for job step 2, which evaluates individual strategies.

(e) Update the CONREG File - Any exceptions to the emission standards simulated in step (d) must be entered into the CONREG disk file at this point. In addition, any errors in the file should be corrected.

(f) Run the Control Strategy Model, Job Step 2 (CSTAB) - The combinations of emission standards which make up the individual strategies are evaluated here. This step may be repeated several times, using different combinations of emission standards, evaluating the effects of regional growth, etc.

4.2 PROGRAM EXTENSIONS

Before these regions could be evaluated, a number of program extensions (beyond the IPP baseline design described in Reference 1) were found to be necessary. Briefly, these consisted of the following items:

(a) The Atmospheric Diffusion Model was expanded to handle 2000 sources. It was also restructured to allow running a region in separate segments, thereby cutting down the computer time required for any individual run.

(b) The Cost Model source/device applicability matrix was expanded to cover twice as many source types.

(c) Several emission standards were added to the Control Strategy Model Job Step 1 (CØNREG). This involved a substantial increase in the capabilities of this module (See Table 4-1).

4.3 DEMONSTRATION RESULTS

The data presented here is only a small portion of the output generated for each region. Copies of the complete Control Strategy Model output have been delivered to NAPCA Rockville; the Atmospheric Diffusion and Cost Model output is on file at TRW Washington Operations. At the time of writing of this report, analysis and simulation was still in progress on all three of these regions; consequently the results presented here should be considered as representative but not final.

4.3.1 National Capitol Interstate Air Quality Control Region

The National Capitol Air Quality Control Region (Washington AQCR) consists of 10 political jurisdictions: the District of Columbia (jurisdiction No. 1); Montgomery County (No. 2) and Prince Georges County (No. 3), Maryland; and Alexandria (No. 4), Arlington County (No. 5), Fairfax County (No. 6), Fairfax City (No. 7), Falls Church (No. 8), Loudoun County (No. 9), and Prince William County (No. 10), Virginia. The regional emissions, existing control level and fuel use data are summarized in Tables 4-2, 4-3, and 4-4, respectively. As can be seen in Table 4-4, virtually all of the combustion sources are burning fuel with a sulfur content of less than 1%.

The regression lines used to calibrate the Atmospheric Diffusion Model are shown in Figures 4-1 and 4-2. The SO_2 air quality measurements in the lower left-hand corner of Figure 4-1 give a y-intercept which is more compatible with assumption of a near-zero SO_2 background than was found to be the case in the other regions.

The emission standards considered for the Washington region are listed in Tables 4-5 and 4-6. Tables 4-7 through 4-9 describe how these emission standards were assembled into control strategies.

The results of the first-level screening of control strategies are shown in Table 4-10. Each strategy is described by a single line in this table.

TABLE 4-1

CONTROL REGULATION MODULE (CØNREG)

FUEL COMBUSTION REGULATIONS

POLLUTANT	REG NO./SUBROUTINE NAME	TYPE	NAMELIST INPUTS	PRESET VALUES
SO_2	1/SØFCO1	Heat capacity	SLFCO1 - Source capacity limit, 10^6BTU/HR SMFCO1 - Maximum emission LBS/10^6 BTU, for sources with heat capacity equal to or greater than SLFCO1 SULF1 - Fuel sulfur limitation, DECIMAL, for sources with heat capacity less than SLFCO1	2000. 2.3 .02
SO_2	2/SØFCO2	Fuel sulfur limitation plus minimum control on power plants	SEFCO2 - Minimum collection, DECIMAL, efficiency on power plants SULF2 - Fuel sulfur limitation, DECIMAL, for all sources (other than power plants)	.85 .02
SO_2	3/SØFCO3	Fuel sulfur limitation	SULF3 - Fuel sulfur limitation, DECIMAL	.01
SO_2	4/SOFCO4	Maximum technology	None - Most efficient device selected for each source	
SO_2	5/SØFCO5	Maximum technology-no fuel switching	None - Most efficient device other than fuel switching, is selected for each source	
SO_2	6/SØFCO6	Equivalent fuel sulfur limitation	SULF6 - Fuel sulfur limitation, DECIMAL, to be achieved by switching fuels or a device giving equivalent emissions	.0125
PART	1/PFCO1	Heat capacity	TAB01 - Four X, Y pairs defining curve of heat capacity, 10^6 BTU/HR vs. allowable emissions, LBS/HR	Kentucky Heat Capacity Curve (Figure 4-26)
PART	2/PFCO2	Heat capacity, with stack height parameter	TABC2 - Three X, Y pairs defining curve of heat capacity, 10^6 BTU/HR vs. allowable emissions, LBS/HR, for each of the following seven stack heights; (meters) 22.9, 38.2, 57.25, 80.2, 137.2, 213.5	ASME Heat Capacity Curve (Figure 4-27)
PART	3/PFCO3	Heat capacity, stack height parameter, plus minimum efficiency for power plants	TAB03 - Three X, Y pairs for each of seven stack heights (see PFCO2) PEFCO3 - Minimum collection for efficiency power plants	ASME Heat Capacity Curve (Figure 4-27) 0.85
PART	4/PFCO4	Heat capacity	TAB04 - Four X, Y pairs defining curve of heat capacity, 10^6 BTU/HR, vs. allowable emission, LBS/HR	Ironton Curve (Figure 4-29)
PART	5/PFCO5	Potential emission	TAB05 - Five X, Y pairs defining curve of potential emission, LBS/HR, vs. allowable emission, LBS/HR	Pennsylvania Potential Emissions Curve (Figure 4-30)
PART	6/PFCO6	Maximum technology	None - Most efficient device is selected for each source	
PART	7/PFCO7	Maximum technology-no fuel switching	None	
PART	8/PFCO8	Exhaust gas concentration limit	EFC - Allowable emission, coal, LBS/BTU EFO - Allowable emission, oil, LBS/BTU EFG - Allowable emission, gas, LBS/BTU	5.8E-7 5.0E-7 3.7E-7
PART	9/PFCO9	Effective stack height	TAB16- Twelve X, Y pairs defining curve of effective stack height, FT, vs. allowable emission, LBS/HR AT* - Ambient temperature for plume rise computation, °K AP* - Ambient pressure for plume rise computation, Millibars WS* - Wind speed for plume rise computation, M/SEC	Cincinnati Regulation 293. 1000. 5.

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*First value of three-dimensional array.

TABLE 4-1

CONTROL REGULATION MODULE (CØNREG) (Continued)

SOLID WASTE REGULATIONS

POLLUTANT	REC NO/SUBROUTINE NAME	TYPE	NAMELIST INPUTS	PRESET VALUES
SO_2	None	--	--	
PART	1/PSW01	Exhaust gas concentration limit plus no open burning	PLSW01 - Burning rate limit, LBS/HR	200.
			PMSW01 - Two-dimensional variable; first value is exhaust gas concentration limit Gr/SCF, for incinerators less than PLSW01 LBS/HR burning rate, second is for those greater than PLSW01 LBS/HR	0.2, 0.3
PART	2/PSW02	Exhaust gas concentration limit plus no open burning	PLSW02 - Burning rate limits, LBS/HR	1000.
			PMSW02 - Two-dimensional variable; first value is exhaust gas concentration limit, Gr/SCF, for incinerators less than PLSW02 LBS/HR burning rate, second is for those greater than PLSW02 LBS/HR	0.2, 0.35
PART	3/PSW03	Exhaust gas concentration limit plus no open burning	PLSW03 - Burning rate limit, LBS/HR	1000.
			PMSW03 - Two values; first value is exhaust gas concentration limit, LBS/10^3LBS exhaust gas, for incinerators less than PLSW03 LBS/HR, second is for those greater than PLSW03 LBS/HR	0.4, 0.4
PART	4/PSW04	Potential emission plus no open burning	TAB13 - Five X, Y pairs defining curve of potential emission, LBS/HR, vs. allowable emission, LBS/HR	**S.F. Process Weight Rate Curve (Figure 4-31)**
PART	5/PSW05	Maximum technology plus no open burning	None --	
PART	6/PSW06	Exhaust gas concentration limit plus no open burning	PLSW06 - Burning rate limit, LBS/HR	1000.
			PMSW06 - Two values; first is exhaust gas concentration limit, LBS/10^3 LBS exhaust gas, for incinerators less than PLSW06 LBS/HR, second is for those greater than PLSW06 LBS/HR	0.4, 0.7
PART	7/PSW07	Exhaust gas concentration limit plus no open burning	PLSW07 - Burning rate limit, LBS/HR	1000.
			PMSW07 - Two values; first is exhaust gas concentration limit, LBS/10^3 LBS exhaust gas, for incinerators less than PLSW07 LBS/HR, second is for those greater than PLSW07 LBS/HR	0.4, 0.4
PART	8/PSW08	Sanitary landfill for all solid waste disposal; all emission set to zero	None	
PART	9/PSW09	Effective stack height	TAB17 - Twelve X, Y pairs defining curve of effective stack height FT, vs. allowable emission LBS/HR	
			AT* - Ambient temperature for plume rise computation, $^{\circ}$K	293.
			AP* - Ambient pressure for plume rise computation, MILLIBARS	1000.
			WS* - Wind speed for plume rise computation, M/SEC	5.0

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* Third value of three-dimensional array.

TABLE 4-1

CONTROL REGULATION MODULE (CØNREG) (Continued)

INDUSTRIAL PROCESS REGULATIONS

POLLUTANT	REG NO./SUBROUTINE NAME	TYPE	NAMELIST INPUTS	PRESET VALUES
SO_2	1/SØIP01	Exhaust gas concentration limit	SMIP01 - Maximum allowable exhaust gas concentration, PPM	2000.
SO_2	2/SØIP02	Exhaust gas concentration limit	SMIP02 - Maximum allowable exhaust gas concentration, PPM	500.
SO_2	3/SØIP03	Maximum technology	None - most efficient device is selected for each source	
SO_2	4/SØIP04	Potential emission	TAB15 - Five X, Y pairs defining curve of potential emission, LBS/HR HR, vs. allowable emission, LBS/HR	S.F. Process Weight Rate Curve (Figure 4-31)
PART	1/PIP01	Process weight rate, plus exception for foundries	TAB06 - Three X, Y pairs defining curve of process weight rate, LBS/HR, vs. allowable emission, LBS/HR	S.F. Process Weight Rate Curve (Figure 4-31)
			PEIP01 - Minumum collection efficiency for foundries, DECIMAL	0.85
			PMIP01 - Two-dimensional variable first value is maximum emission concentration G/SCF	0.4, 0.3
PART	2/PIP02	Process weight rate, plus exceptions for foundries, catalytic crackers, cement manufacture and blast furnaces	TAB07 - Four X, Y pairs defining curve of process weight rate, LBS/HR, vs. allowable emission, LBS/HR, for foundries with PLIP02 LBS/HR or greater process rate	Missouri Foundry Curve
PART	2/PIP02	Process weight rate, plus exceptions for foundries, catalytic crackers, cement manufacture and blast furnaces.	TAB08 - Three X, Y pairs defining curve of process weight rate, LBS/HR, vs. allowable emission, LBS/HR, for foundries with process rate less than PLIP02 LBS/HR	S.F. Process Weight Rate Curve (Figure 4-31)
			TAB09 - Three X, Y pairs defining curve of process weight rate, LBS/HR, vs. allowable emission LBS/HR, for all particulate process sources other than foundries	S.F. Process Weight Rate Curve (Figure 4-31)
			PEIP02 - Two-dimensional variable; both values, DECIMAL are minimum collection efficiencies, first for catalytic crackers, second for cement manufacture	0.997. 0.997
			PLIP02 - Foundry process weight rate limit LBS/HR for use of curve TAB07; if process weight rate is greater than this value, use TAB08	20000.
			PMIP02 - Two-dimensional variable, maximum emission limits, G/SCF; first value is for blast furnances, second in for cement manufacture	0.1, 0.1
PART	3/PIP03	Process weight rate	TAB10 - Three X, Y pairs defining curve of process weight rate, LBS/HR, vs. allowable emission, LBS/HR	S.F. Process Weight Rate Curve (Figure 4-31)
PART	4/PIP04	Potential Emission	TAB11 - Five X, Y pairs defining curve of potential emission, LBS/HR, vs. allowable emision, LBS/HR	Pennsylvania Potential Emission Curve (Figure 4-30)
PART	5/PIP05	Maximum technology	None - most efficient device is selected for each source	
PART	6/PIP06	Process weight rate	TAB12 - Eleven X, Y pairs defining curve of process weight rate; LBS/HR , vs. allowable emission, LBS/HR	L.A. Process Weight Rate Curve (Figure 4-19)
PART	7/PIP07	Effective stack height	TAB14 - Twelve X, Y pairs defining curve of effective stack height, FT vs. allowable emission, LBS/HR	Cincinnati regulation
			AT* - Ambient temperature for plume rise computation; ^oK	293.
			AP* - Ambient pressure for plume rise computation, MILLIBARS	1000.
			WS* - Wind speed for plume rise computation, M/SEC	5.0

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*Second value of three-dimensional array.

TABLE 4-2

WASHINGTON AQCR EMISSIONS SUMMARY

POLLUTANT EMISSIONS BY SOURCE CATEGORY (TONS/YEAR)

WASHINGTON

5/05/70

POLLUTANT	POLITICAL JURISDICTION	FUEL COMBUSTION	INDUSTRIAL PROCESS	WASTE DISPOSAL	TOTAL
SO2	1	33635	0	327	33962
SO2	2	58723	0	255	58978
SO2	3	65110	0	0	65110
SO2	4	16536	0	109	18645
SO2	5	1933	0	73	2006
SO2	6	984	0	0	984
SO2	7	0	0	0	0
SO2	8	0	0	0	0
SO2	9	[illegible]	0	0	0
SO2	10	18614	0	73	18687
	TOTAL	197535	0	837	198372

TABLE 4-2

WASHINGTON AQCR EMISSIONS SUMMARY (Continued)

POLLUTANT EMISSIONS BY SOURCE CATEGORY (TONS/YEAR)

WASHINGTON

5/05/70

POLLUTANT	POLITICAL JURISDICTION	FUEL COMBUSTION	INDUSTRIAL PROCESS	WASTE DISPOSAL	TOTAL
PARTICULATE	1	8348	0	2955	11303
PARTICULATE	2	4379	0	438	4817
PARTICULATE	3	3719	255	511	4485
PARTICULATE	4	1057	948	693	2698
PARTICULATE	5	290	73	146	509
PARTICULATE	6	584	2554	0	3138
PARTICULATE	7	0	0	0	0
PARTICULATE	8	0	0	0	0
PARTICULATE	9	0	0	0	0
PARTICULATE	10	2992	0	145	3137
	TOTAL	21369	3830	4888	30087

TABLE 4-3

WASHINGTON EXISTING CONTROL LEVEL

NUMBER OF CONTROLLED AND UNCONTROLLED SOURCES BY SOURCE CATEGORY
WASHINGTON
5/05/70

POLITICAL JURISDICTION	COMBUSTION		INDUSTRIAL		WASTE	
	C	UC	C	UC	C	UC
1	16	48	0	0	0	7
2	2	17	0	0	3	3
3	3	24	1	0	0	2
4	5	14	1	1	0	3
5	1	10	1	0	1	0
6	1	5	1	3	0	0
7	0	0	0	0	0	0
8	0	0	0	0	0	0
9	0	0	0	0	0	0
10	3	2	0	0	0	2
11	0	0	0	0	0	0
TOTAL	31	120	4	4	4	17

TABLE 4-4

WASHINGTON AWCR FUEL SULFUR LEVELS

NUMBER OF COMBUSTION SOURCES BY SULFUR CONTENT OF FUEL
WASHINGTON
5/05/70

POLITICAL JURISDICTION	UNDER 0.5%	OVER 0.5% UNDER 1%	OVER 1% UNDER 2%	OVER 2%
1	2	59	3	0
2	1	16	0	2
3	3	21	0	3
4	1	18	0	0
5	2	9	0	0
6	0	6	0	0
7	0	0	0	0
8	0	0	0	0
9	0	0	0	0
10	1	4	0	0
TOTAL	10	133	3	5

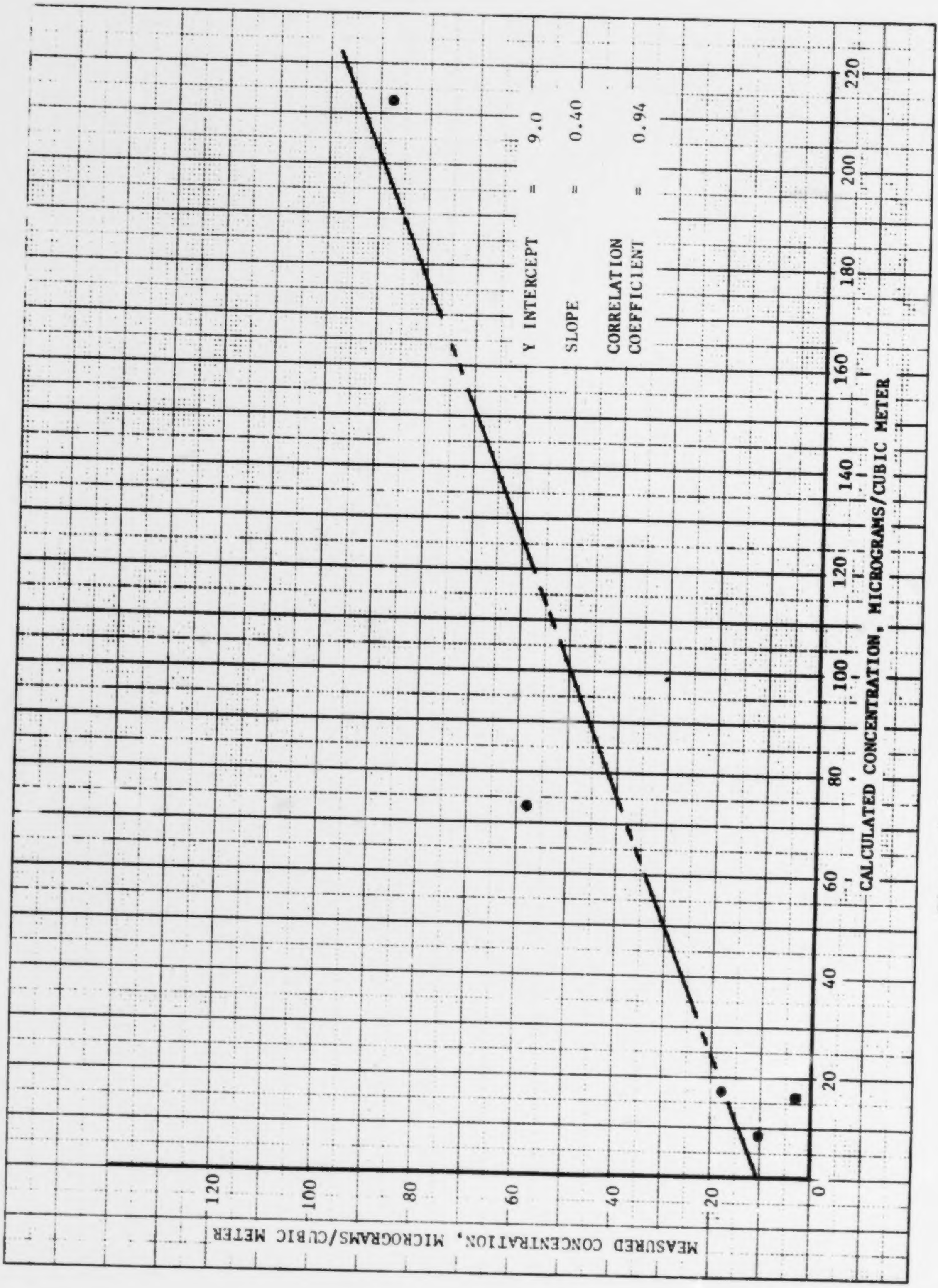

Figure 4-1. Washington AQCR SO_2 Calibration

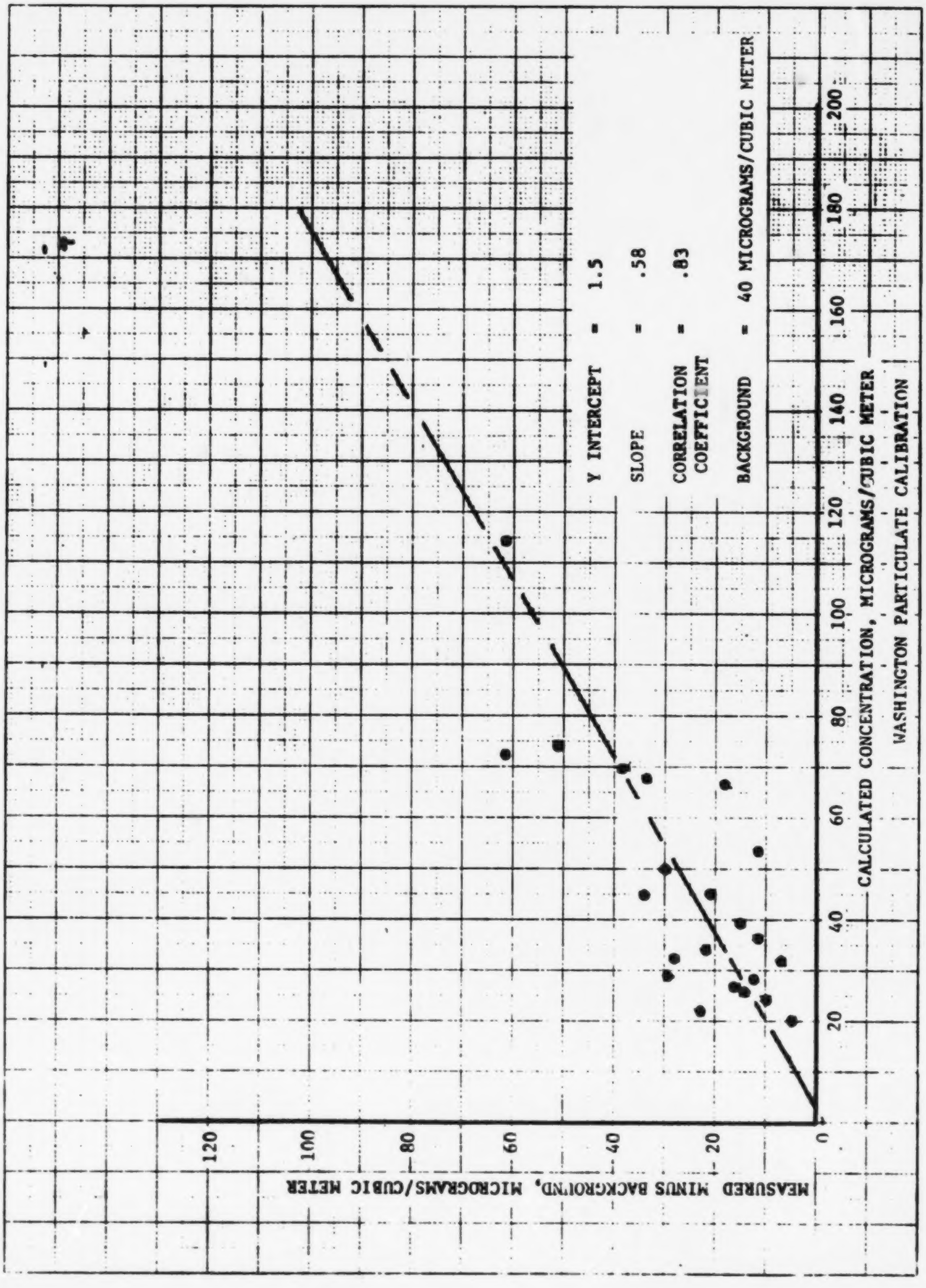

Figure 4-2. Washington AQCR Particulate Calibration

TABLE 4-5

PARTICULATE EMISSION STANDARDS FOR THE WASHINGTON AQCR

FUEL COMBUSTION

0. Null
1. Heat input curve (Figure 4-3)
2. All emissions zero
3. Heat input curve (Figure 4-4)
4. Heat input curve (Figure 4-5)
5. Potential emissions (NOT USED)
6. Maximum technology
7. Maximum technology - no fuel switching
8. .03 G/SCF

INDUSTRIAL PROCESS

0. Null
1. .03 G/SCF
2. Process weight curve (Figure 4-6 - curve C)
3. Process weight curve (Figure 4-6 - curve B)
4. All emissions zero
5. Maximum technology
6. Process weight curve (Figure 4-6 curve A)

SOLID WASTE DISPOSAL

0. Null
1. All units, .345 G/SCF; no open burning
2. > 200 LBS/HR., 0.2 GR/SCT; < 200 LBS/HR., .3G/SCF
3. .03 G/SCF
4. Potential emissions (NOT USED)
5. Maximum technology - no open burning
6. >200 LBS/HR., .159 G/SCF; < 200 LBS/HR., .345 G/SCF; max 250 LB/HR
7. All 0.03 G/SCF, except 4953 nnnl which are 0.2 G/SCF
8. All emissions zero

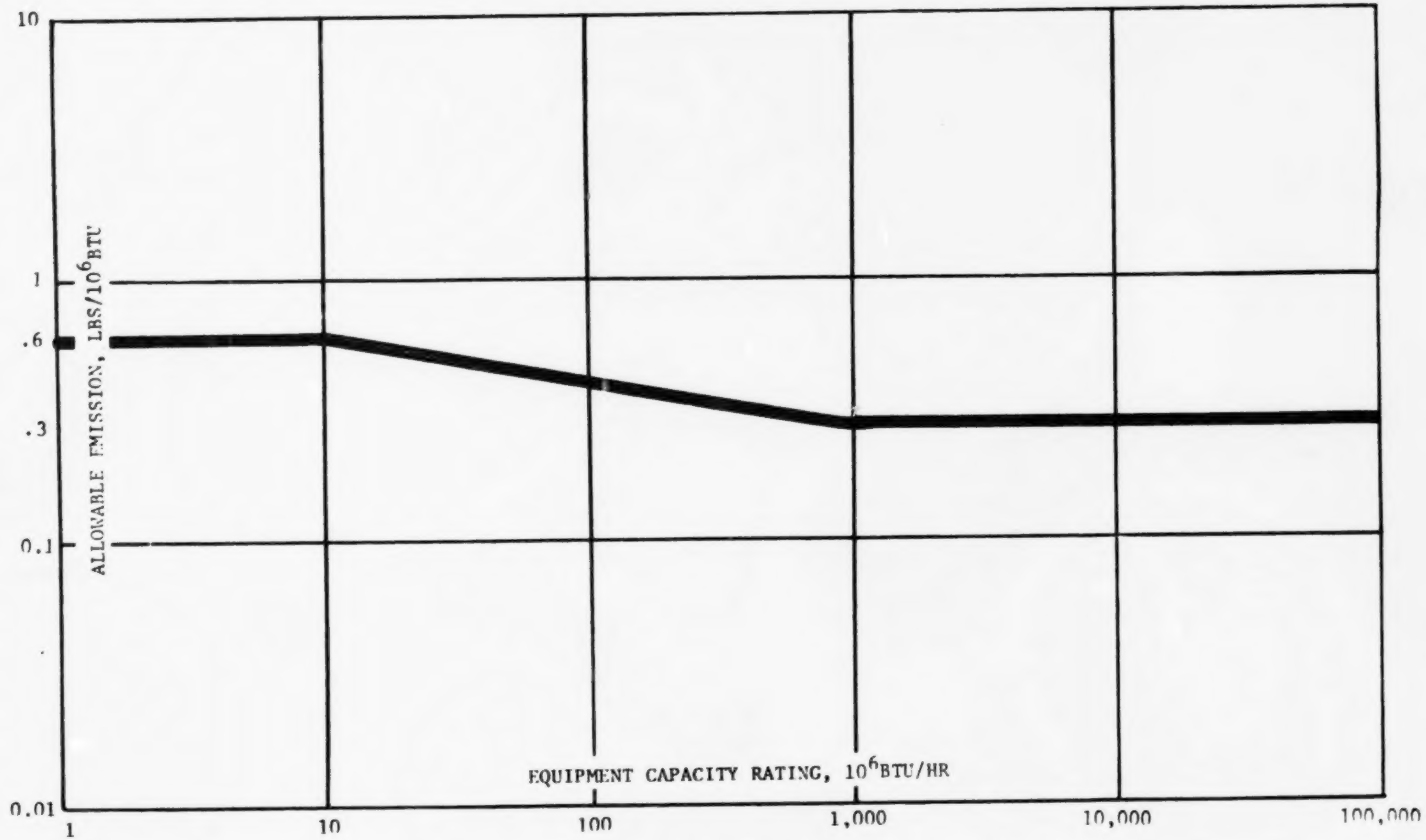

Figure 4-3. Washington AQCR Heat Input Curve - FC Emission Standard No. 1

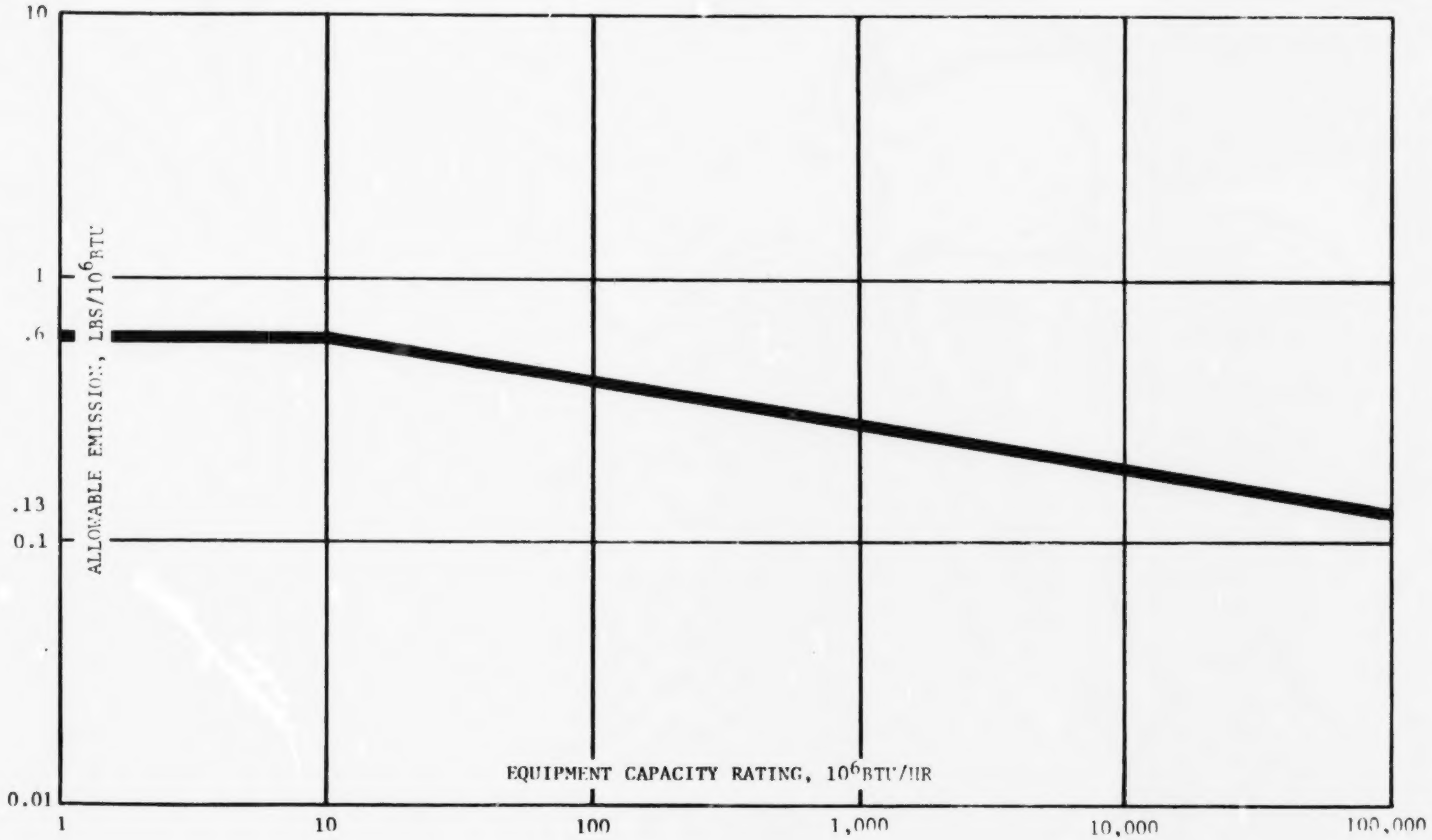

Figure 4-4. Washington AQCR Heat Input Curve - FC Emission Standard No. 3

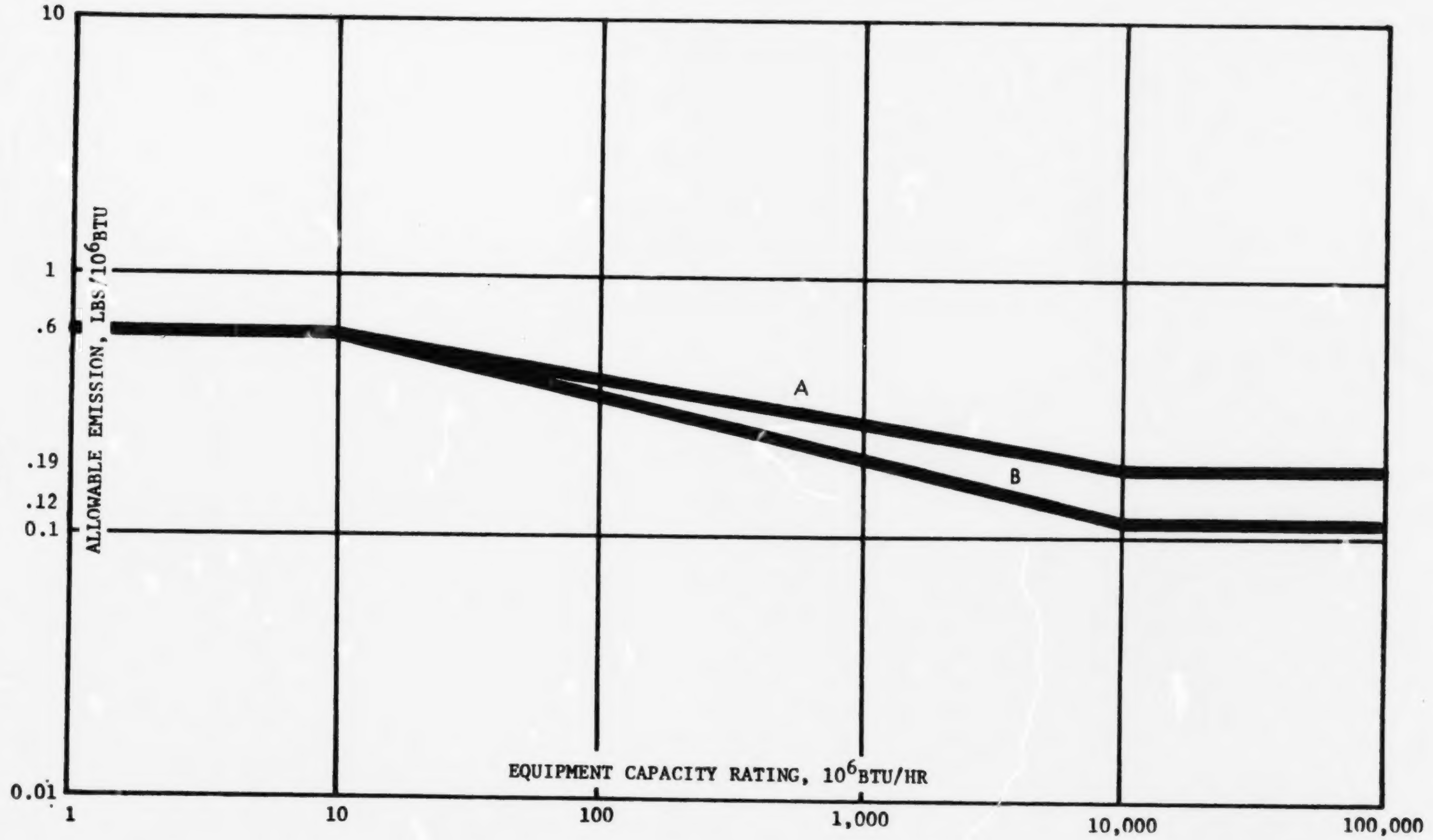

Figure 4-5. Washington AQCR Heat Input Curve - FC Emission Standard No. 4

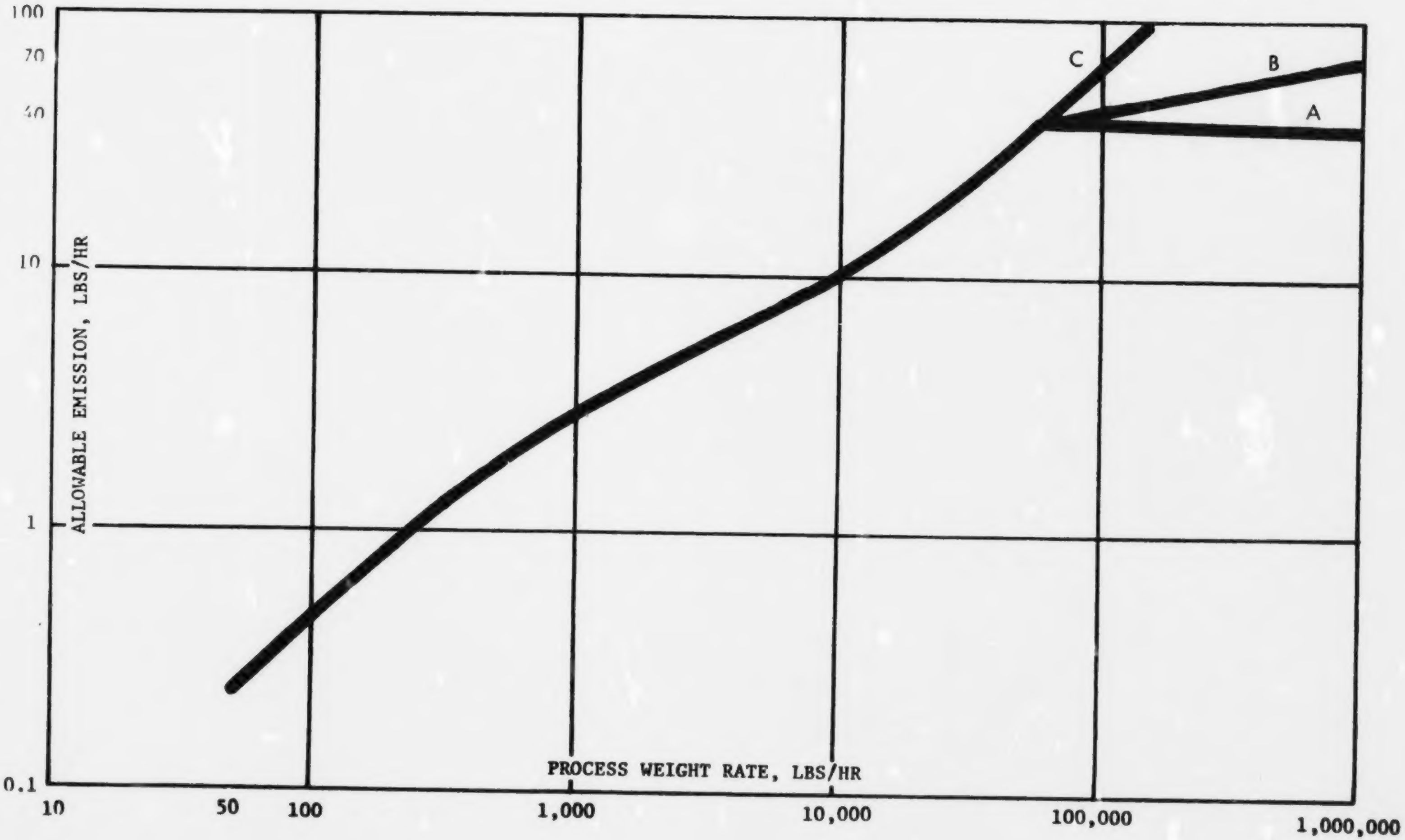

Figure 4-6. Washington AQCR Process Weight Rate Curve - IP Emission Standard Nos. 2, 3 and 4

TABLE 4-6

SULFUR DIOXIDE EMISSION STANDARDS FOR THE WASHINGTON AQCR

FUEL COMBUSTION

0. Null
1. All emissions zero
2. 1.0% sulfur - all fuel
3. 0.7% sulfur - all fuels
4. Maximum technology
5. Maximum technology - no fuel switch (NOT USED)
6. 0.5% sulfur - all fuel

INDUSTRIAL PROCESSES

NO REGULATIONS

TABLE 4-7

WASHINGTON AQCR CONTROL STRATEGIES

SO_2

1S - Existing Regulations
2S - Maximum Technology
3S - Existing Regulations in D.C. and Virginia - No Emissions in Maryland
4S - Existing Regulation in D.C. and Maryland - No Emissions in Virginia
5S - Maximum of 0.7% Sulfur Content Fuel
6S - Maximum of 0.5% Sulfur Content Fuel
7S - Existing Regulations with Virginia Counties of Loudoun and Prince William Adopting Proposed Regulations.

Particulates

1P - Existing Regulations
2P - Maximum Control Technology
3P - Existing Regulations in D.C. and Virginia - No Emissions in Maryland
4P - Existing Regulations in D.C. and Maryland - No Emissions in Virginia
5P - Maximum Technology - No Fuel Switching
6P - 0.03 Gr/SCF on All Sources of Particulate Emission
7P - Same as 6P with Municipal Incinerators Allowed Emissions of 0.2 GR/SCF
8P - Existing Control Regulation with Auto Particulate Emissions Reduced 50%
9P - Existing Control Regulations Including the Adoption of Proposed Regulations in the Virginia Counties of Loudoun and Prince William

TABLE 4-8

STRUCTURE* OF THE WASHINGTON AQCR SO_2 CONTROL STRATEGIES

SO_2 AREA SOURCE CONTROL IN ( )**

	10S	11S	12S	13S	14S	15S	16S
District of Columbia	2-0 (0)	4-0 (94)	2-0 (0)	2-0 (0)	3-0 (22)	6-0 (39)	2-0 (0)
Montgomery County	2-0 (0)	4-0 (74)	1-0 (100)	2-0 (0)	3-0 (12)	6-0 (22)	2-0 (0)
Prince George County	2-0 (0)	4-0 (74)	1-0 (100)	2-0 (0)	3-0 (11)	6-0 (20)	2-0 (0)
Prince William County	0-0 (0)	4-0 (63)	0-0 (0)	1-0 (100)	3-0 (5)	6-0 (9)	7-0 (0)
Loudoun County	0-0 (0)	4-0 (73)	0-0 (0)	1-0 (100)	3-0 (8)	6-0 (13)	7-0 (0)
Alexandria City	2-0 (0)	4-0 (79)	2-0 (0)	1-0 (100)	3-0 (13)	6-0 (24)	2-0 (0)
Arlington County	7-0 (0)	4-0 (80)	7-0 (0)	1-0 (100)	3-0 (16)	6-0 (29)	7-0 (0)
Fairfax County	7-0 (0)	4-0 (76)	7-0 (0)	1-0 (100)	3-0 (14)	6-0 (25)	7-0 (0)
Fairfax City	7-0 (0)	4-0 (52)	7-0 (0)	1-0 (100)	3-0 (6)	6-0 (12)	7-0 (0)
Falls Church	7-0 (0)	4-0 (68)	7-0 (0)	1-0 (100)	3-0 (9)	6-0 (17)	7-0 (0)

---

*Emission standards are listed in the order: Fuel Combustion (FC)-Industrial Process (IP)-Solid Waste Disposal (SWD). Thus 2-3-6 means FC standard number 2, IP standard number 3, and SWD standard number 3.

**For example, (94) means 94 percent of area source SO_2 emissions are controlled.

TABLE 4-9

STRUCTURE* OF THE WASHINGTON PARTICULATE CONTROL STRATEGIES

PARTICULATE AREA SOURCE CONTROL IN ( )*

	1P	2P	3P	4P	5P	6P
District of Columbia	4-6-2 (28)	6-5-5 (66)	4-6-2 (28)	4-6-2 (28)	7-5-5 (66)	8-1-3 (66)
Montgomery County	3-2-6 (10)	6-5-5 (30)	2-4-8 (100)	3-2-6 (10)	7-5-5 (30)	8-1-3 (30)
Prince Georges County	4-2-6 (24)	6-5-5 (48)	2-4-8 (100)	4-2-6 (24)	7-5-5 (48)	8-1-3 (48)
Prince William County	0-0-0 (0)	6-5-5 (33)	0-0-0 (0)	2-4-8 (100)	7-5-5 (33)	8-1-3 (33)
Loudoun County	0-0-0 (0)	6-5-5 (54)	0-0-0 (0)	2-4-8 (100)	7-5-5 (54)	8-1-3 (54)
Alexandria City	1-3-1 (18)	6-5-5 (41)	1-3-1 (18)	2-4-8 (100)	7-5-5 (41)	8-1-3 (41)
Arlington County	1-3-1 (15)	6-5-5 (37)	1-3-1 (15)	2-4-8 (100)	7-5-5 (37)	8-1-3 (37)
Fairfax County	1-3-6 (6)	6-5-5 (56)	1-3-6 (6)	2-4-8 (100)	7-5-5 (56)	8-1-3 (56)
Fairfax City	1-3-6 (	6-5-5 (30)	1-3-6 (9)	2-4-8 (100)	7-5-5 (30)	8-1-3 (20)
Falls Church	1	6-5-5 (44)	1-3-6 (15)	2-4-8 (100)	7-5-5 (44)	8-1-3 (44)

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*Emission standards are listed in the o Fuel Combustion (FC)-Industrial Process (IP)-Solid Waste Disposal (SWD). Thus 2-3-6 means FC s d number 2, IP standard number 3, and SWD standard number 3.

**For example - (28) means 28 percent of a source particulate emission are controlled.

TABLE 4-9

STRUCTURE* OF THE WASHINGTON PARTICULATE CONTROL STRATEGIES (Continued)

PARTICULATE AREA SOURCE CONTROL IN ( )*

	7P	8P	9P
District of Columbia	8-1-7 (66)	4-6-2 (39)	4-6-2 (28)
Montgomery County	8-1-7 (30)	3-2-6 (39)	3-2-6 (10)
Prince Georges County	8-1-7 (48)	4-2-6 (45)	4-2-6 (24)
Prince William County	8-1-7 (33)	0-0-0 (22)	1-3-5 (10)
Loudoun County	8-1-7 (54)	0-0-0 (12)	1-3-6 (9)
Alexandria City	8-1-7 (41)	1-3-1 (42)	1-3-1 (18)
Arlington County	8-1-7 (37)	1-3-1 (40)	1-3-1 (15)
Fairfax County	8-1-7 (36)	1-3-6 (22)	1-3-6 (6)
Fairfax City	8-1-7 (30)	1-3-6 (41)	1-3-6 (9)
Falls Church	8-1-7 (44)	1-3-6 (38)	1-3-6 (15)

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*Emission standards are listed in the order: Fuel Combustion (FC)-Industrial Process (IP)-Solid Waste Disposal (SWD). Thus 2-3-6 means FC standard number 2, IP standard number 3, and SWD standard number 3.

**For example - (28) means 28 percent of area source particulate emissions are controlled.

TABLE 4-10

CONTROL STRATEGY SIMULATION RESULTS - NATIONAL CAPITAL INTERSTATE AIR QUALITY CONTROL REGION

STRATEGY NUMBER	TOTAL % REDUCTION IN EMISSIONS	POINT SOURCES NEW EMISSION RATE (TONS/DAY)	AREA SOURCES		CONTROL COST		RESULTING AIR QUALITY ($\mu g/m^3$) at SELECTED RECEPTORS ($\mu g/m^3$)	
			% REDUCTION	NEW EMISSION RATE (TONS/DAY)	$ X 10^6	$/TON REMOVED	#111	#192
SO_2 Strategies		(542.0)*		(118.6)*			(71.9)*	(59.2)*
10	36.4	344.8	0	118.6	-3.5	-48	70.1	58.2
11	100.0	0.0	82.3	21.0	113.2	563	36.6	37.4
12	65.9	184.7	23.8	90.4	44.7	343	68.0	52.7
13	56.6	235.0	14.3	101.6	34.2	305	65.7	56.5
14	51.7	251.6	25.5	88.4	6.1	57.6	62.5	53.9
15	67.4	176.6	31.3	81.4	14.6	109	56.2	50.1
16	36.4	344.8	0	118.6	-3.5	-48	70.1	58.2
Particulates Strategies		(84.7)*		(75.6)*			(75.4)*	(80.0)*
1	36.3	54.0	18.0	62.0	0.5	47	68.0	78.7
2	87.4	10.7	51.4	36.7	13.9	514	55.3	69.2
3	62.2	32.0	47.5	39.7	4.0	210	64.1	67.5
4	52.7	40.1	48.6	38.9	4.4	267	6[illegible].5	75.4
5	86.4	11.6	51.4	36.7	7.2	271	55.5	69.4
6	78.2	18.4	51.4	36.7	7.0	289	56.0	69.7
7	78.2	18.4	51.4	36.7	7.0	289	56.0	69.7
8	36.3	54.0	36.25	48.2	0.5	47	64.8	75.4
9	41.0	50.0	18.6	61.6	0.6	44	66.8	78.1
17			51.4	36.7				

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*Existing Conditions.

4.3.2 Philadelphia Interstate Air Quality Control Region

The Philadelphia AQCR is by far the largest region, in terms of number of point sources, for which control strategy simulation has been undertaken. The region consists of the jurisdictions of the States of Delaware (jurisdiction No. 1), New Jersey (No. 2), Pennsylvania (No. 3), and the City of Philadelphia (No. 4).

The regional emissions, existing control level, and fuel use data are summarized in Tables 4-11, 4-12 and 4-13, respectively. The size of the region is apparent when these tables are compared, for example, with those for Washington; Philadelphia regional emissions are greater by at least a factor of four in all categories.

The regression lines used to calibrate the Atmospheric Diffusion Model are shown in Figures 4-7 and 4-8. Work is continuing on the SO_2 calibration with the aim of finding the cause of the very large y-intercept.

The emission standards considered for the Philadelphia AQCR are listed in Tables 4-14 and 4-15. Figures 4-9 through 4-22 describe the various curves used in these emission standards.

A description of the strategies run for the Philadelphia AQCR is given in Table 4-16; the simulation results are presented in Table 4-17.

4.3.3 Cincinnati Interstate Air Quality Control Region

The Cincinnati AQCR was the first region for which control strategy simulation was attempted. This work was done under a separate contract and will be described in the final report for that contract. The simulation results are summarized here as a further demonstration of the application of the RAPA program.

The Cincinnati AQCR consists of four political jurisdictions, the City of Cincinnati (jurisdiction No. 1), the State of Indiana (No. 2), the State of Kentucky (No. 3), and the State of Ohio (No. 4).

The regional emissions, existing control level and fuel use data are summarized in Tables 4-18, 4-19 and 4-20, respectively. The Cincinnati AQCR is the smallest of the regions described in this report; on the basis of regional emissions (Table 4-18), it is less than one-half the size of the Washington AQCR.

TABLE 4-11

PHILADLEPHIA AQCR EMISSION SUMMARY

POLLUTANT EMISSIONS BY SOURCE CATEGORY (TONS/YEAR)

PHILADELPHIA

06/05/70

POLLUTANT	POLITICAL JURISDICTION	FUEL COMBUSTION	INDUSTRIAL PROCESS	WASTE DISPOSAL	TOTAL
SO2	1	201673	30839	0	232512
SO2	2	157683	18320	0	176003
SO2	3	236655	26751	471	263877
SO2	4	197580	10836	473	208889
	TOTAL	793591	86746	944	881281
PARTICULATE	1	25396	17443	364	43203
PARTICULATE	2	12741	2618	472	15831
PARTICULATE	3	25264	18813	2599	46666
PARTICULATE	4	17309	2948	1275	21532
	TOTAL	80710	41822	4700	127232

TABLE 4-12

PHILADELPHIA AQCR EXISTING CONTROL LEVEL

NUMBER OF CONTROLLED AND UNCONTROLLED SOURCES BY SOURCE CATEGORY
PHILADELPHIA
06/05/70

POLITICAL JURISDICTION	COMBUSTION		INDUSTRIAL		WASTE	
	C	UC	C	UC	C	UC
1	6	81	15	19	0	4
2	13	190	57	21	3	4
3	21	220	47	34	7	8
4	20	128	12	25	8	1
TOTAL	60	619	131	99	18	17

TABLE 4-13

PHILADELPHIA AQCR FUEL SULFUR LEVELS

NUMBER OF COMBUSTION SOURCES BY SULFUR CONTENT OF FUEL PHILADELPHIA 06/05/70				
POLITICAL JURISDICTION	UNDER 0.5%	OVER 0.5% UNDER 1%	OVER 1% UNDER 2%	OVER 2%
1	3	9	27	47
2	12	87	62	36
3	8	26	67	93
4	5	18	54	63
TOTAL	28	140	210	239

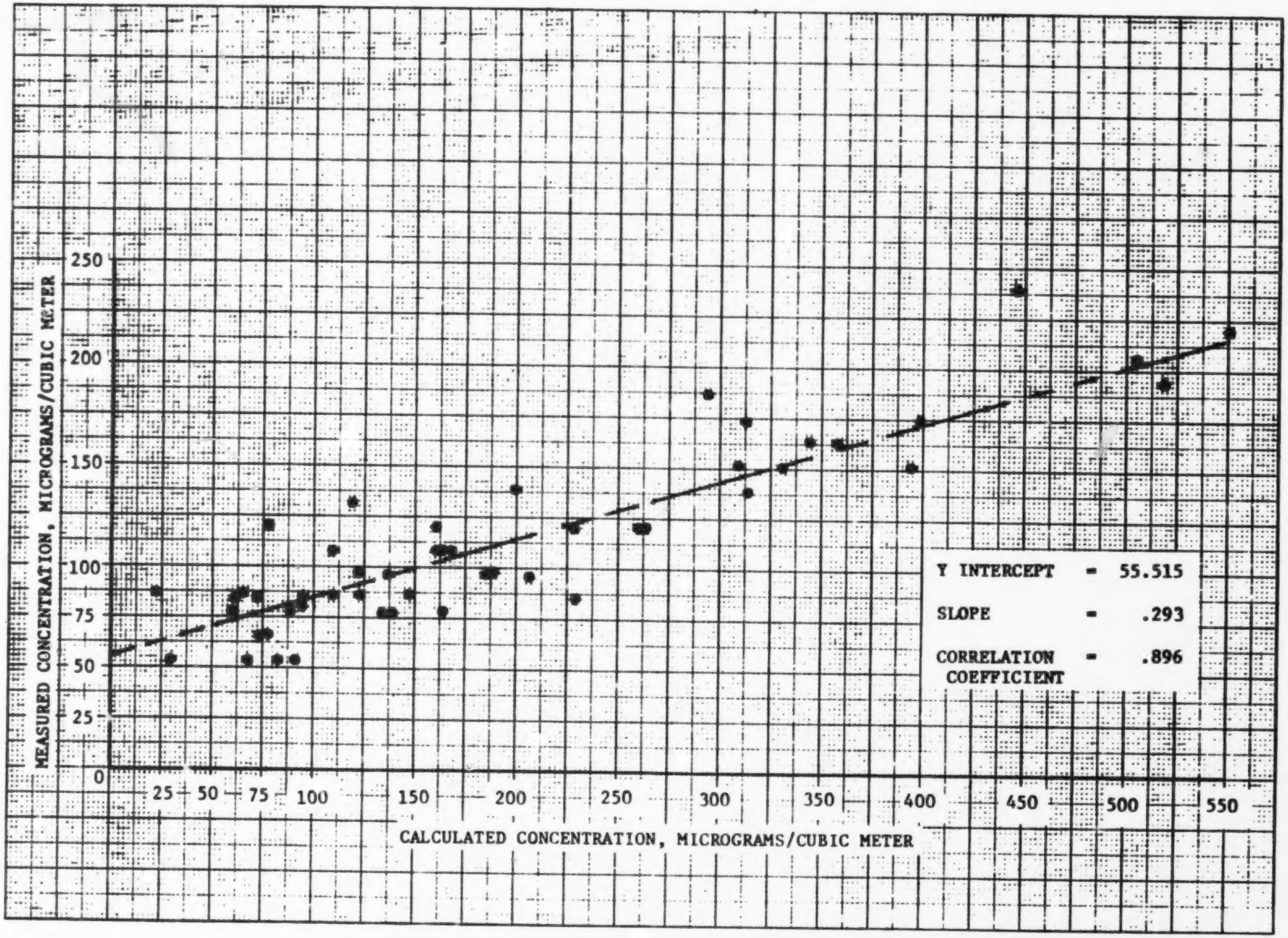

Figure 4-7. Philadelphia AQCR SO_2 Calibration

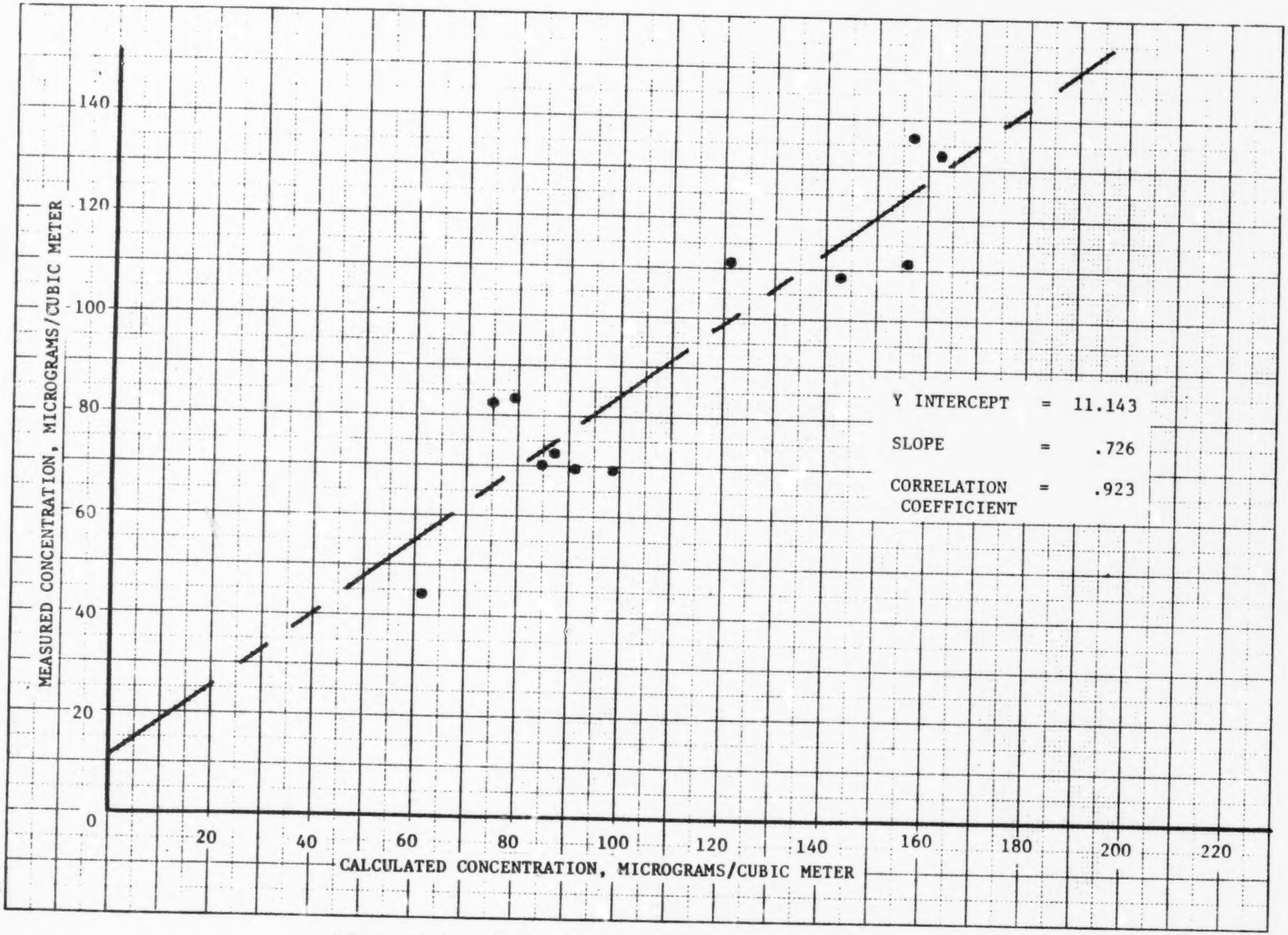

Figure 4-8. Philadelphia AQCR Particulate Calibration

TABLE 4-14

PHILADELPHIA AQCR SO_2 EMISSION STANDARDS

Fuel Combustion

0. Null
1. Heat Input Curve - (Figure 4-9)
2. Fuel Sulfur Limitation - 0.8%
3. Full Sulfur Limimtation - 0.3%
4. Maximum Technology
5. Maximum Technology - No Fuel Switch
6. Fuel Sulfur Limitation - 0.2%
8. Heat Input Curve - (Figure 4-10)
9. Heat Input Curve - (Figure 4-11)

Industrial Processes

0. Null
1. Exhaust gas concentration limit - 2000 PPM
2. Exhaust gas concentration limit - 500 PPM
3. Maximum Technology
4. Potential Emissions - (Figure 4-12)
5. Potential Emissions - (Figure 4-13)
6. Potential Emissions - (Figure 4-14)
7. Potential Emissions - (Figure 4-15)
8. New Jersey Regulation V111 - (Manual input)

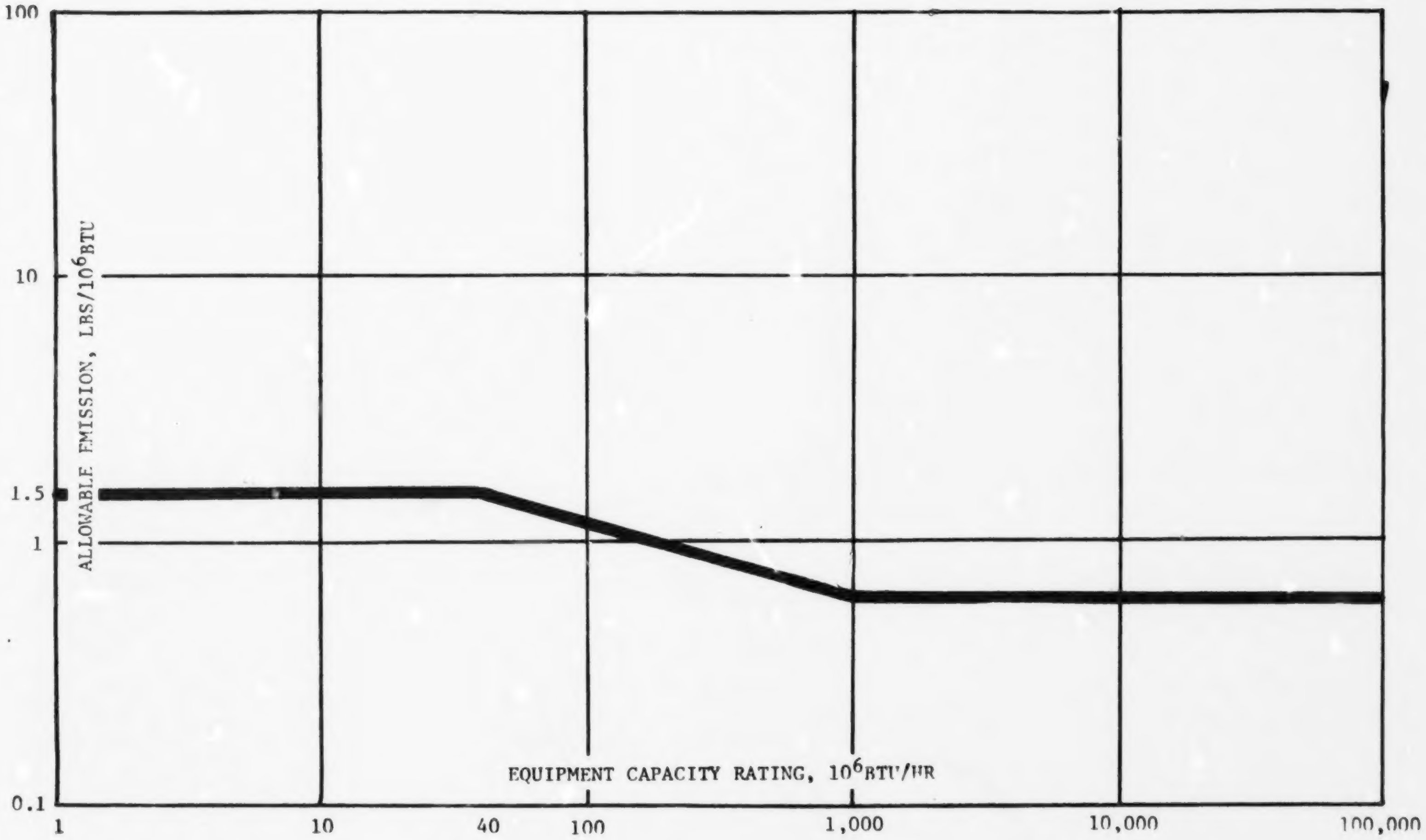

Figure 4-9. Philadelphia AQCR Heat Input Curve - FC Emission Standard No. 1

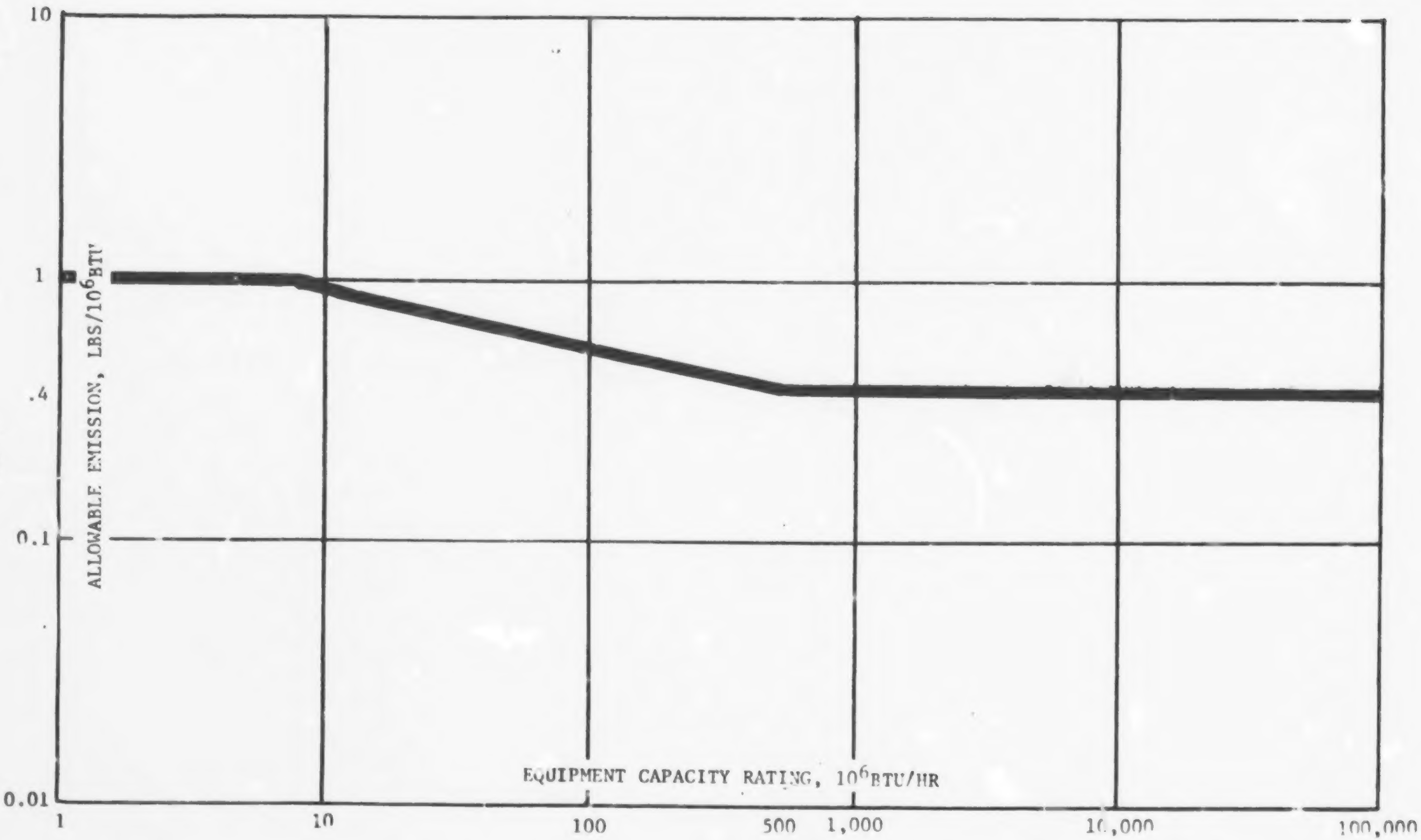

Figure 4-10. Philadelphia AQCR Heat Input Curve - FC Emission Standard No. 8

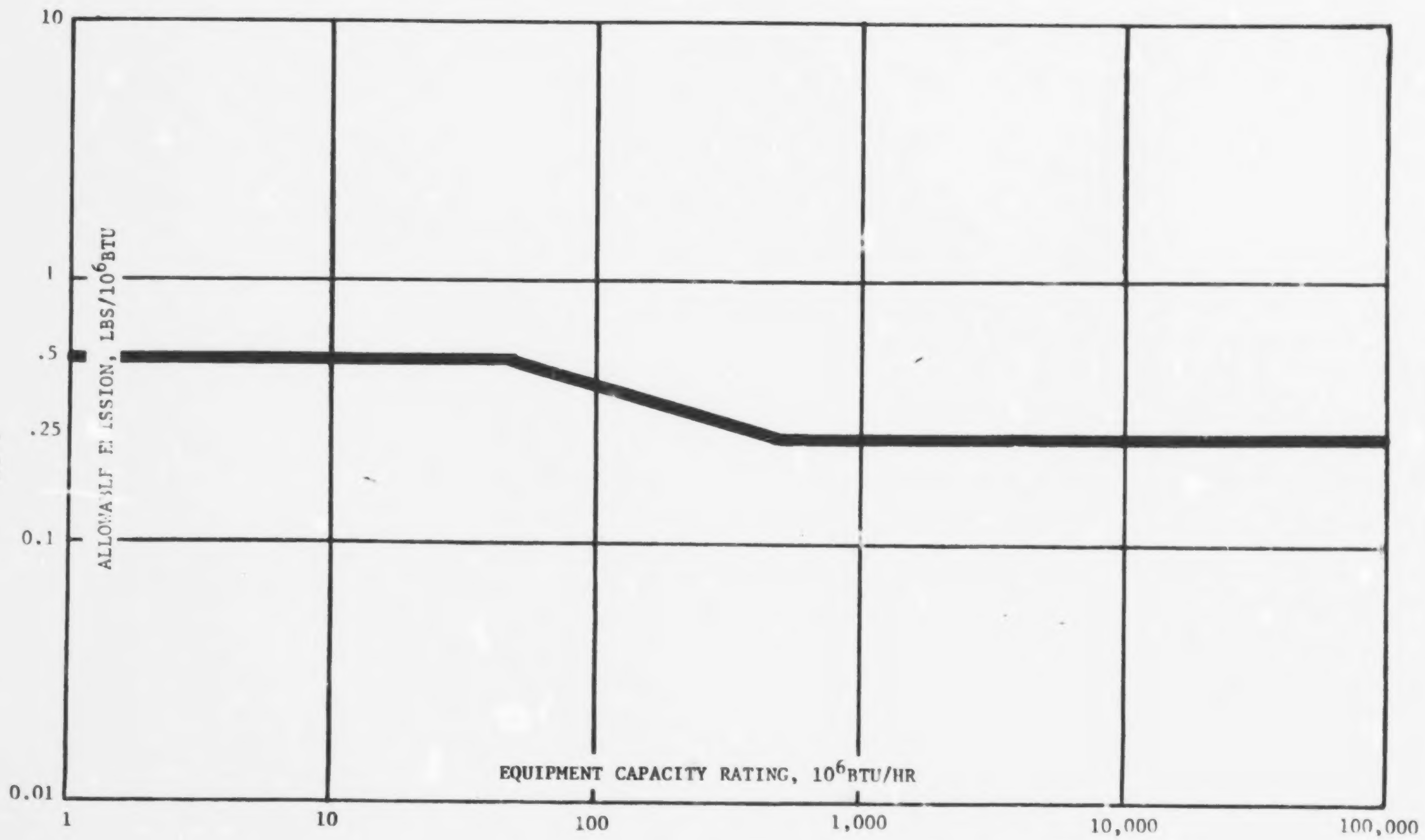

Figure 4-11. Philadelphia AQCR Heat Input Curve - FC Emission Standard No. 9

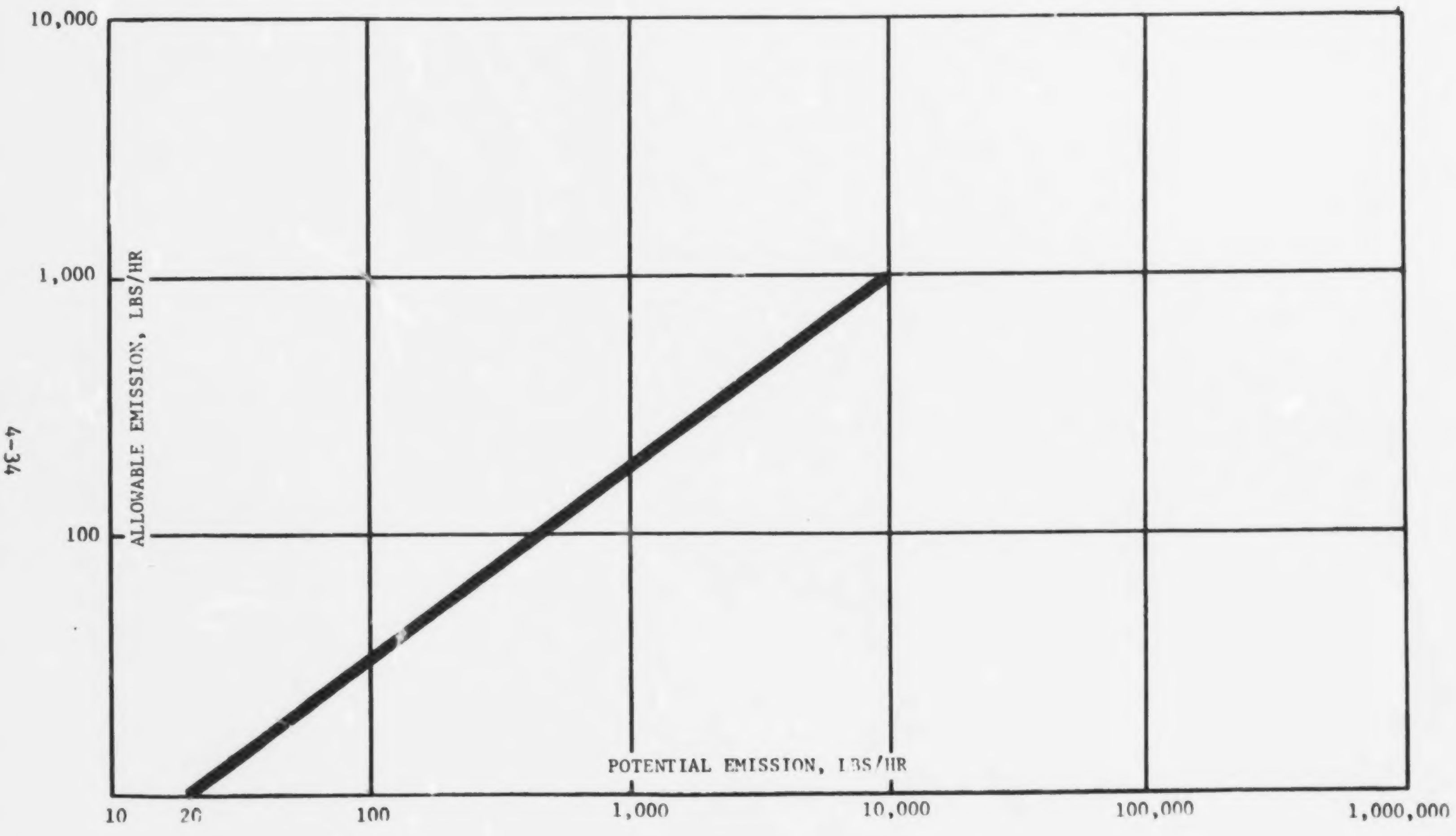

Figure 4-12. Philadelphia AQCR Potential Emission Curve - IP Emission Standard No. 4

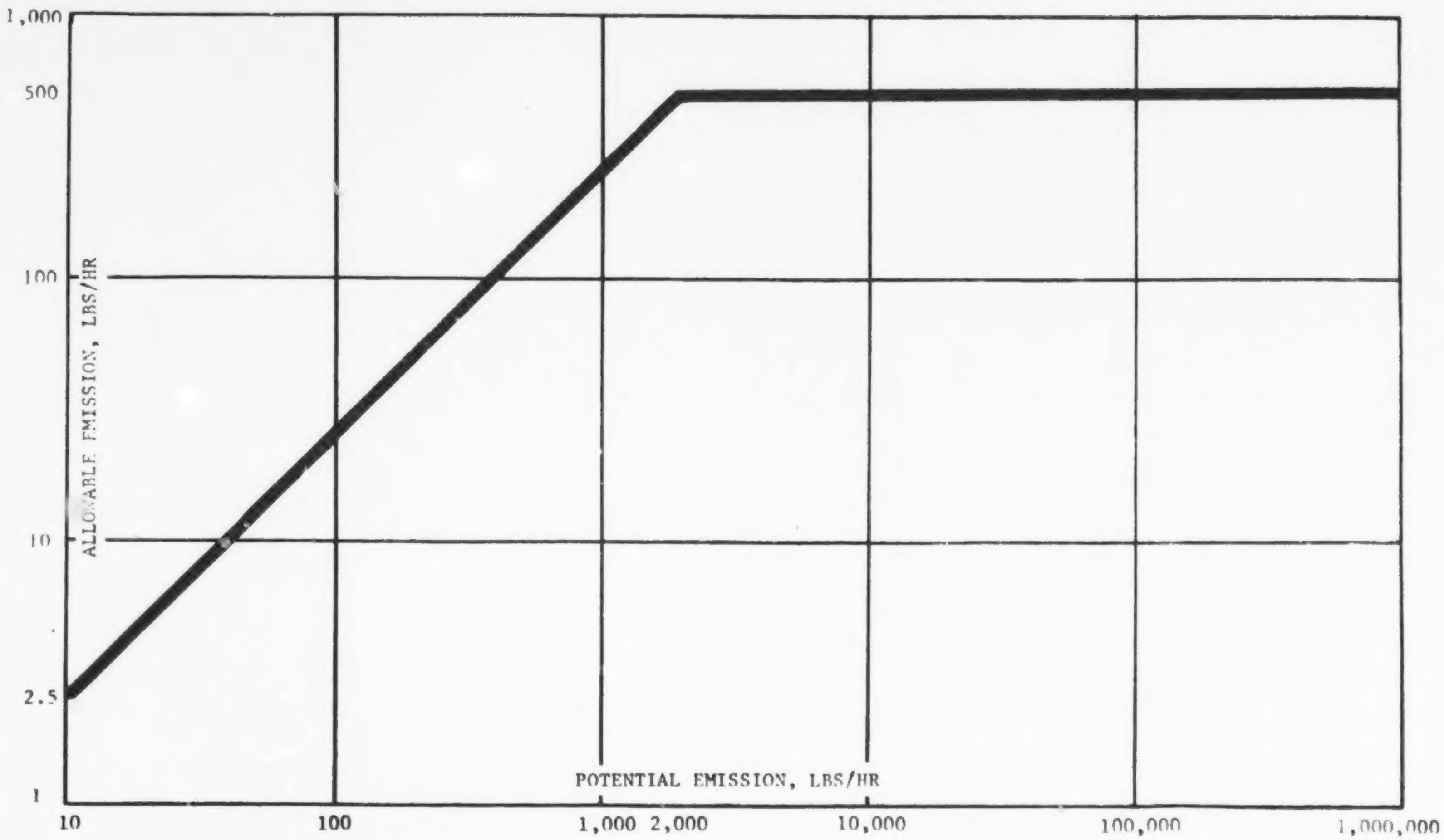

Figure 4-13. Philadelphia AQCR Potential Emissions Curve - IP Emission Standard No. 5

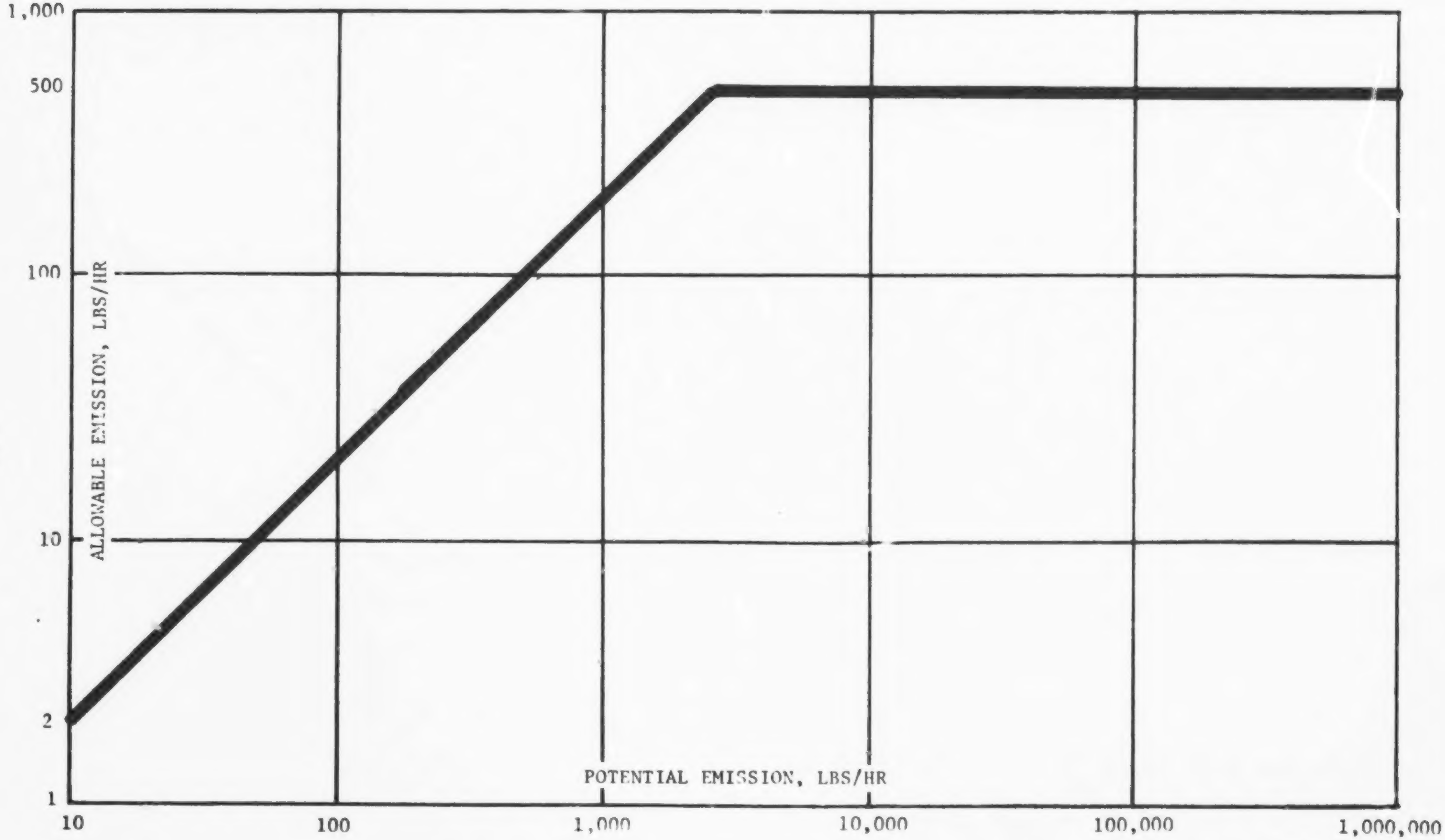

Figure 4-14. Philadelphia AQCR Potential Emissions Curve - IP Emission Standard No. 6

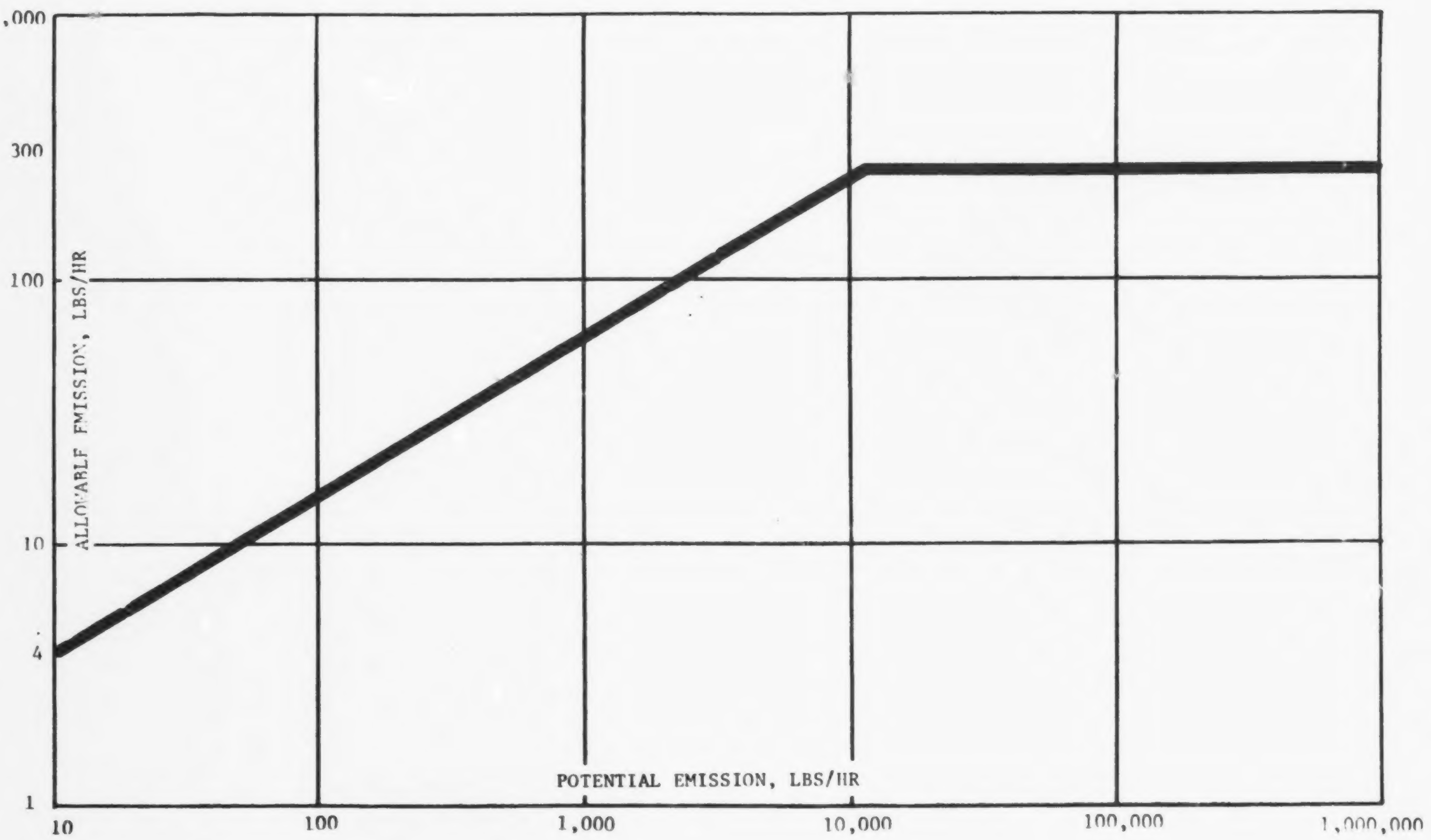

Figure 4-15. Philadelphia AQCR Potential Emissions Curve - IP Emission Standard No. 7

TABLE 4-15

PHILADELPHIA AQCR PARTICULATE EMISSION STANDARDS

FUEL COMBUSTION

0. Null
1. Heat Input Curve (Figure 4-16, Curve A)
2. Maximum Emission, 0.8 LBS/10^6 BTU
3. Maximum Emission, 0.3 LBS/10^6 BTU
4. Heat Input Curve (Figure 4-15, Curve B)
5. Potential Emissions Curve (Figure 4-17)
6. Maximum Technology
7. Not Used
8. Maximum Emission, 0.85 LBS/10^6 BTU
9. Effective Stack Height Curve (Figure 4-18)
10. Maximum Emission, 0.56 LBS/10^3 BTU stack gas
11. Maximum Emission, 0.15 LBS/10^6 BTU
12. Maximum Emission, 0.20 LBS/10^3 LBS stack gas

INDUSTRIAL PROCESSES

0. Null
1. Not Used
2. Not Used
3. Not Used
4. Potential Emissions Curve (Figure 4-17)
5. Maximum Technology
6. Process Weight Curve (Figure 4-19)
7. Effective Stack Height Curve (Figure 4-18)
8. Potential Emissions Curve (Figure 4-20)
9. Potential Emissions Curve (Figure 4-20)
10. Potential Emissions Curve (Figure 4-21)
11. New Jersey Regulation VII (Manual Computation)

TABLE 4-15

PHILADELPHIA AQCR PARTICULATE EMISSION STANDARDS (Continued)

SOLID WASTE DISPOSAL

0. Null
1. Maximum Emission, 0.2 g/set
2. Maximum Emission, 0.2 g/set, incinerators less than 2000 LBS/HR; 0.1 g/set, larger incinerators
3. Maximum Emission, 0.2 LBS/10^3 LBS stack gas
4. Potential Emissions Curve (Figure 4-14)
5. Maximum Technology
6. Maximum Emission, 0.4 LBS/10^3 LBS stack gas
7. Maximum Emission, 1.0 LBS/10^6 BTU
8. Sanitary Landfill - all emissions set to zero
9. Effective Stack Height Curve (Figure 4-18)
10. Potential Emissions Curve (Figure 4-20)
11. Potential Emissions Curve (Figure 4-23)

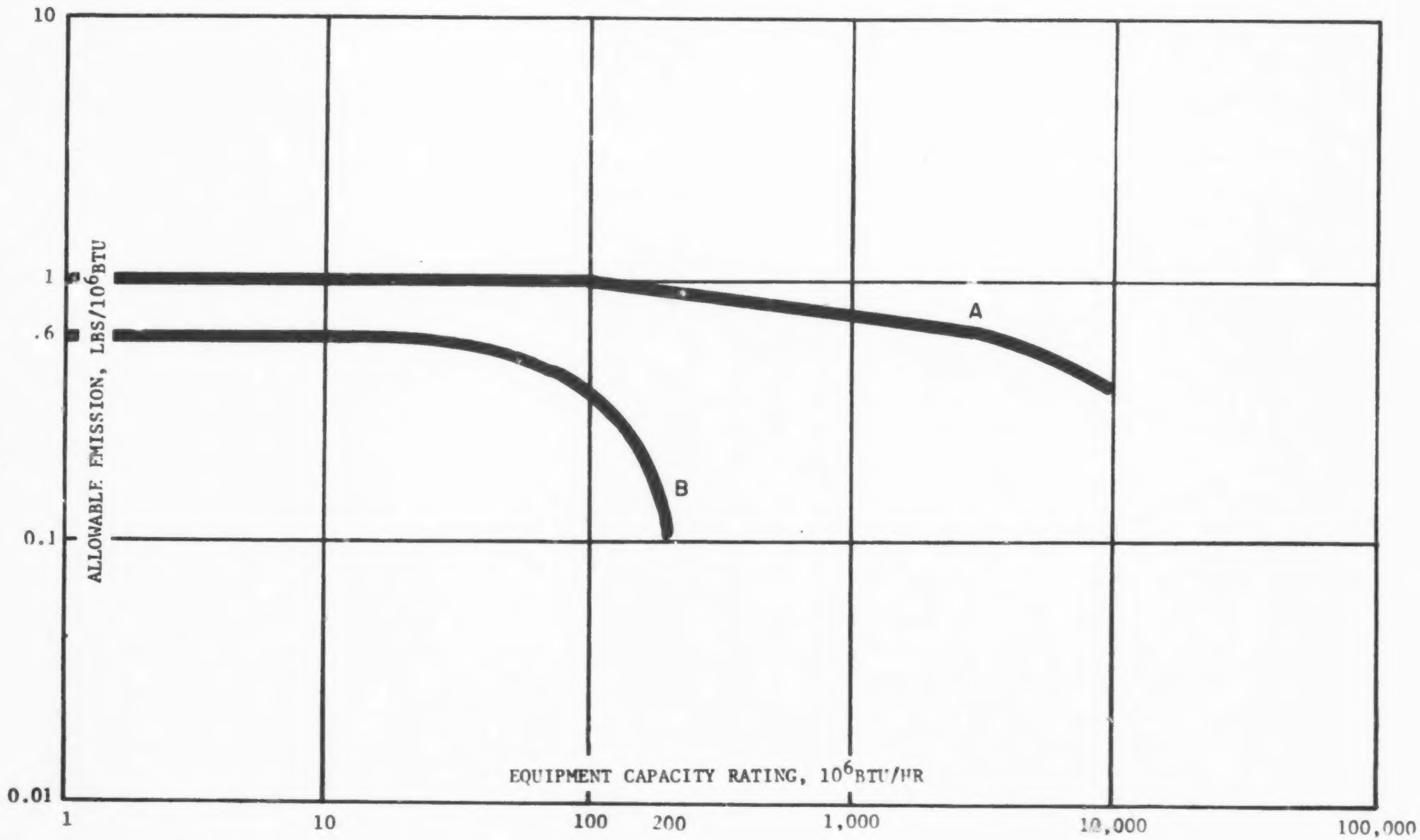

Figure 4-16. Philadelphia AQCR Heat Input Curve - FC Emission Standard Nos. 1 and 4

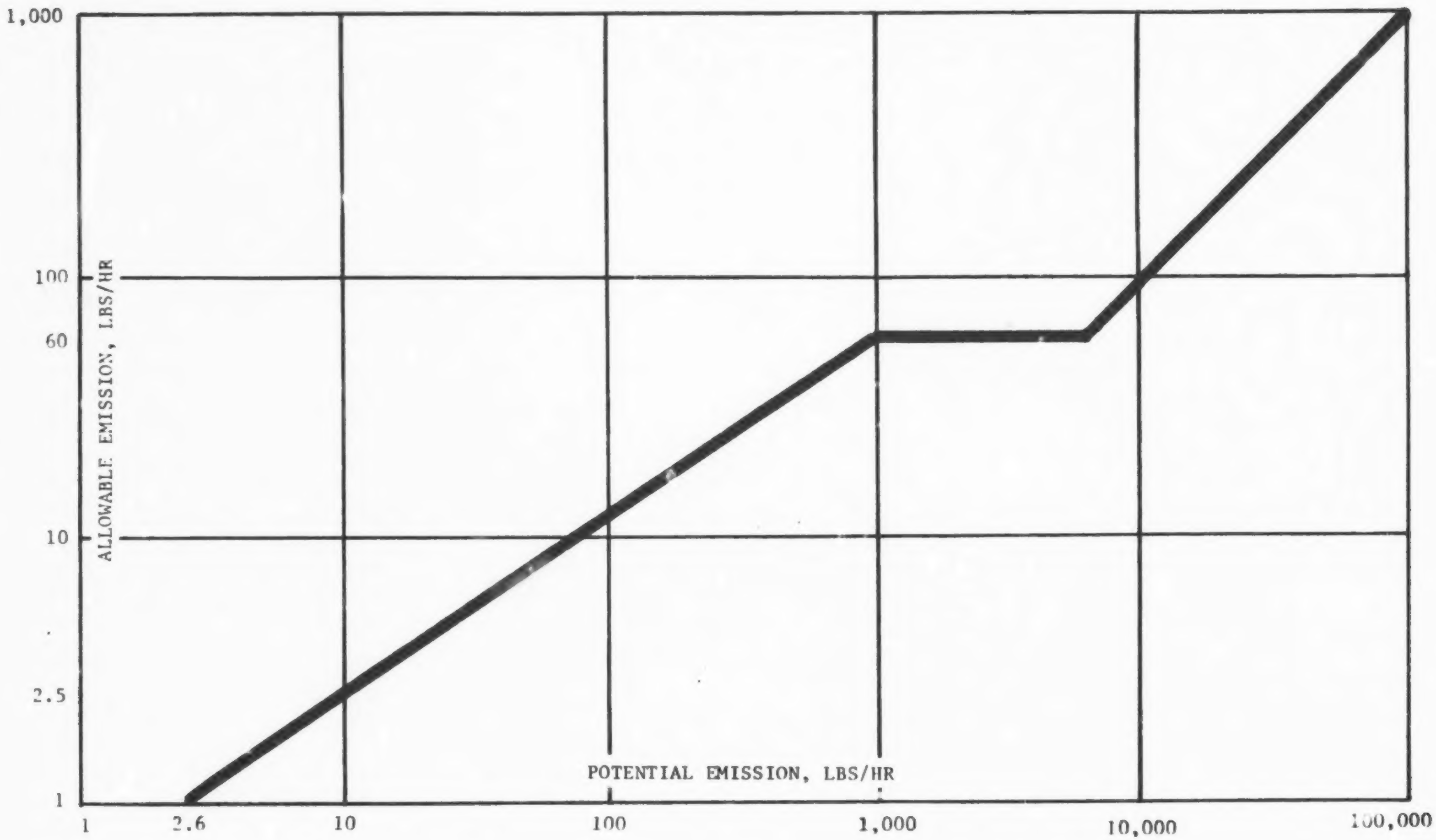

Figure 4-17. Philadelphia AQCR Potential Emissions Curve - FC Emission Standard No. 5 and IP No. 4

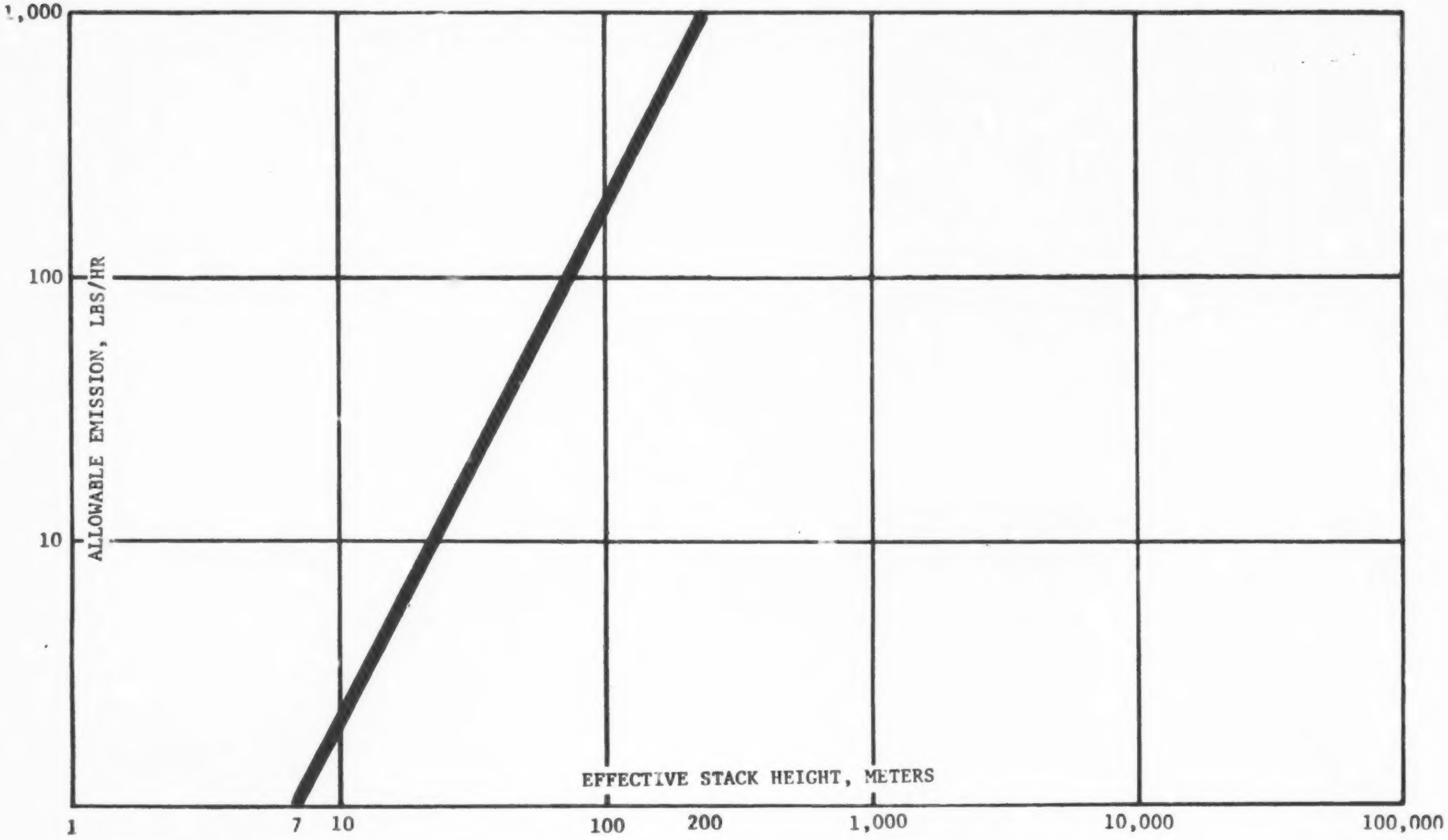

Figure 4-18. Philadelphia AQCR Effective Stack Height Curve - FC Emission Standard No. 7, IP No. 7 and SWD No. 9

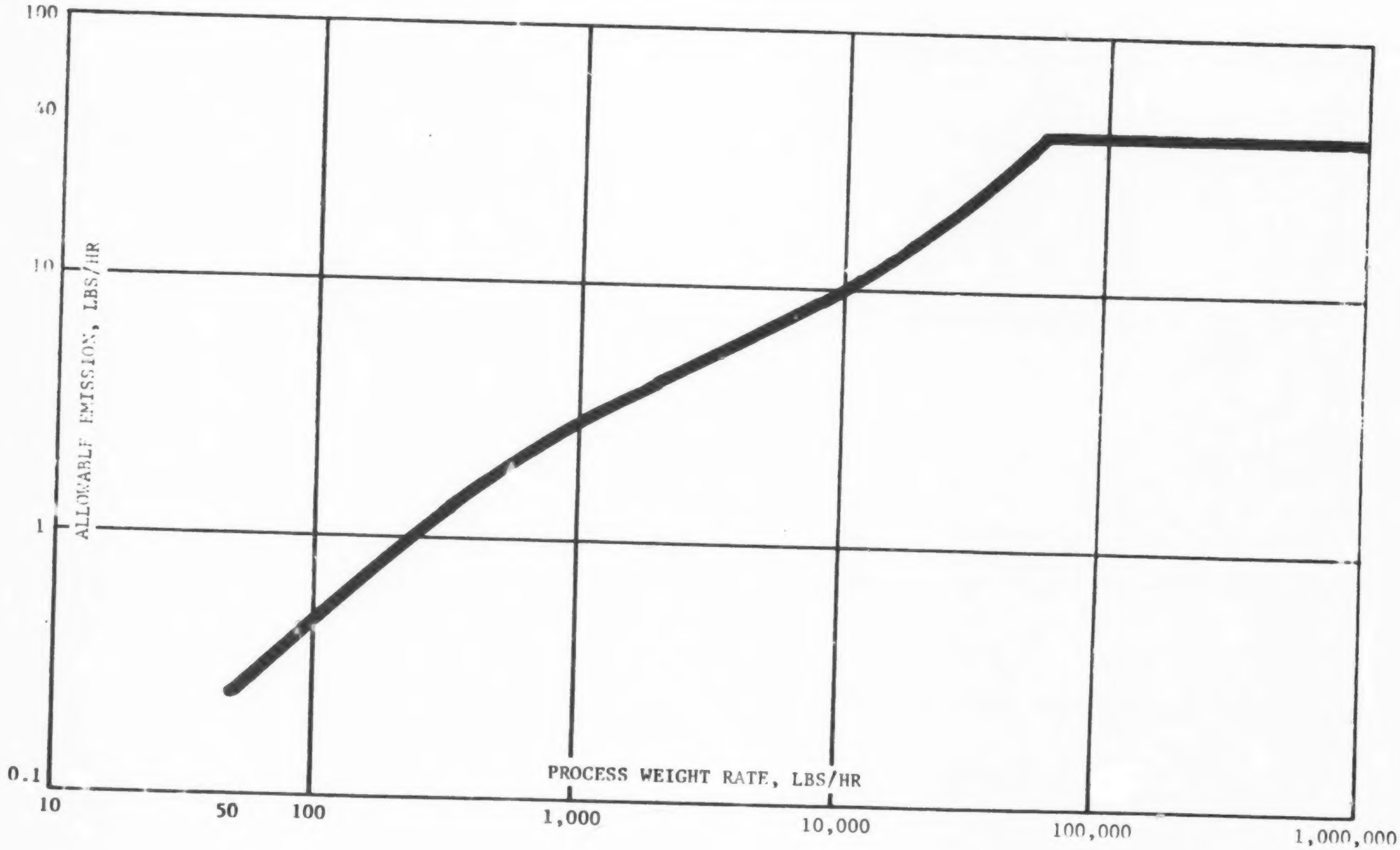

Figure 4-19. Philadelphia AQCR Process Weight Curve - IP Emission Standard No. 6

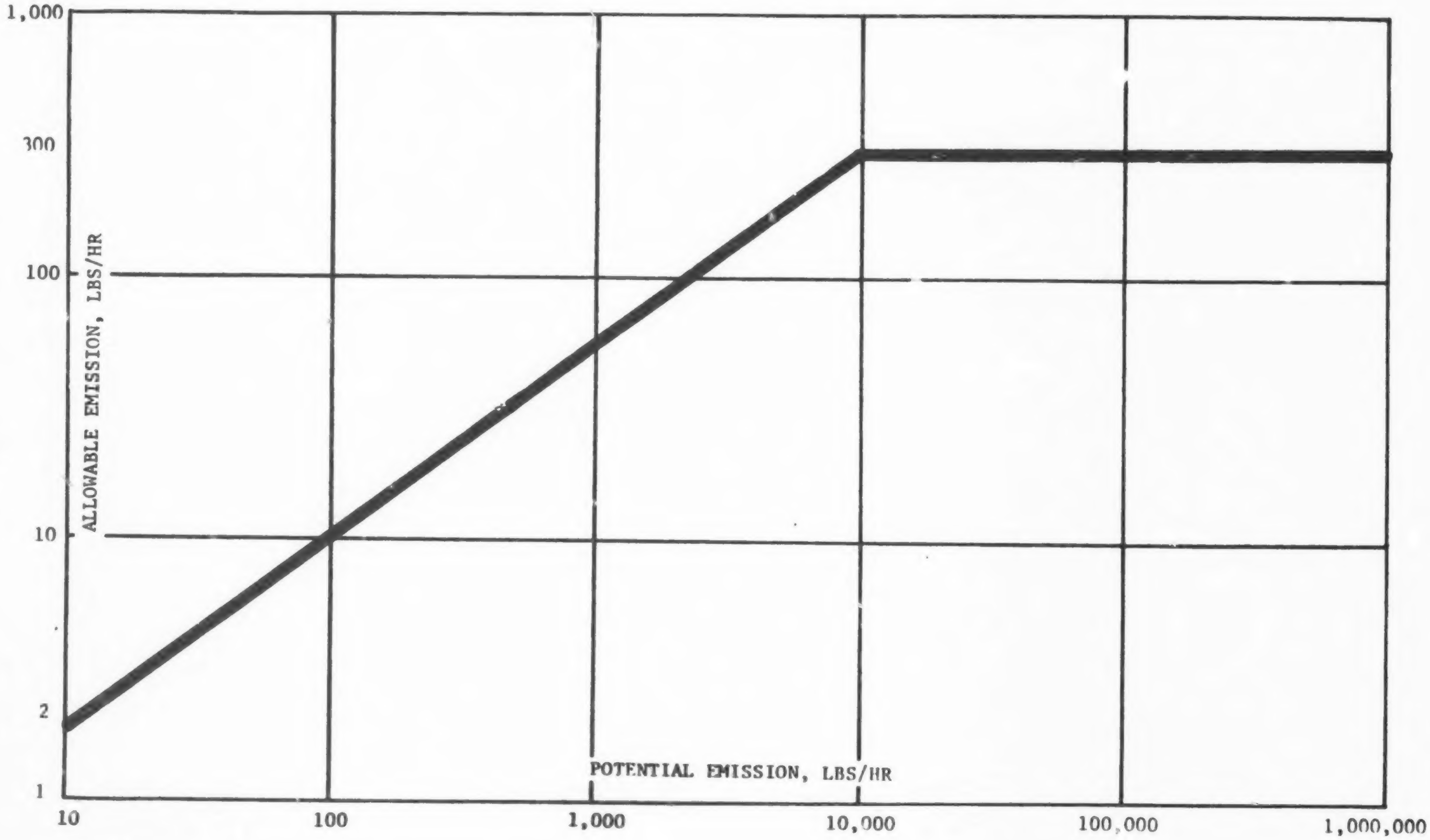

Figure 4-20. Philadelphia AQCR Potential Emissions Curve - IP Emission Standard No. 9 and SWD No. 10

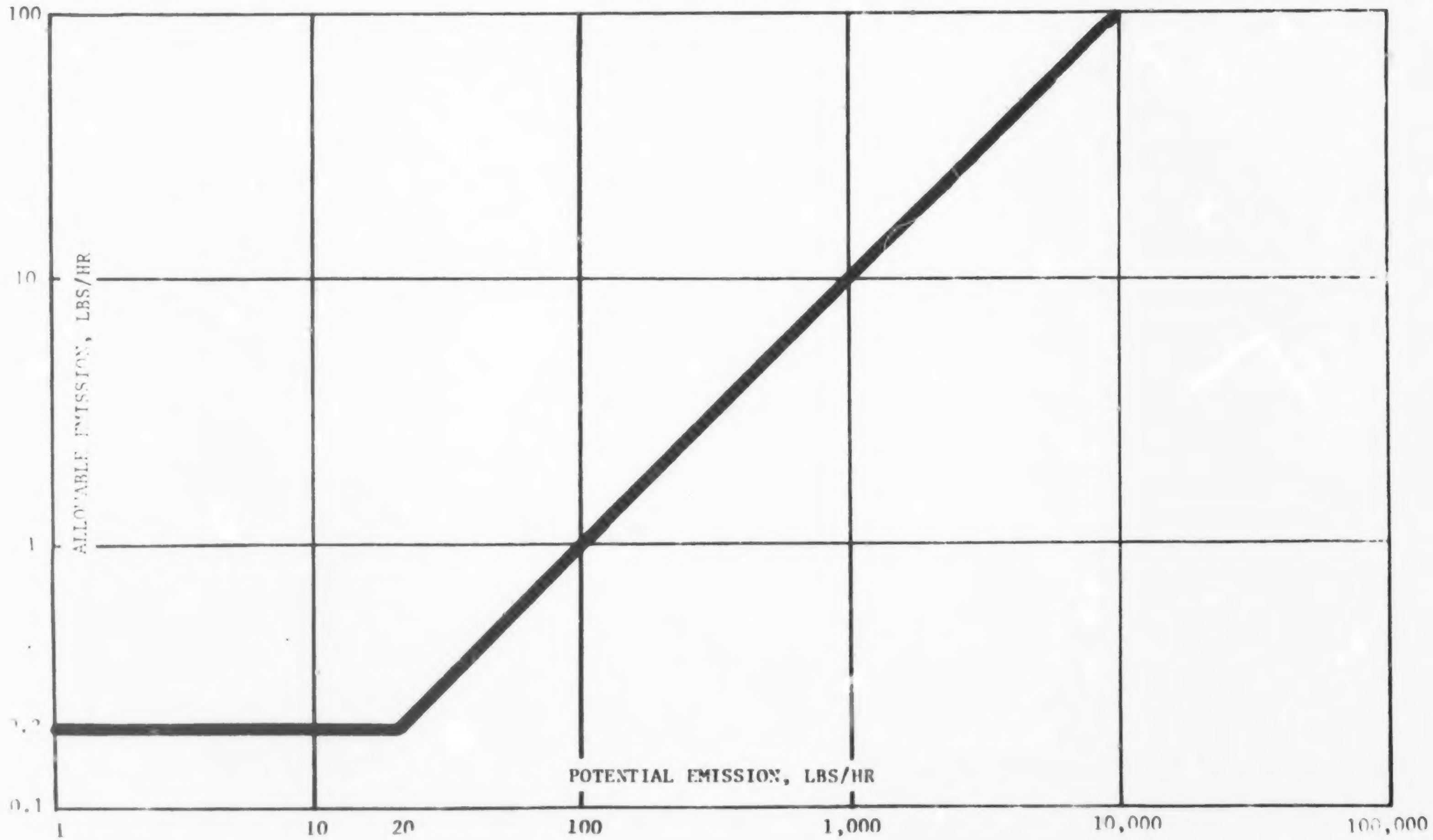

Figure 4-21. Philadelphia AQCR Potential Emissions Curve - IP Emission Standard No. 10

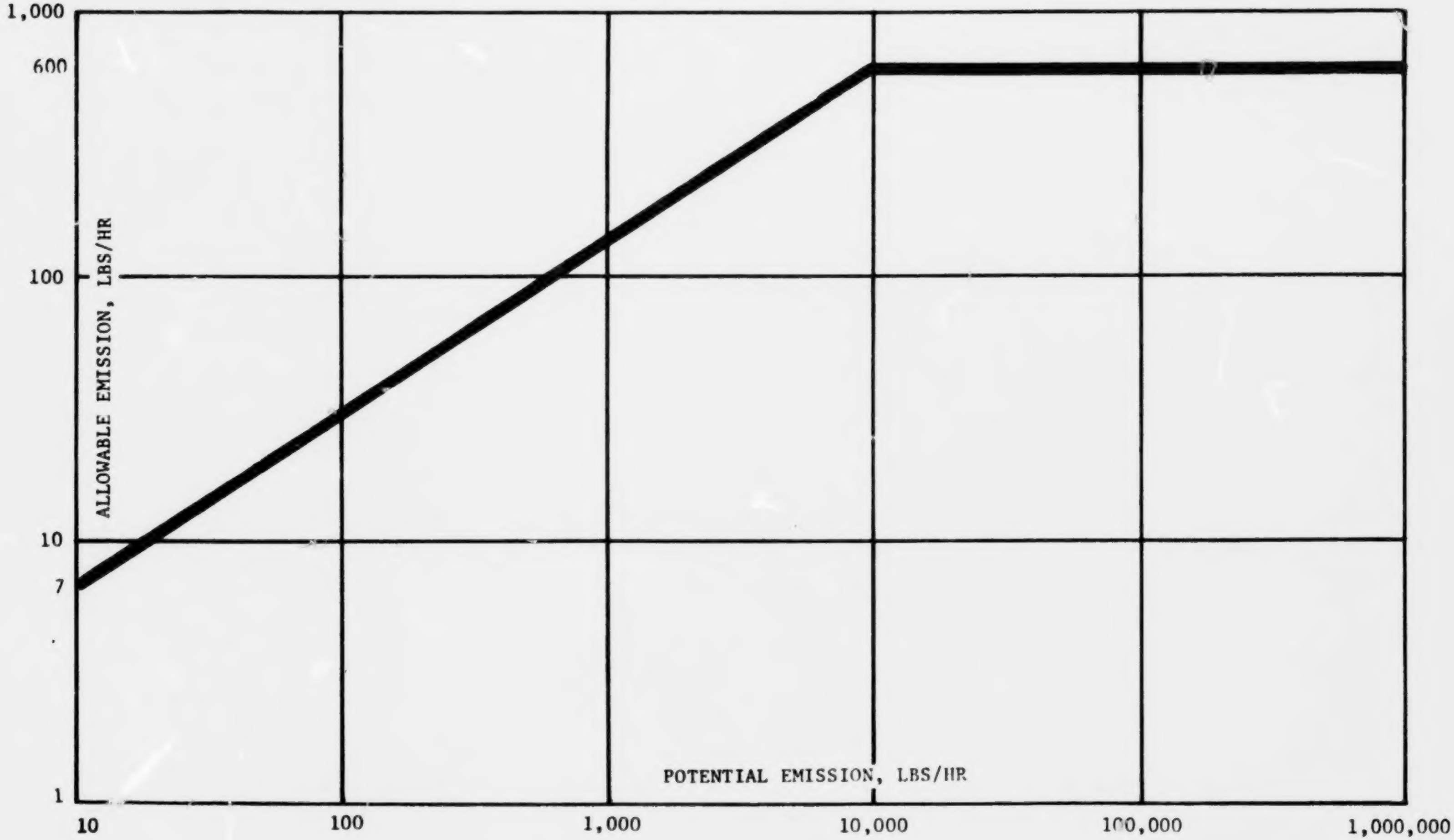

Figure 4-22. Philadelphia AQCR Potential Emission Curve - SWD Emission Standard No. 11

TABLE 4-16

STRUCTURE* OF THE PHILADELPHIA AQCR PARTICULATE CONTROL STRATEGIES

PARTICULATE AREA SOURCE CONTROL IN ( ) **

	IP	IIP	IIIP	IVP	VP	VIP
Delaware	2-0-7 (0)	2-0-7 (0)	3-8-10 (7)	3-5-10 (7)	10-9-11 (7)	11-8-10 (7)
New Jersey	1-11-1 (16)	1-11-1 (16)	4-10-1 (16)	4-5-1 (16)	4-10-1 (16)	4-10-1 (16)
Pennsylvania	9-7-9 (3)	5-4-4 (3)	5-4-4 (3)	5-5-4 (3)	5-4-4 (3)	5-5-5 (3)
Philadelphia	8-0-3 (0)	8-0-3 (0)	12-6-3 (21)	12-5-3 (21)	12-6-3 (21)	12-6-3 (21)

	VIIP	VIIIP	IXP
Delaware	3-8-10 (7)	3-8-10 (7)	6-5-5 (52)
New Jersey	4-10-1 (16)	4-10-1 (16)	6-5-5 (48)
Pennsylvania	5-4-4 (3)	9-7-9 (3)	6-5-5 (68)
Philadelphia	5-4-4 (0)	12-6-3 (21)	6-5-5 (65)

---

*Emission standards are listed in the order: Fuel Combustion (FC)-Industrial Process (IP)-Solid Waste Disposal (SWD). Thus 2-3-6 means FC standard number, IP standard number 3, and SWD standard number 3.

**For example, (94) means 94 percent of area source SO_2 emissions are controlled.

TABLE 4-16

STRUCTURE* OF THE PHILADELPHIA AQCR SO_2 CONTROL STRATEGIES

SO_2 AREA SOURCE CONTROL IN ( )** (Continued)

	IS	IIS	IIIS	IVS	VS
Delaware	0-0 (0)	7-5 (31)	7-5 (31)	7-5 (31)	7-5 (31)
New Jersey	7-8 (32)	7-6 (51)	6-8 (57)	3-8 (51)	6-8 (57)
Pennsylvania	0-0 (0)	8-7 (25)	1-4 (25)	1-4 (25)	1-4 (25)
Philadelphia	0-0 (0)	7-8 (56)	6-2 (62)	6-2 (62)	3-2 (56)

	VIS	VIIS	VIIIS	IXS
Delaware	8-6 (33)	9-7 (43)	4-3 (81)	7-5 (31)
New Jersey	3-8 (51)	3-8 (51)	4-3 (67)	3-8 (51)
Pennsylvania	1-4 (25)	1-4 (25)	4-3 (93)	1-4 (25)
Philadelphia	3-2 (56)	3-2 (56)	4-3 (75)	1-4 (17)

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*Emission standards are listed in the order: Fuel Combustion (FC)-Industrial Process (IP)-Solid Waste Disposal (SWD). Thus 2-3-6 means FC standard number, IP standard number 3, and SWD standard number 3.

**For example, (94) means 94 percent of area source SO_2 emissions are controlled.

TABLE 4-17

CONTROL STRATEGY SIMULATION RESULTS - PHILADELPHIA AQCR

The control strategy simulation for Philadelphia is still in progress; this table will be sent to all recipients of this report when it is available.

The regression lines used to calibrate the Atmospheric Diffusion Model are shown in Figures 4-23 and 4-24. Because of a shortage of air quality measurements, the diffusion model was calibrated for the winter season and the resulting regression constants were used to predict annual concentrations. Consequently, all points plotted on Figures 4-22 and 4-23 are for the winter seasons.

The emission standards considered for the Cincinnati AQCR are listed in Tables 4-21 and 4-22. A general description of the strategies run is given in Table 4-23, and a detailed description may be found in Table 4-24. The results of the control strategy simulations are listed in Table 4-25.

4.4 RECOMMENDATIONS

This demonstration involved exhaustive use of the RAPA program, and valuable experience was gained in handling the problems encountered in control strategy simulation. The following list summarizes this experience.

(a) The single most vexing problem was the existence of errors in the emission inventory. A tremendous quantity of data is involved, and an error in any single data item may be enough to cause the program to operate improperly. Data file error checks should be extended to point out, at an early stage in the analysis, where errors exist.

(b) Insofar as model credibility is concerned, the calibration of the Atmospheric Diffusion Model was the critical item. Particulates presented little difficulty, since an abundance of reasonably good air quality measurements are usually available. On the other hand, SO_2 data is scarce and unreliable. The SO_2 regression lines for all regions studied showed very high y-intercepts and low slopes. The high intercept would indicate a substantial background concentration which seems to invalidate the usual assumption of a zero background. This could be due to an incomplete emission inventory, SO_2 drifting in from other regions, or simply very poor measurement quality in the lower concentrations (below 30 $\mu g/m^3$) which tend to determine where the regression line crosses the y-axis.

(c) When dealing with very large volumes of data, some of the input formats were found to be cumbersome. The CØNREG update module, which was used extensively, is particularly bad in this respect. Inputs need to be redesigned to make things easier for the user.

TABLE 4-18

CINCINNATI AQCR EMISSION SUMMARY

POLLUTANT EMISSIONS BY SOURCE CATEGORY (TONS/YEAR)

CINCINNATI

06/01/70

POLLUTANT	POLITICAL JURISDICTION	FUEL COMBUSTION	INDUSTRIAL PROCESS	WASTE DISPOSAL	TOTAL
SO2	1	4922	0	398	5320
SO2	2	125158	0	0	125158
SO2	3	73	0	0	73
SO2	4	221031	8612	253	229896
	TOTAL	351184	8612	651	360447
PARTICULATE	1	3133	364	1531	5028
PARTICULATE	2	24527	73	474	25074
PARTICULATE	3	72	7444	511	8027
PARTICULATE	4	80180	16897	5510	102587
	TOTAL	107912	24778	8026	140716

TABLE 4-19

CINCINNATI AQCR EXISTING CONTROL LEVELS

NUMBER OF COMBUSTION SOURCES BY SULFUR CONTENT OF FUEL
CINCINNATI
06/01/70

POLITICAL JURISDICTION	UNDER 0.5%	OVER 0.5% UNDER 1%	OVER 1% UNDER 2%	OVER 2%
1	0	17	2	0
2	0	1	1	1
3	0	5	0	0
4	0	22	10	11
TOTAL	0	45	13	12

TABLE 4-20

CINCINNATI AQCR FUEL SULFUR LEVELS

NUMBER OF CONTROLLED AND UNCONTROLLED SOURCES BY SOURCE CATEGORY
CINCINNATI
06/01/79

POLITICAL JURISDICTION	COMBUSTION		INDUSTRIAL		WASTE	
	C	UC	C	UC	C	UC
1	15	4	1	4	5	2
2	3	0	1	0	0	2
3	6	0	2	10	0	1
4	35	9	8	8	3	7
TOTAL	59	13	12	22	8	12

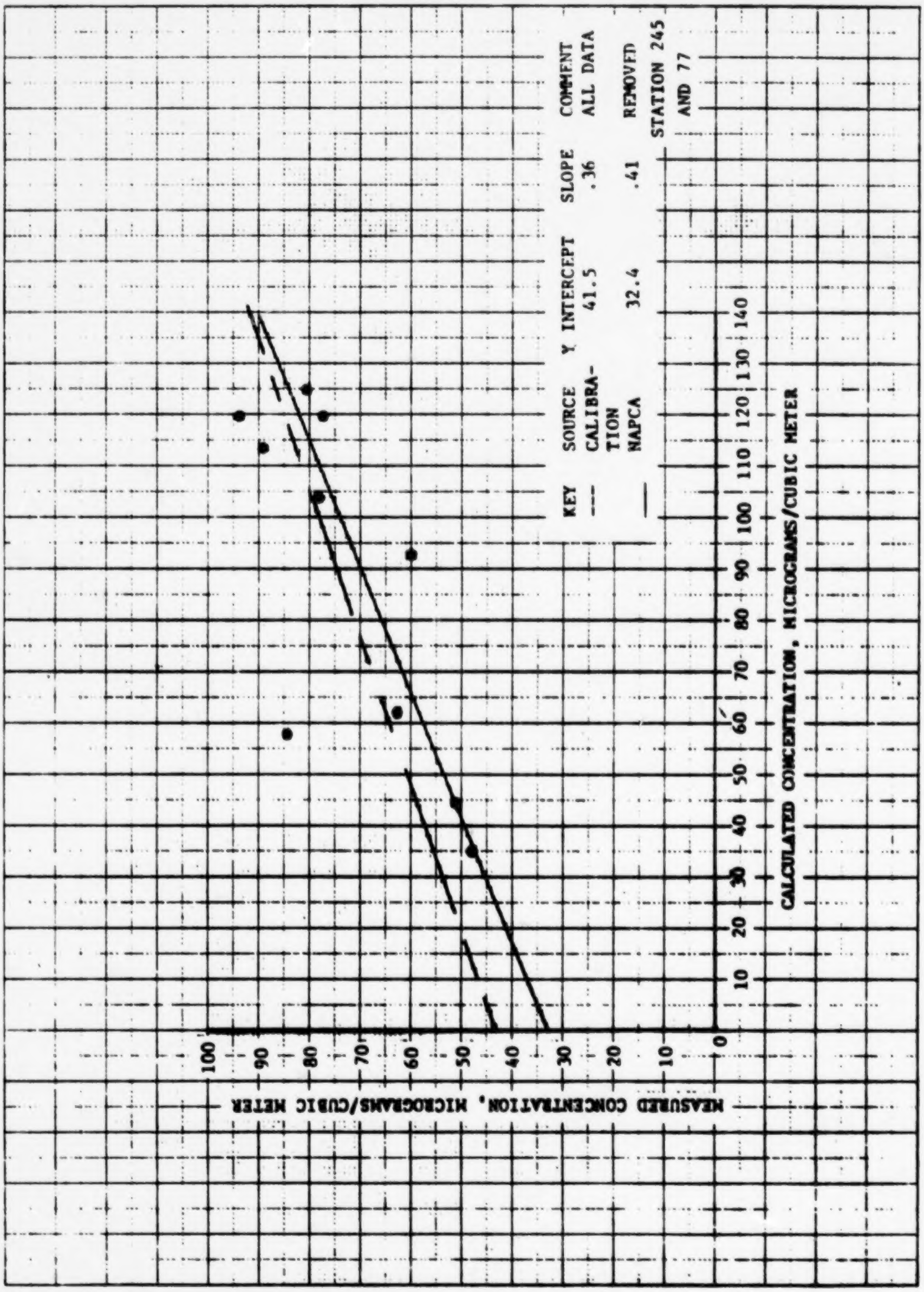

Figure 4-23. Cincinnati AQCR SO_2 Calibration

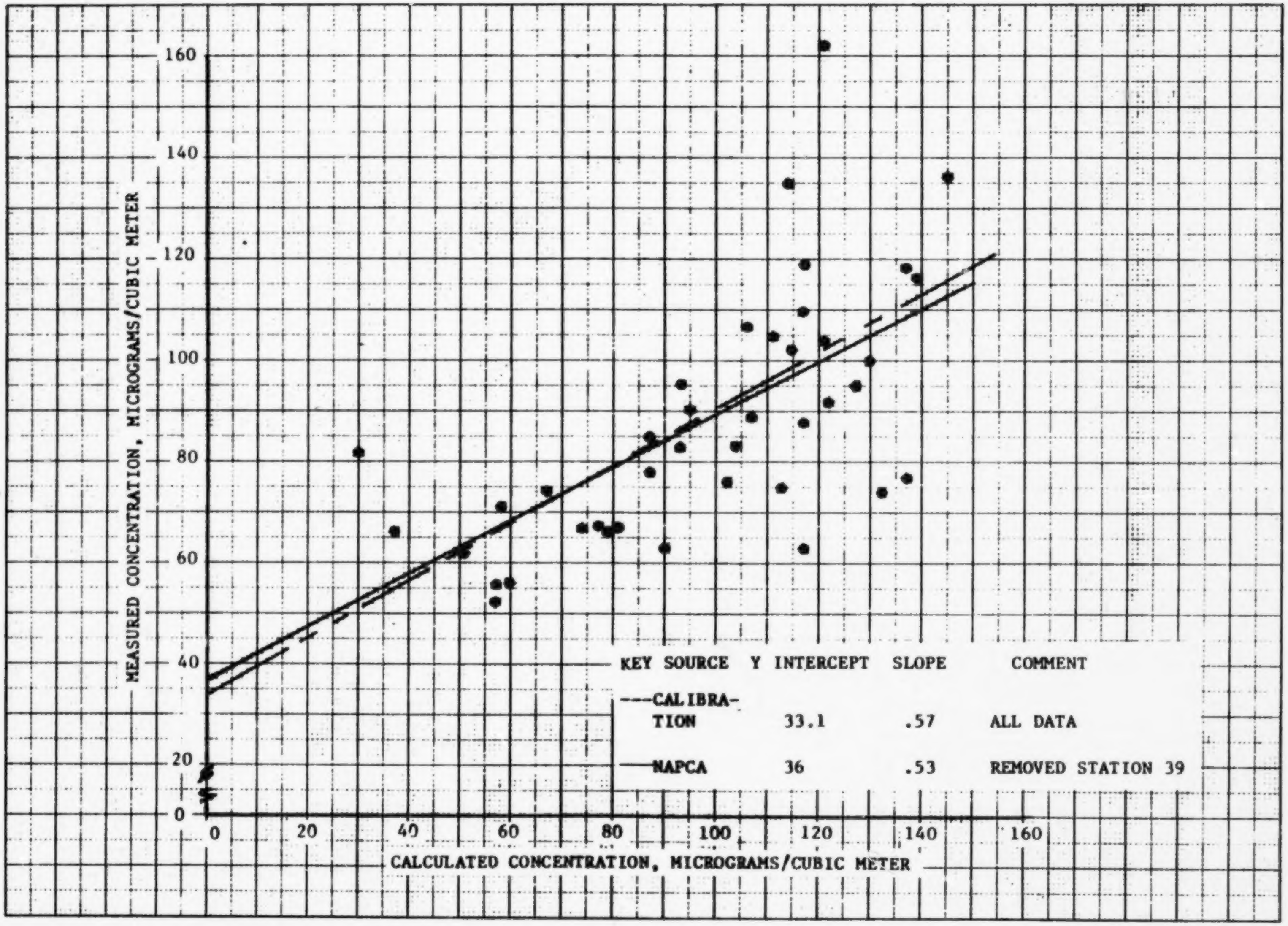

Figure 4-24. Cincinnati AQCR Particulate Calibration

TABLE 4-21

CINCINNATI AQCR SO_2 EMISSION STANDARDS

FUEL COMBUSTION

0. Null
1. Not Used
2. Fuel Sulfur Limitation - 0.8%
3. Fuel Sulfur Limitation - 0.3%
4. Maximum Technology
5. Maximum Technology - No Fuel Switch
6. Cincinnati - Equivalent Fuel Sulfur Limitation - 1.25%

INDUSTRIAL PROCESS

0. Null
1. Exhaust Gas Concentration Limit - 2000 ppm
2. Exhaust Gas Concentration Limit - 500 ppm
3. Maximum Technology
4. Potential Emission (Figure 4-25)

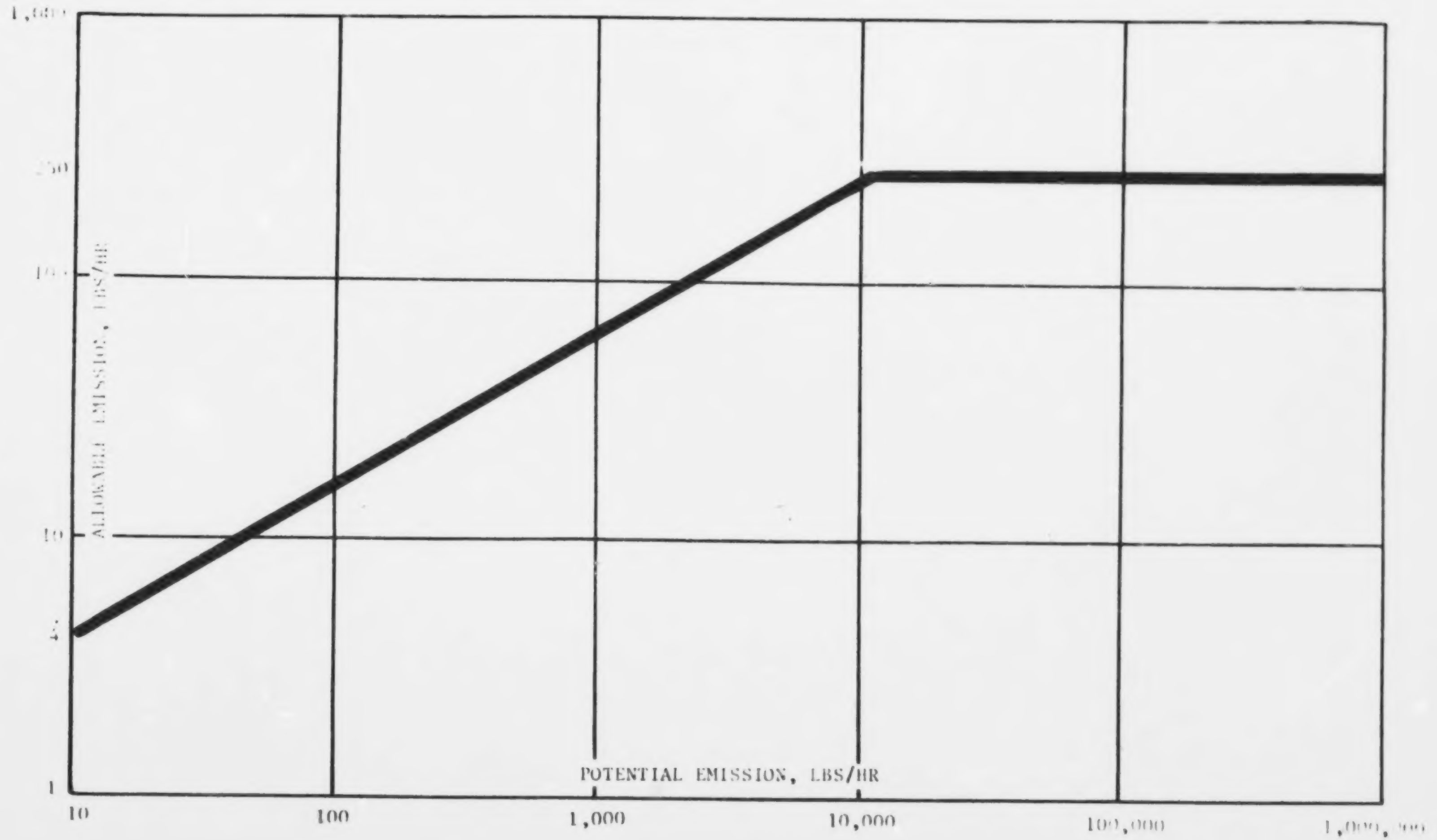

Figure 4-25. Cincinnati AQCR Potential Emissions Curve - NAPCA Recommendation - IP Emission Standard No. 4

TABLE 4-22

CINCINNATI AQCR PARTICULATE EMISSION STANDARDS

FUEL COMBUSTION

0. Null
1. Kentucky Heat Input Curve (Figure 4-26)
2. Indiana Heat Input Curve (Figure 4-27)
3. NAPCA Heat Input Curve (Figure 4-28)
4. Ironton Heat Input Curve (Figure 4-29)
5. Potential Emission (Figure 4-30)
6. Maximum Technology - no fuel switching
8. Cincinnati, .4 lb/10^3 lb exhaust gas

INDUSTRIAL PROCESSES

0. Null
1. Indiana Process Weight Curve (Figure 4-31)
2. San Francisco Process Weight Curve (not used)
3. San Francisco Process Weight Curve (Figure 4-31)
4. Potential Emissions Curve (Figure 4-30)
5. Maximum Technology
6. Los Angeles Process Weight Curve (Figure 4-19)
7. Cincinnati Effective Stack Height

SOLID WASTE DISPOSAL

0. Null
1. NAPCA, .05G/SCF
2. NAPCA, .2G/SCF
3. Cincinnati
4. Potential Emission (Figure 4-30)
5. Maximum Technology
6. Indiana
7. Kentucky
8. Sanitary Landfill, all emissions set to zero

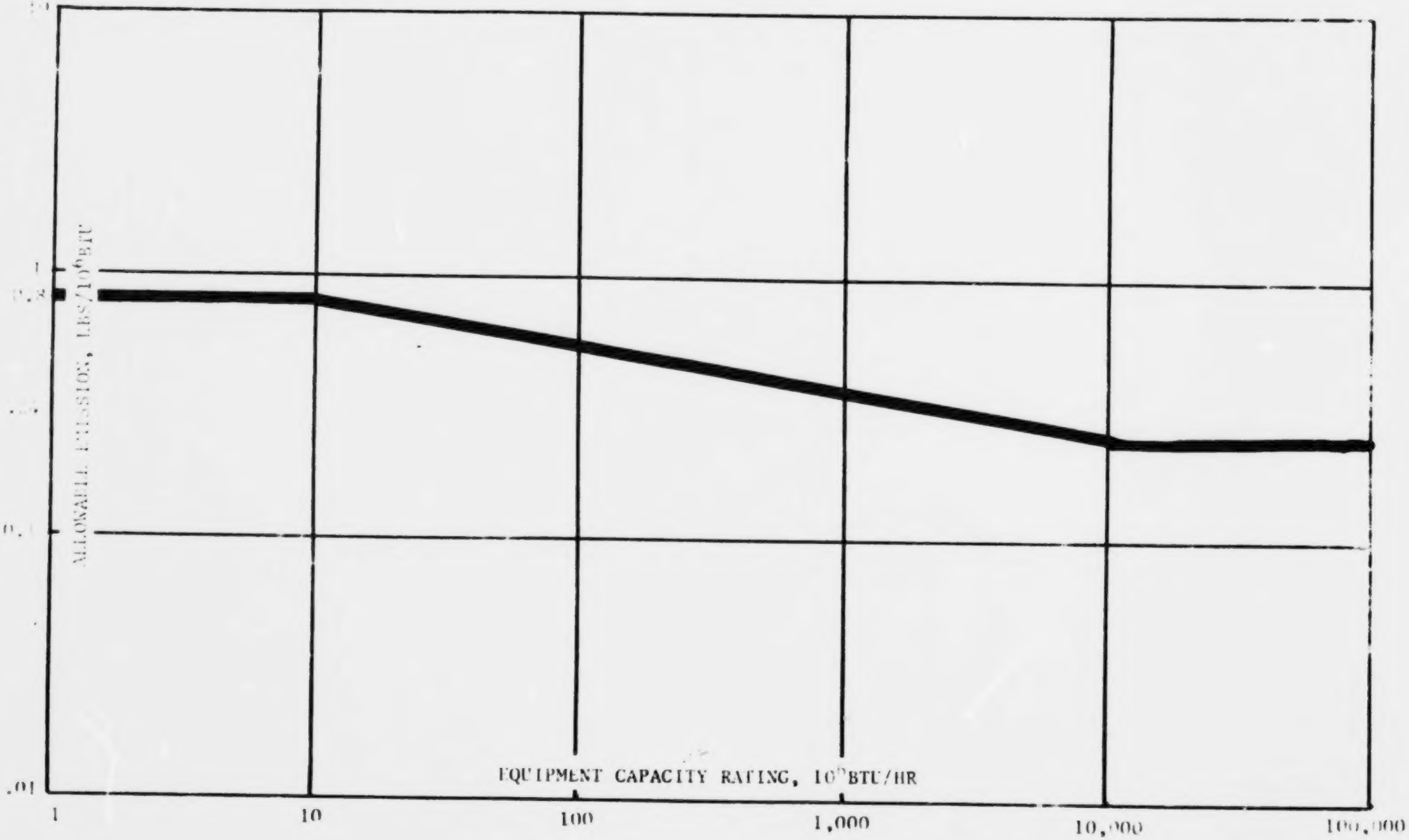

Figure 4-26. Cincinnati AQCR Heat Input Curve (Kentucky) - FC Emission Standard No. 1

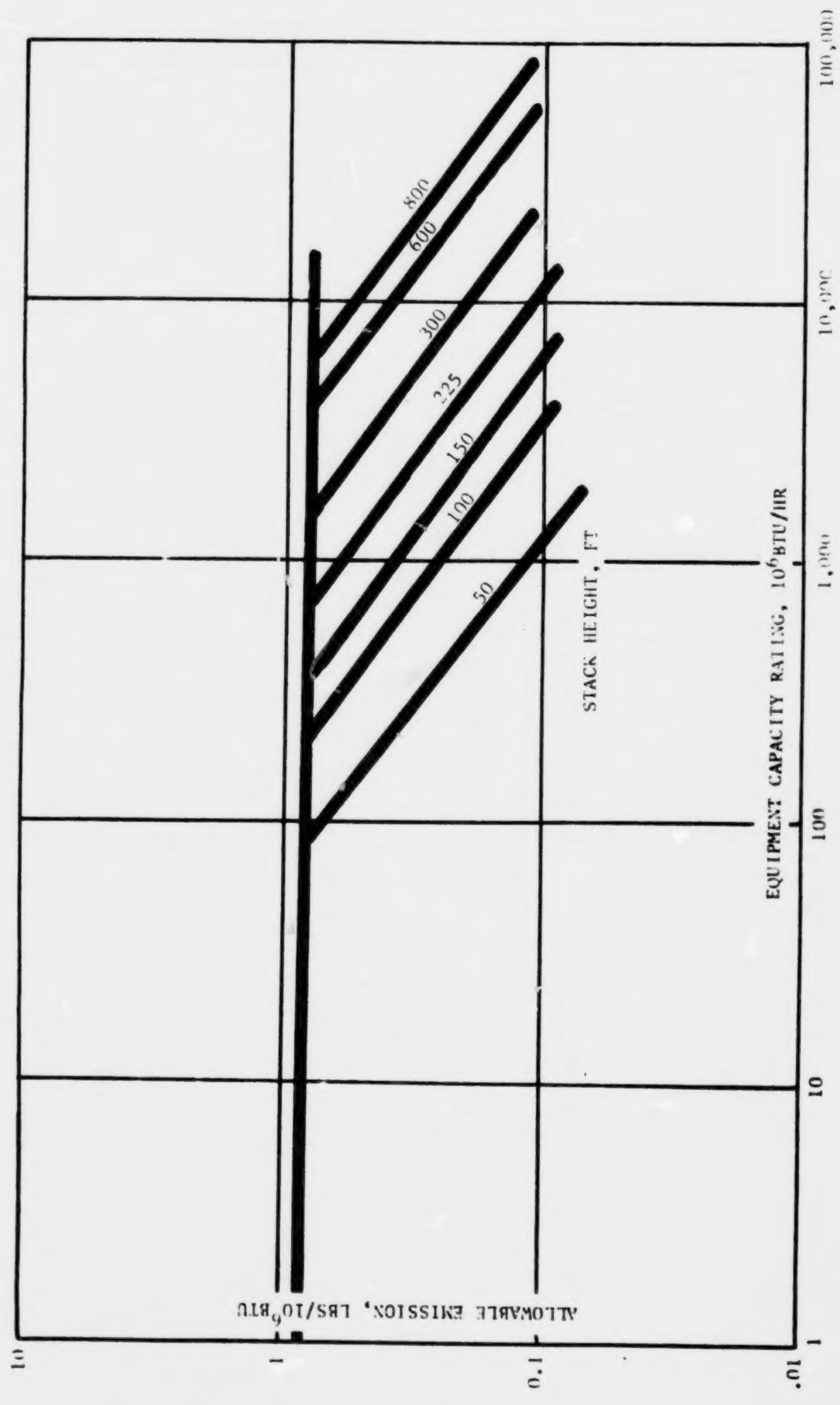

Figure 4-27. Cincinnati AQCR Heat Input Curve - Stack Height Parameter - FC Emission Standard No. 2

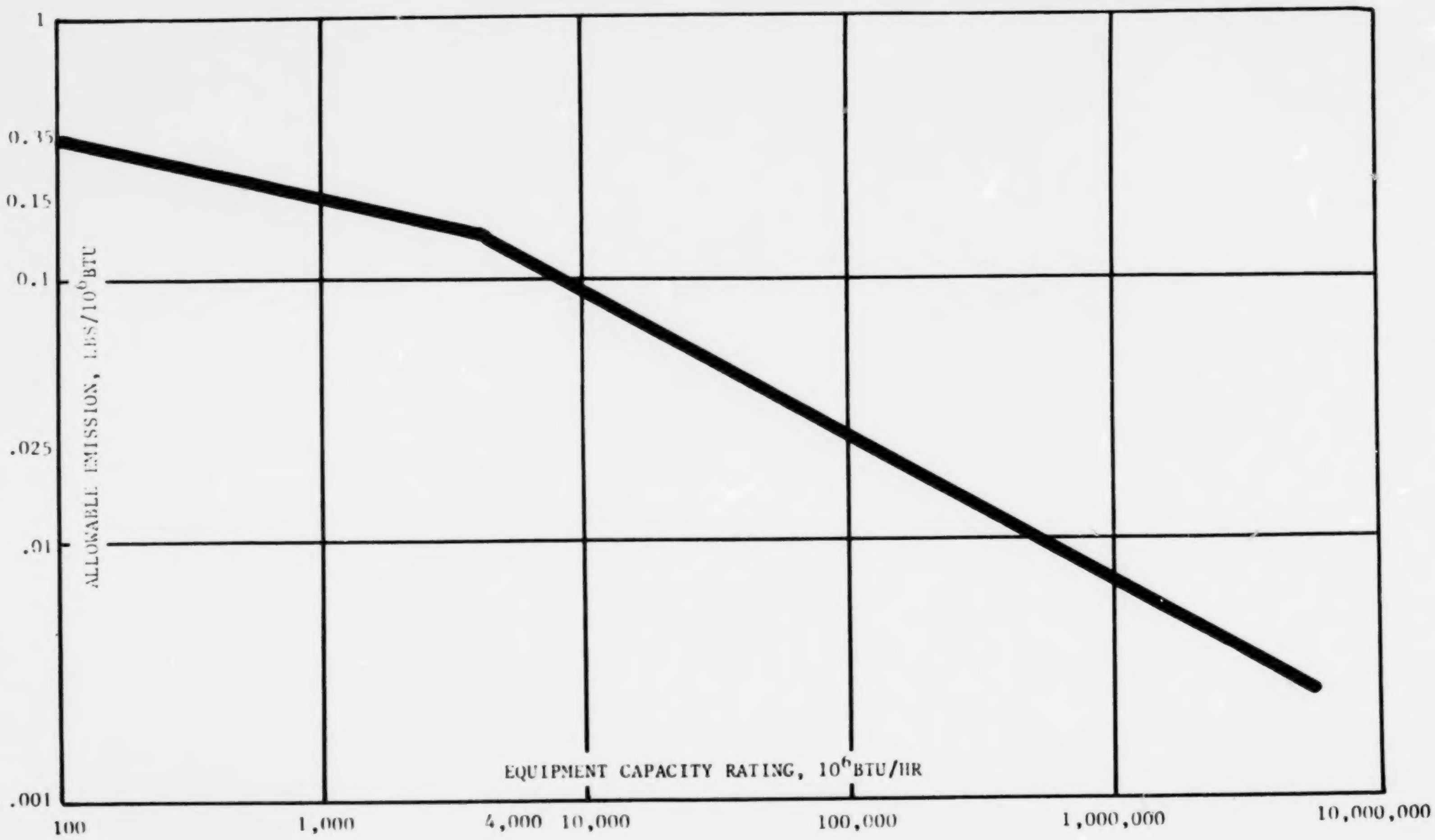

Figure 4-28. Cincinnati AQCR Heat Capacity Curve - NAPCA Recommendation - FC Emission Standard No. 3

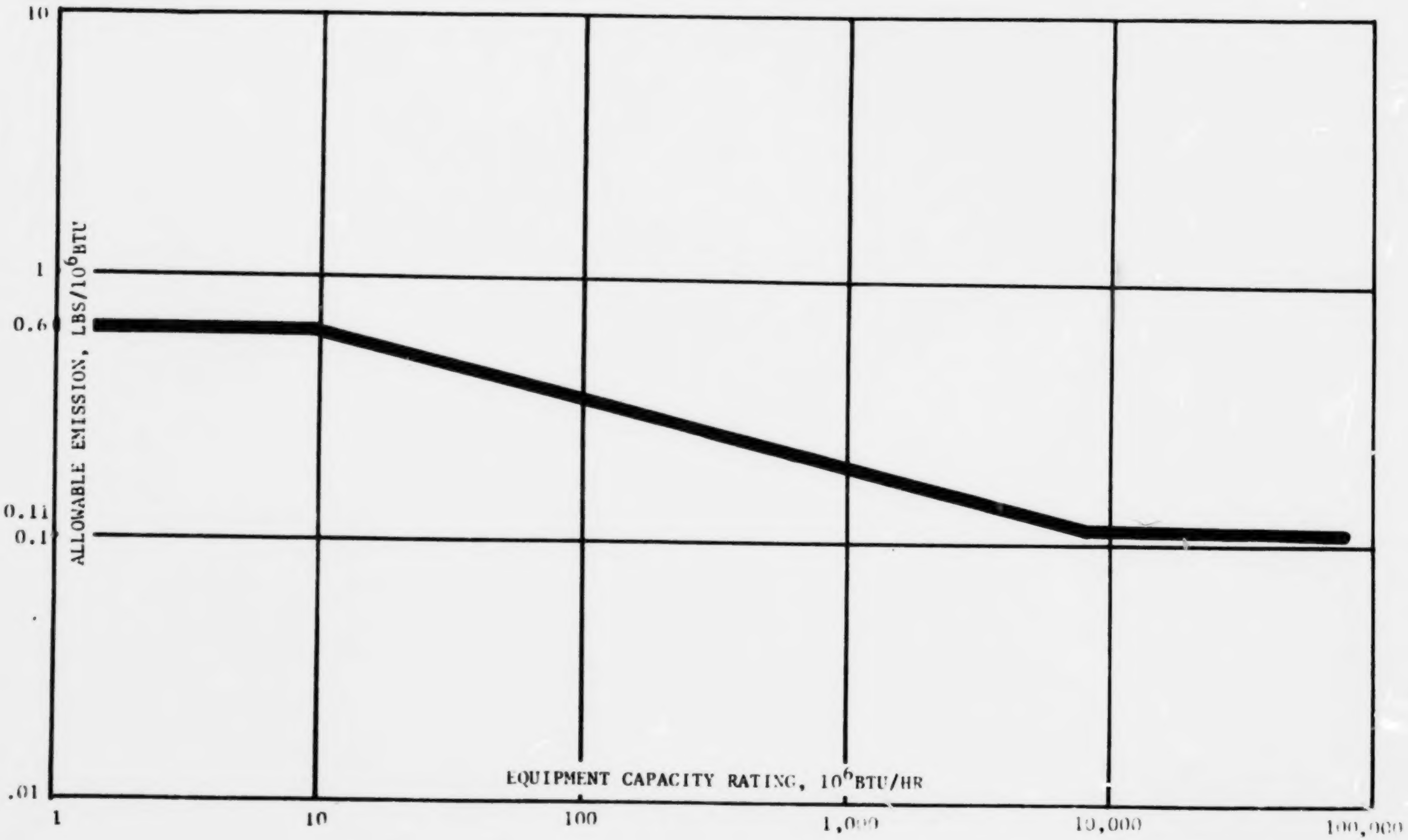

Figure 4-29. Cincinnati AQCR Heat Capacity Curve (Ironton) - FC Emission Standard No. 4

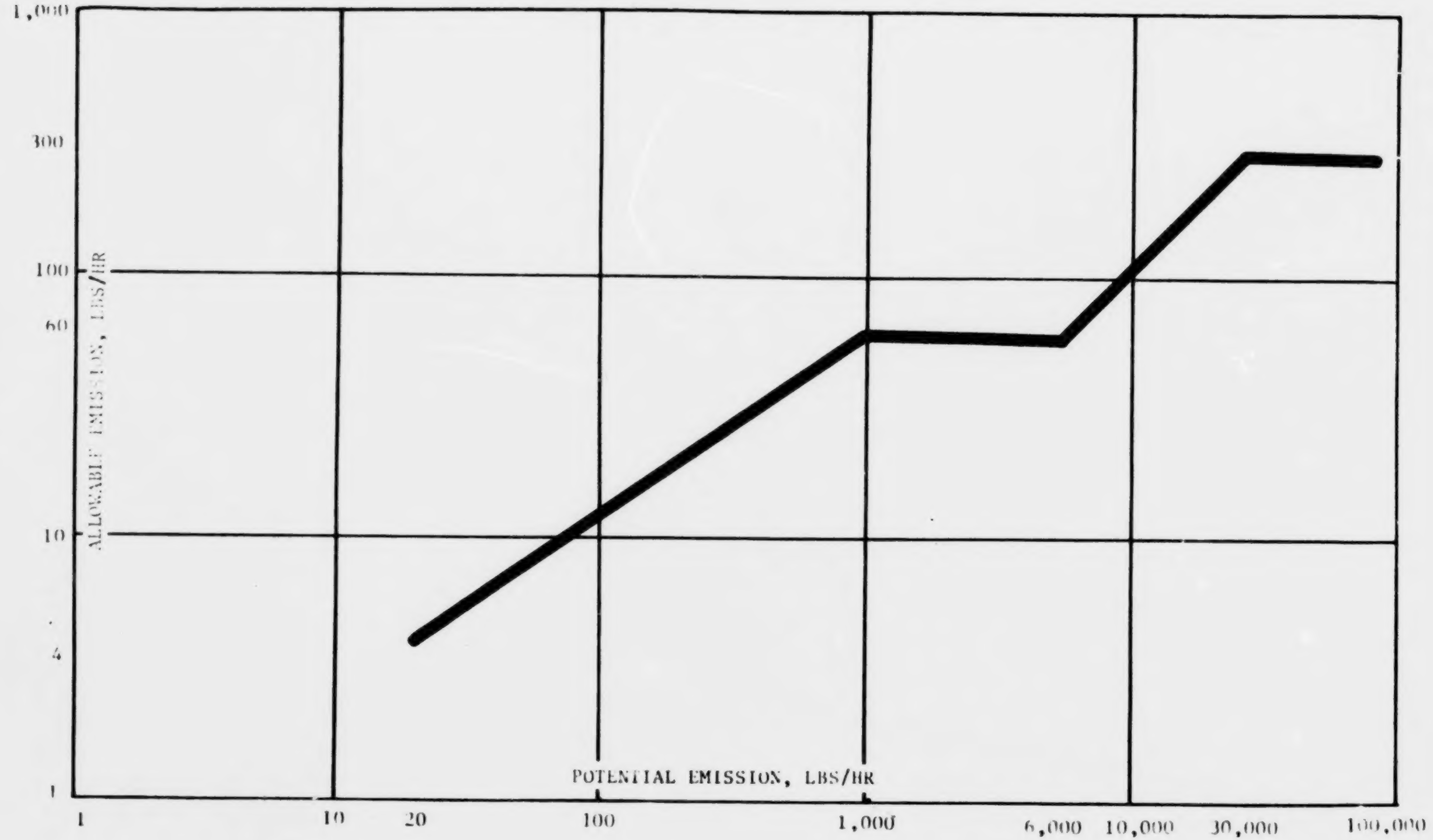

Figure 4-30. Cincinnati AQCR Potential Emission Curve - FC Emission Standard No. 5, IP No. 4 and SWD No. 4

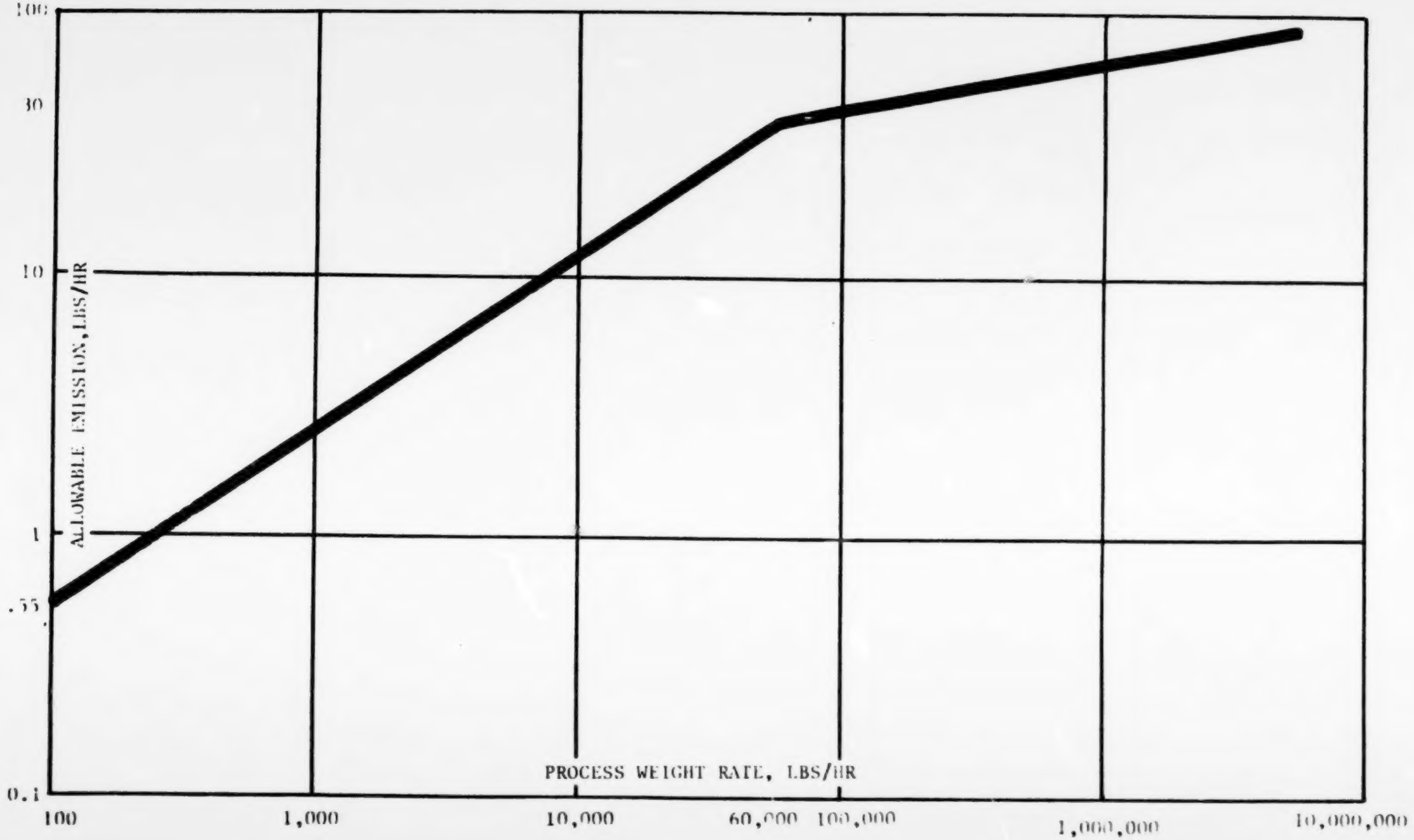

Figure 4-31. Cincinnati Process Weight Rate Curve (Bay Area APCD) - IP Emission Standard Nos. 1 and 3

TABLE 4-23

CINCINNATI AQCR CONTROL STRATEGIES

GROUP I - Strategies Composed of Existing Emission Standards

SO_2

1S - Existing standards - all jurisdictions
2S - Most stringent existing standards, applied throughout the region

Particulates

IP - Existing standards - all jurisdictions
2P 5P - Combinations of 2 most stringent FC standards, 2 most stringent IP standards and the single most stringent SWD standard (a total of four strategies; any given selection of standards is applied throughout the region)

GROUP II - Strategies Including Maximum Technology Emission Standards

SO_2

6S - Maximum Technology
7S - Maximum Technology - no fuel switching
9S - #7S with most stringent IP standard replacing IP maximum technology

Particulates

6P - Maximum Technology
7P - Maximum Technology - no fuel switching
8P - #7P with most stringent SWD standard replacing SWD Maximum Technology
9P - #8P with most stringent IP standard replacing IP Maximum Technology

GROUP III - Strategies Based on NAPCA Recommended Emission Standards

SO_2

10S 13S All combinations of FC standards 2 and 3, and IP standards 2 and 4, plus 9.3% reduction on area sources

Particulates

10P 17P All combinations of FC standards 2 and 3, and IP standards 2 and 4, plus 33% reduction on area sources

18P - Potential Emissions Curve

TABLE 4-24

STRUCTURE* OF THE CINCINNATI AQCR SO_2 STRATEGIES

SO_2 AREA SOURCE CONTROL IN ( )**

	1S	2S	6S	7S	9S
Cincinnati	6-1 (0)	6-1 (0)	4-3 (0)	5-3 (0)	5-1
Indiana	0-0 (0)	6-1 (0)	4-3 (0)	5-3	5-1
Kentucky	0-0 (0)	6-1 (0)	4-3 (0)	5-3	5-1
Ohio	6-1 (0)	6-1 (0)	4-3 (0)	5-3	5-1

	10S	11S	12S	13S
Cincinnati	2-2 (9.3)	2-4 (9.3)	3-2 (9.3)	3-4 (9.3)
Indiana	2-2 (9.3)	2-4 (9.3)	3-2 (9.3)	3-4 (9.3)
Kentucky	2-2 (9.3)	2-4 (9.3)	3-2 (9.3)	3-4 (9.3)
Ohio	2-2 (9.3)	2-4 (9.3)	3-2 (9.3)	3-4 (9.3)

NOTE: Strategies 3S, 4S and 5S are identical to 2S; 8S is the same as 7S.

	1P	2P	3P	4P	5P	6P
Cincinnati	8-7-3 (0)	1-1-3 (0)	1-7-3 (0)	8-1-3 (0)	8-7-3 (0)	6-5-8 (0)
Indiana	2-1-6 (0)	1-1-3 (0)	1-7-3 (0)	8-1-3 (0)	8-7-3 (0)	6-5-8 (0)
Kentucky	1-0-7 (0)	1-1-3 (0)	1-7-3 (0)	8-1-3 (0)	8-7-3 (0)	6-5-8 (0)
Ohio	8-7-3 (0)	1-1-3 (0)	1-7-3 (0)	8-1-3 (0)	8-7-3 (0)	6-5-8 (0)

	7P	8P	9P	10P	11P	12P
Cincinnati	7-5-5 (0)	7-5-3 (0)	7-1-3 (0)	3-3-1 (33)	3-3-2 (33)	3-6-1 (33)
Indiana	7-5-5 (0)	7-5-3 (0)	7-1-3 (0)	3-3-1 (33)	3-3-2 (33)	3-6-1 (33)
Kentucky	7-5-5 (0)	7-5-3 (0)	7-1-3 (0)	3-3-1 (33)	3-3-2 (33)	3-6-1 (33)
Ohio	7-5-5 (0)	7-5-3 (0)	7-1-3 (0)	3-3-1 (33)	3-3-2 (33)	3-6-1 (33)

	13P	14P	15P	16P	17P	18P
Cincinnati	3-6-2 (33)	4-3-1 (33)	4-3-2 (33)	4-6-1 (33)	4-6-2 (33)	5-4-4 (0)
Indiana	3-6-2 (33)	4-3-1 (33)	4-3-2 (33)	4-6-1 (33)	4-6-2 (33)	5-4-4 (0)
Kentucky	3-6-2 (33)	4-3-1 (33)	4-3-2 (33)	4-6-1 (33)	4-6-2 (33)	5-4-4 (0)
Ohio	3-6-2 (33)	4-3-1 (33)	4-3-2 (33)	4-6-1 (33)	4-6-2 (33)	5-4-4 (0)

*Emission standards are listed in the order: Fuel Combustion (FC)-Industrial Process (IP)-Solid Waste Disposal (SWD). Thus 2-3-6 means FC standard number 2, IP standard number 3, and SWD standard number 3.

**For example, (94) means 94 percent of area source SO_2 emissions are controlled.

TABLE 4-25

CONTROL STRATEGY SIMULATION RESULTS - CINCINNATI AQCR

Strategy Number	Total% Reduction In Emissions	New Point Source Emission Rate (Tons/day)	AREA SOURCES		POINT SOURCE CONTROL COST		Resulting Air Quality ($\mu g/m^3$) at Selected Receptor	
			% Reduction	New Emission Rate (tons/day)	$ x 10^6	$/ton Removed	#305	#315
SO_2 Strategies		(986.)		(369.4)			(97.)	(117.)
1S	22.8	676.5	--	369.4	2.0	18.	72.	74.
2S	38.9	458.2	--	369.4	3.9	20.	59.	73.
6S	72.2	7.2	--	369.4	55.5	155.	35.	41.
7S	51.1	293.9	--	369.4	8.9	35.	51.	75.
9S	51.1	293.9	--	369.4	8.9	35.	51.	75.
10S	69.9	73.4	9.3	335.0	4.3	47.	36.	68.
11S	69.9	73.4	9.3	335.0	4.3	47.	36.	68.
12S	66.1	123.9	9.3	335.0	7.1	23.	40.	52.
13S	66.1	123.9	9.3	335.0	7.1	23.	40.	52.
							#264	#269
Particulate Strategies (386.)				(212.6)			(134.8)	(141.8)
1P	53.4	130.2	--	212.6	1.7	18.	80.0	83.7
2P	64.5	75.7	--	212.6	3.0	27.	79.5	82.9
3P	64.0	77.0	--	212.6	2.7	24.	79.0	82.6
4P	58.0	106.5	--	212.6	2.2	22.	79.8	83.3
5P	58.0	108.7	--	212.6	1.9	18.	79.3	83.0
6P	79.5	5.7	--	212.6	20.0	150.	63.5	66.5
7P	75.7	23.8	--	212.6	6.1	44.	67.7	70.9
8P	74.0	28.7	--	212.6	5.9	45.	71.0	75.7
9P	73.8	30.2	--	212.6	5.6	43.	71.5	76.6
10P	80.4	1.2	33.	142.0	7.0	54.	65.1	68.5

NOTE: Existing conditions are enclosed in parentheses.

TABLE 4-25

CONTROL STRATEGY SIMULATION RESULTS - CINCINNATI AQCR (Continued)

Strategy Number	Total % Reduction In Emission	New Point Source Emission Rate (Tons/day)	AREA SOURCES		POINT SOURCE CONTROL COST		Resulting Air Quality ($\mu g/m^3$) at Selected Receptor	
			% Reduction	New Emission Rate (tons/day)	$ x 10^6	$/ton Removed	#264	#269
Particulate Strategies		(386.)2		$(212.6)^2$			(134.8)	(141.8)
11P	80.2	2.5	33.0	142.4	6.9	53.	65.3	68.9
12P	80.6	1.0	33.0	142.4	7.2	56.	64.9	68.3
13P	80.4	1.5	33.0	142.4	7.0	54.	65.1	68.8
14P	78.7	8.7	33.0	142.4	3.7	29.	58.8	61.5
15P	78.5	9.6	33.0	142.4	3.5	28.	64.5	67.8
16P	78.9	7.7	33.0	142.4	3.9	30.	64.2	67.2
17P	78.7	10.8	33.0	142.4	3.7	29.	64.3	67.6
18P	70.9	45.8	--	212.6	3.6	29.	70.6	73.9

(d) Area source control should be at the political jurisdiction level, rather than only regionwide as at present. In addition, certain area source control strategies should be handled automatically by the model, e.g., prohibition of soft coal for residential space heating.

4.5 RAPA EXPERIMENTS

Although the RAPA experiments task was deleted during the contract year, a generalized experiment plan was made for the Diffusion Model. This plan, described below, was the necessary first step in the overall experiments task. The overall task consisted of the following:

(1) Determine the output sensitivity to errors and uncertainties of selected input variables for the three RAPA models: Diffusion Model, Control Cost Model and Control Strategies Model.

(2) Using (1) as a basis, develop the output sensitivity of the Cost and Control Strategies Models with respect to the input variables for the Diffusion Model.

(3) Using an allowable output variability (determined by NAPCA-TRW agreement), determine the allowable input uncertainty for each model.

The sensitivity analysis of the Diffusion Model was concerned with the following input data:

- Source data (emission rates, source location, stack parameters and area source size)
- Ambient conditions (temperature and pressure)
- Pollutant half-life values
- Relative frequency of occurrence for each combination of the 16 wind direction, 5 stability, and 6 wind speed categories
- Representation of the standard deviation of the plume concentration distribution in the cross-plume direction
- Size and shape of the receptor pattern
- Statistical parameters for short-term calculations (e.g., geometric standard deviation and averaging times).

As a result of the myriad of variables involved, a completely generalized sensitivity study was rejected from both the real time and computer cost standpoints. Instead a "typical" region would be selected to provide a baseline order of magnitude for the variables of interest. The sensitivity study could then be made of the baseline variables.

The input variables to be tested (described above) may be divided into the following categories:

(1) Variables for optimum size determination

- Receptor size and shape
- Area source size

(2) Variables for numerical-statistical analysis

- Emission rates
- Stack parameters
- Pollutant half-life values
- Relative frequency of occurrence for each meteorological condition
- Source locations
- Ambient conditions
- Standard deviation of plume concentration

(3) Variables for analytical study

- Statistical parameters

Investigation of the sensitivity of both the receptor system and the area source size may be accomplished by systematically varying the configuration and assessing the changes in the isopleth contour plots. Since previous experience with the Air Quality Control Region study has shown that the rectangular grid pattern produces a more consistent regionwide isopleth map than a radial pattern, it remains only to determine the optimum density of the rectangular grid pattern. The experiment, then, would consist of varying the grid density and observing the change in air quality resolution of the contour maps. The optimum spacing would be the maximum receptor spacing which resulted in a minimum degradation of the isopleth map resolution. In a similar manner the optimum area source size could be determined.

The sensitivity of the variables in category 2 could be determined by conducting a statistical analysis relating the change in the input variables to the mean and standard deviation of the error in the set of air quality output values. This procedure, described in detail by Hilst[2], would consist of the following sequence of operations:

(1) Run the baseline case using the optimum grid receptor spacing and area source size.

(2) Perform iterative solutions of the model, varying each of the input parameters in turn in a stepwise fashion.

(3) Construct the mean and standard deviation of the error of each solution with respect to the baseline solution.

The results of this procedure would be presented in the form of plots, for each input variable, showing the relationship between the error in the input variable (both random and systematic) and the variation of the output air quality.

The sensitivity of the statistical parameters (category 3) is classified as a purely analytical problem, since several authors[3,4] have investigated the fundamental "log-normal" approach of Larsen[5]. The user-oriented results of these investigations may be obtained from a straightforward analysis of the work cited.

4.6 BENEFIT MODEL

The RAPA Benefit Model task was deleted from the Phase II activity, because of a lack of suitable damage models and the need to divert resources into the regional analysis task. However, some preliminary work which was done in this area is described below.

Reasonable qualitative estimates of air pollution control costs and the effect of these expenditures on air quality are available. In general, this cost relationship is non-linear, with the rate of air quality improvement diminishing as investment in air pollution control increases. The relationship between air pollution damage expenses and air quality levels is less well understood and may be linear or also non-linear, with the rate of decrease in pollution damage remaining constant or diminishing for each additional increase in air quality levels.

Both of these cost relationships must be analyzed simultaneously to determine society's total expenses in air pollution control and air pollution damage. The results of this analysis will indicate the atmospheric management strategy which balances the rate of air pollution control expenditures with the rate of air quality benefits.

As has been described above, the Control Strategy Model contains the emission standards which, in most cases, require the application of the control measures identified by the Control Cost Model. The outputs illustrate the actual control measures selected for each source, together with the annual costs and data on the degree to which emissions have been controlled. The Model also displays the conversion from emission reductions in annual and expected maximum ground-level concentrations for selected emission standards. A regional summary culminates the Program's output and includes a description, by political jurisdiction, of regional costs, regional emission rates, number of sources affected by the control strategy, and figures of merit for each strategy (e.g., cost per ton of pollutant removed, cost per microgram/cubic meter of reduced ground-level concentrations). In addition, a ground-level concentration contour map is provided which highlights the effect of the control strategy.

The ability of the Program to determine the consequences of specific emission regulations on the basis of computer simulations provides the user with a means for objectively comparing pollution control strategies before they are put into effect. The Program assists the local air pollution control official in selecting, from the large number of control strategy alternatives available to a region, that strategy which will be most effective in solving the region's air pollution problems.

Ideally, the regional air pollution control official is expected to strike a balance between regional expenditures on air pollution control and regional savings resulting from increasing air quality levels. Information on air quality savings (or, conversely, the costs of air pollution damage) is much scarcer than cost data on air pollution control discussed previously. However, for illustrative purposes the format of a typical cost-benefit relationship was presented in Figure 2-11. The cost function states the added industrial and power generating costs associated with investments in a maximum sulfur-oxide and particulate matter control technology (Figure 2-5). This strategy has been applied, via the Program, to 75% of the major point sources of pollution in the St. Louis Region (the line between zero added investment and the $55 million maximum added control costs implies a continuous function through intermediate degrees of control). The benefit function states the added per capita cost resulting from the soiling damages caused by particulate matter pollution. This soiling damage function was previously estimated for the Washington, D.C. area in 1968 by R. Wilson and D. Minnotte of the National Air Pollution Control Administration.

It is clear from this type of a diagram that the savings from reducing particulate matter soiling damage alone almost offset the combined costs of maximum sulfur oxide and particulate matter control. If the added savings of human and animal health, increased yield in vegetation, added visability, and the all encompassing esthetic values are added to the savings of soiling damage, then regional air quality savings far outstrip industrial air pollution control costs.

Incidentally, the "optimum" management strategy is represented by a point which balances the marginal utility of additional air pollution control expenditures with additional benefits of air quality. For example, after a point has been reached where the total control cost equals total air quality savings, an additional dollar of control investments may return more than one dollar in air quality benefits.

Unfortunately, quantitative assessemnts of the benefit-cost functions remain in the future. The control costs and regional economic estimates of these industrial investments are but a first step in the development of these functions. As soon as additional air pollution damage data are evaluated, estimates of benefits will be made alongside the estimates of control costs, resulting in more quantitative appriasals of atmospheric resource management strategies.

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2. Hilst, Glenn R (1969): "The Sensitivity of Air Quality Predictions to Input Errors and Uncertainties," presented at NAPCA Symposium, October 27-30, 1969, Chapel Hill, North Carolina.

3. Yao, J.Z., (1969), "Review of Larsen's 'A New Mathematical Model of Air Pollution Concentration Averaging Time and Frequency'," TRW Interoffice Correspondence, 18 August 1969, to M. Frankel.

4. Singpurmulla, N.D., "The Rate of the Lognormal and the Extreme-Value Distributions in a Study of Air Pollutant Concentrations," Serial R-1, December 15, 1969, under NASA grant NGL-09-010-030.

5. Larsen, R.I., "A New Mathematical Model of Air Pollution Concentration Averaging Time and Frequency", APCA J. Vol. 19, No. 1, January 1969.

5.0 ECONOMIC MODELING

5.1 INTRODUCTION

Rapid population growth, high levels of consumption, technological innovation, and an increasing rate of use of natural resources have led to environmental deterioration in the United States. Severe effects have been noted on the vegetation, land, air, and water that sustain biotic and industrial processes. Clean air and clean water have become scarce.

Pure or relatively unpolluted air is no longer a free resource in our society; money outlays must be made to move pollutant sources to areas where air is relatively cleaner or to trap pollutants before they escape into the air. In the past, those who were responsible for pollution did not bear the external costs generated through environmental deterioration. Industries emitting harmful pollutants into the air during the production process have not borne the cost of the resulting social damage and thus have enjoyed the advantages of lower production costs.

With the recent enactment of effective Federal air pollution abatement legislation, business and industry are now being required to control the amount of pollutants that they discharge into the air. To industry, this requirement means that production costs for a given amount of output will be increased in proportion to the required investments in air pollution control equipment. Thus, certain industries and regions that have in the past enjoyed the economic advantage of low-cost production may face some degree of economic decline due to the requirements of pollution abatement.

Offsetting economic effects from air quality control strategies will take two forms in such regions. First, there will be an increased demand for the products of industries that produce pollution control equipment and low-pollutant fuels, leading to increased output, employment, and income in those sectors. Second, a variety of general economic benefits will result from pollution abatement. These gains include increased labor productivity, reduced health expenditures, reduced outlays on physical maintenance of homes and plants, and savings in agricultural production activities. These gains will lead to increased consumption, output, employment and income, and will begin to offset the economic reductions resulting from the pollution control costs.

Air pollution control thus leads to changes in economic output, labor markets, and availability of capital, as well as redistributions within the entire economy. Implementation of air quality control programs will also have a variety of other effects, such as tax-base impacts on communities and changes in land-use and industrialization patterns in various regions.

The development of a regional economic model under the RAPA contract is designed to provide pollution control policy-makers with assessments of the effects of various pollution strategies. The following types of information are provided for public-policy analysis:

(a) Regional economic changes expected to result from enforcement of varying control standards upon high-emission industries.

(b) Regional economic effects expected to result from reduction of industrial damage and growth in air pollution equipment industries.

(c) Data on regional output, investment, employment, income, and consumption under various air quality standards.

(d) Fiscal effects of regional implementation of air quality control programs, including tax base impacts of economic change and the effects of tax credits upon economic change and the rate of achievement of emission standards in terms of the implementation plan.

(e) Statistical indicators, including industry-specific ratios of air pollution control costs to total investment and to value added in production.

5.2 THE ROLE OF THE ECONOMIC MODEL IN RAPA

With alternative abatement strategies at hand, not only the associated direct control costs of each strategy but also the measurement of corresponding economic impacts of chain reactions throughout the region will provide useful information to the different levels of decision-making for the implementation plan. The changes in major economic indicators are the focal point in the analysis. For example, given the fact that control costs directly increase production costs, then as profits decline, new investments may be reduced. Demands for products from other firms and industries and for labor are thus affected, resulting in changes in regional income and unemployment levels. On the other hand, those industries (and individuals) that originally suffer damages and pay additional costs due to the polluted air may start to enjoy some economic gains from air pollution control. Meanwhile, fuel substitutions might occur in some firms, especially in the electric power industry. The demands for pollution control equipment will also result in a new series of product demands within and outside the area. Basically, there is a shift in the economic structure of the region.

Benefits of cleaner air not only cost more to the high-emission industries, but also result in an initial decline in regional employment and income. Decision-making is required in the public sector, based on an economic analysis of the region. The trade-off between the total costs and benefits to the entire regional community may depend upon information concerning the measured economic impact of alternative control strategies. Tax policy and techniques for financing and distributing air pollution control costs can be determined through a full examination of total impact within the community.

During Phase I of the RAPA program, a Regional Economic Model of the effects of air pollution control in the St. Louis metropolitan region was developed on the basis of time series data.* During Phase II of the program, the aim has been to focus on the development of a model which can be applied to major U.S. cities. This has resulted in estimation of a cross-sectional economic model for the 31 largest Standard Metropolitan Statistical Areas (SMSA's) across the country. The roles of Keynsian theory and regional export base theory have been the same as for the St. Louis time series model. However, the entire equation system has been reformulated in order to integrate the cross-sectional structure into the model.

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*CONSAD Research Corporation, Structure Requirements of an Econometric Model of St. Louis SMSA, prepared for TRW Systems, January, 1969, and Final Version of the CONSAD Regional Econometric Model as Applied to St. Louis SMSA for Air Pollution Control Analysis, May, 1969.

The use of economic models for examining policy alternatives has been increasing.* Such a model is illustrated in Figure 5-1. The inputs and outputs of the present model are given in Figure 5-2. Two types of input information are necessary:

(a) Basic regional economic variables, including

  (1) Regional income and consumption of the previous year;

  (2) Output and capital stocks of the previous year by industry;

  (3) Population and migration data; and

  (4) Government expenditure.

(b) Policy instrument variables (direct costs and benefits of air pollution control), including

  (1) Annual cost requirements of air pollution control investment for each control strategy by industry; and

  (2) Direct economic gains of industry and residents from each control strategy.

Given these sets of input, the model provides estimates of regional economic variables and changes in these variables generated by each type of control strategy:

(a) Output of the model;

  (1) Regional income and consumption,

  (2) Investments and capital stock by industry,

  (3) Value added by industry,

  (4) Employment by industry,

  (5) Regional unemployment and labor force.

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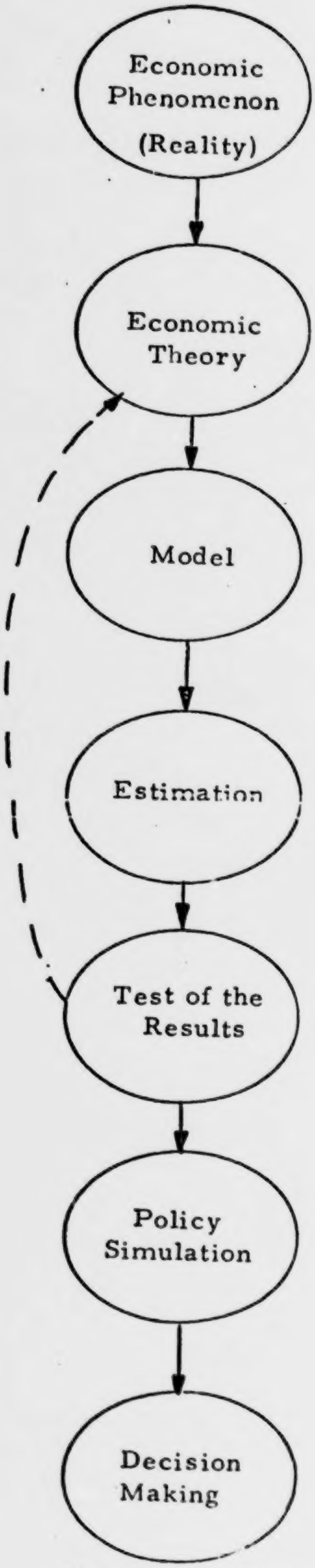
*As a recent example of policy simulations with a complicated Brookings model, see Gary Fromm and Paul Taubman, Policy Simulation with an Econometric Model, The Brookings Institute, 1968.


Economic Phenomenon (Reality)
Economic Theory
Model
Estimation
Test of the Results
Policy Simulation
Decision Making

Assumptions and simplification of relations between key variables

Mathematical formulation of equation system includes:

endogeneous (output)
exogeneous (input)

Data collection, selecting statistical methods, and computations

Test of economic theory by statistical methods to determine level of acceptability

Impact simulation of different strategies

Information from results of policy simulations

Figure 5-1. Economic Model for Policy Simulations

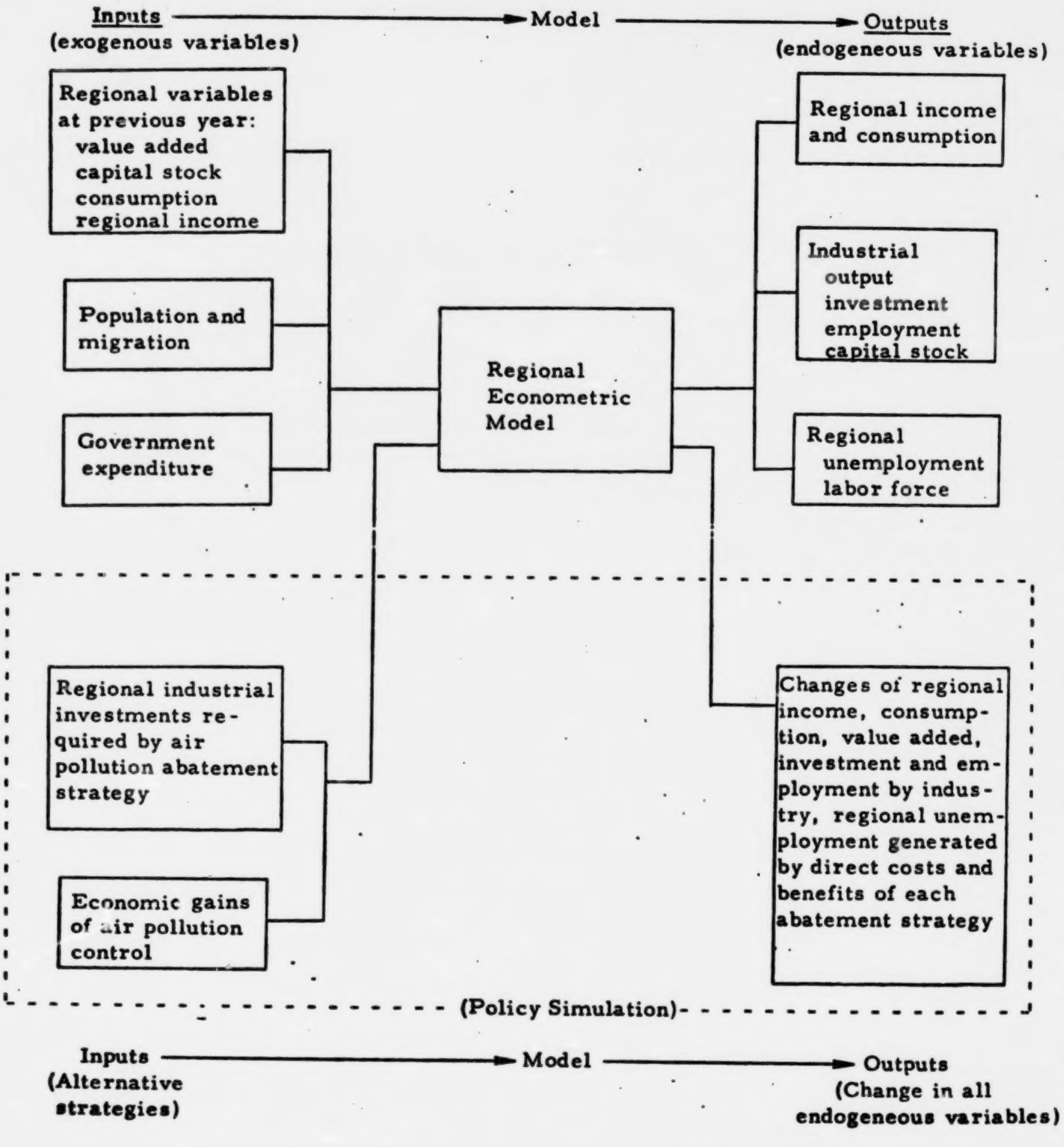

Inputs
(exogenous variables)
Model
Outputs
(endogeneous variables)
Regional variables at previous year: value added capital stock consumption regional income
Population and migration
Government expenditure
Regional Econometric Model
Regional income and consumption
Industrial output investment employment capital stock
Regional unemployment labor force
Regional industrial investments required by air pollution abatement strategy
Economic gains of air pollution control
Changes of regional income, consumption, value added, investment and employment by industry, regional unemployment generated by direct costs and benefits of each abatement strategy
(Policy Simulation)
Inputs
(Alternative strategies)
Model
Outputs
(Change in all endogeneous variables)

Figure 5-2. Inputs and Outputs of Regional Economic Model

(b) Output of policy simulation;

Changes of all output variables as indirect economic impacts generated by direct costs and benefits from each control strategy.

5.3 CITIES INCLUDED IN THE MODEL

The RAPA-II Regional Economic Model has the ability to be applied to major cities throughout the country. The 31 largest SMSA's have been chosen for the present model. The size of the city, in general, is reported to be consistent with the degree of the severity of the air pollution problem.* In the following table, rank according to size of population and severity of air pollution is listed for each of the cities studied.

5.4 GENERAL STRUCTURE OF THE MODEL

At the regional level, single equation cross-sectional models are not uncommon, since the data are usually unavailable in continuous time-series form. However, at the time this effort was initiated, an economy-wide cross-sectional model was more a conceptual work than a well-developed operational model,** owing to the fact that growth models at the national level deal with time series data (unless international comparisons of the growth patterns are the focus). Before the detailed equation system of a cross-sectional regional model is presented, the theoretical framework of a metropolitan economy needs further elaboration to provide a useful guide to the model formulation.

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*Middleton, John,T., "Air Pollution: How 65 Metropolitan Areas Rank in Severity," Nation's Cities, Vol. 5, No. 8, August, 1967, pp. 8-11.

**Some Keynsian-type cross-sectional models have been suggested in Carl F. Christ, Econometric Models and Methods, New York, 1966, and a cross-sectional income model has been estimated in J. M. Mattila and W. R. Thompson, "Toward An Econometric Model of Urban Economic Development," (Appendix), Issues in Urban Economics, edited by H. S. Perloff and L. Wingo, Jr., Resources for the Future, 1968.

Code	SMSA	Rank of City*	Rank of Severity**
1	New York, New York	1	1
2	Chicago, Illinois	2	2
3	Los Angeles/Long Beach, California	3	4
4	Philadelphia, Pa. - New Jersey	4	3
5	Detroit, Michigan	5	9
6	San Francisco/Oakland, California	6	35
7	Pittsburgh, Pennsylvania	8	6
8	St. Louis, Missouri - Illinois	9	10
9	Washington, D. C. - Maryland - Va.	10	18
10	Cleveland, Ohio	11	5
11	Baltimore, Maryland	12	13
12	Newark, New Jersey	13	8
13	Minneapolis/St. Paul, Minnesota	14	32
14	Houston, Texas	15	43
15	Buffalo, New York	16	30
16	Milwaukee, Wisconsin	17	20
17	Cincinnati, Ohio - Indiana - Kentucky	18	19
18	Paterson/Clifton/Passaic, New Jersey	19	21
19	Dallas, Texas	20	57
20	Seattle/Everett, Washington	21	36
21	Kansas City, Kansas	22	25
22	San Diego, California	23	61
23	Atlanta, Georgia	24	51
24	Indianapolis, Indiana	25	14
25	Miami, Florida	26	63
26	Denver, Colorado	27	27
27	New Orleans, Louisiana	28	59
28	Portland, Oregon - Washington	29	49
29	Columbus, Ohio	33	46
30	Rochester, New York	34	53
31	Dayton, Ohio	35	26

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*Rank of city is based on the size of the population reported in Current Population Reports: Technical Studies, Series P-23, No. 23, October, 1967. Boston SMSA (ranked 7th), Providence-Pawtucket-Warwick SMSA (ranked 30th), San Bernardino-Riverside-Ontario SMSA (ranked 31st) and Tampa-St. Petersburg SMSA have been excluded from the present study because of changes in definition of SMSA or lack of certain data.

**See John T. Middleton, ibid.

5.4.1 Export-Oriented Industry, Economic Base and Value Added

There is a fundamental difference between the economy of metropolitan regions and the national economy. The former is based on an open economy where growth and development are closely related to its capability to carry on external trade with other regions. The latter is by nature more self-contained. In spite of Keynsian macro-economic theory, the concept of "export-base" or economic base theory has been the core of the analytical frameworks for urban economies since its appearance in 1928.* However, measurement of the economic base multipliers originally estimated by calculating the ratio between export-oriented (or basic) employment and local-oriented (or service) employment has been changed by using the concept of "value added" instead of employment.**

This latter development now takes into account different production structures among local industries, so that factor intensity of capital and labor may contribute different weight to the multipler. Since manufacturing industries are more capital-intensive, the role of manufacturing in regional growth becomes decisive. That industrial group usually dominates the value of exports and represents more than 80 percent of the total value added.*** It is therefore conceptually acceptable to treat manufacturing industries as export-oriented industries in the regional growth model. Also, some of the recent successful regional econometric models embodied, explicitly or implicitly, a casual relationship between manufacturing industry and overall regional growth.*

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*Haig, Robert M., Major Economic Factors in Metropolitan Growth and Arrangement, Vol. I, Regional Survey of New York and Environs, New York, 1928. See also W. R. Thompson, A Preface to Urban Economics, Resources for the Future, 1965.

**As an early example, see Charles L. Leven, "Measuring the Economic Base," Papers and Proceedings, Vol. 2, The Regional Science Association, 1965.

***Ibid. In a case study of five midwest metropolitan areas, Leven reported that manufacturing industry dominates 80 to 96 percent of export measured by value added, while it only accounts for 45 to 71 percent if measured by employment.

Since the main interest in this model is focused on those manufacturers contributing to pollution and on air pollution control, the sectoral structure must be based on criteria that reflect the major air pollution industries being considered in the present study. In this model, 13 classes of manufacturing industries have been chosen (see Figure 5-3).** The industry title is listed in the table together with the Standard Industrial Classification (SIC) code of the related industry groups being analyzed in the study.

The change in measurement from employment to value added in economic base theory not only has improved the applicability of the regional multiplier in recent regional growth analysis but also seems to be consistent with the familiar Keynsian-type trade multiplier in the open economic system. However, economic base theory is only a partial phenomenon, in that it explains the demand side of urban growth without proper inclusion of the supply side. The labor and capital markets, the relation between export-oriented industry and other local service industry, are also interrelated. Details are discussed in an equation-to-equation explanation in Appendix A, along with the data, source listings and other basic information from which the model was developed.

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*A typical and successful model of this nature is presented in Frederick W. Bell's "An Econometric Forecasting Model for a Region," Journal of Regional Science, Vol. 7, No. 2, 1967.

**See CONSAD Research Corporation, Progress Report on Sectoral Structure and Data Collection, prepared for TRW Systems, Inc., September 20, 1969. This classification has been based on the information available in R.L. Duprey, Compilation of Air Pollution Emission Factors, Washington, D.C., Department of Health, Education and Welfare, Public Health Service, 1968.

Industry Title	Related SIC Codes
Canning and grain mill products	203, 204
Fabrics, yarn and thread mills	221, 222, 223, 224, 226, 228
Paper and allied products	26
Chemicals and selected chemical products	281, 286, 287, 289
Petroleum refining and related industries	29
Rubber and plastics products	30
Glass and glass products	321-323
Stone and clay products	324-329
Primary iron and steel, copper and aluminum manufacturing	331, 332, 3391, 3399, 3331, 3351, 3362, 2819, 3334, 3352, 3361
Other non-ferrous metals manufacturing	3332, 3333, 3339, 3341, 3356, 3357, 3369, 3392
Motor vehicles and equipment	371
Aircraft and other transportation equipment	372-379
Other manufacturing industries	20-39, except included in above classification

Figure 5-3. Classes of Manufacturing Industries

6.0 BUDGET RESOURCE ALLOCATION MODELS

Two resource allocation models were developed under the RAPA contract. The first, developed by TRW, is designed to assist NAPCA in allocating funds among its major independent goals; this model has been designated the Major Goals Model. Leading up to the attainment of each of these major goals is a complex interdependent series of events, usually described by a PERT chart. The second resource allocation model (RAM) developed by the Research Analysis Corporation (RAC) under subcontract to TRW, is designed to allocate resources among nodes in the PERT chart. The allocation can be determined either with the policy of minimizing the time to completion of a major goal, or, for a fixed time, maximizing the number of program objectives completed within the major goal.

6.1 MAJOR GOALS MODEL

The solution methodology used in the Major Goals Model is dynamic programming, which allows nonlinear element payoff functions and the use of tabular payoff functions. Dynamic programming is ideally suited to sensitivity analysis of element payoff functions. It is easily adapted to changing payoff functions and is readily alterable to include additional program elements. It provides solutions for multiple budget levels in a computationally efficient manner. It can provide alternate optimal solutions (if they exist) and sub-optimal (i.e., less than optimal) solutions -- information that is extremely valuable to top-level management. Finally, dynamic programming is iterative and especially designed for use with a computer.

6.1.1 Problem Statement

The resource allocation problem is as follows:

We have N program elements, or areas of development, such as personnel training, surveillance, and stationary source control, into which we would like to invest money. Each of these program elements will provide us with a return, or payoff, in terms of enhancement of air quality. We have a limited budget and seek to get the most for our money, that is, we want to maximize the total return by judiciously allocating our budget to the N program elements.

Mathematically the problem is formulated as follows: Let I be the total amount of our budget, and c_i the portion of the budget allocated to the i^{th} program element for $i = 1, \ldots, N$. Then, maximize $R(c_1, \ldots, c_N)$ subject to the condition that

$$c_1 + \ldots + c_N \leq B. \tag{1}$$

Dynamic programming transforms the initial problem of maximizing the total return of the 63 program costs into 63 similar problems of two variables, the cost of one program element and the cost of the remaining program elements. This transformation is based on Bellman's "principle of optimality," which essentially states that an optimal set of decisions has the property that, whatever the first decision is, the remaining decisions must be optimal with respect to the outcome which results from the first decision.

The dynamic programming approach imposes certain constraints on the total return function $R(c_1, \ldots, c_N)$ or, more specifically, on the interrelations of the element payoff functions. Since the exact form of the return function is not known, these constraints must be explained in some detail. This is best done by considering various general forms of the return function, beginning with the simplest.

6.1.2 Additive Returns

If the element payoff functions are independent, the total return function has the form

$$R(c_1, \ldots, c_N) = r_N (c_N) + \ldots + r_1 (c_1), \tag{2}$$

where r_i is the i^{th} payoff function and c_i is the money spent on the i^{th} program element for $i = 1, \ldots, N$. Our problem, them, is to:

$$\text{Maximize} \quad \sum_{i=1}^{N} r_i (c_i)$$

$$\text{subject to} \quad \sum_{i=1}^{N} c_i \leq B. \tag{3}$$

We break this problem down into N subproblems as follows:

For each $n = 1, \ldots, N$, let x_n be the budget available for allocation to the first n program elements. Set

$$f_1 (x_1) = r_1 (x_1) \text{ and}$$

$$f_{n+1} (x_{n+1}) = \max_{c_{n+1}} \left\{ r_{n+1} (c_{n+1}) + f_n (x_{n+1} - c_{n+1}) \right\}. \tag{4}$$

Then, the functions f_n $(n=1, \ldots, N)$ are well defined, and it can be shown that

$$f_n (B) = \max_{c_1, \ldots, c_N} R(c_1, \ldots, c_N). \tag{5}$$

After tabulating values for each f_n, we can proceed iteratively with $n = N, N-1, \ldots, 1$ to solve the N functions f_n for c_n*, the optimal decision value which maximizes f_n. In view of (5), our initial problem is solved, since

$$R(c_1^*, \ldots, c_N^*) = \max_{c_1, \ldots, c_n} R(c_1, \ldots, c_N).$$

6.1.3 Payoff Functions

The element payoff functions required for this model are presently envisioned to be based on six-year intervals. However, since the budget allocation cycle will utilize one-year payoffs, several problems arise. The first of these is the problem of altering six-year payoff (return) functions with low initial payoff (due to large startup costs), to ensure that funding is begun in the early years even though the payoff is negligible for these years. A second concern is the effect of changes in the return functions based on accumulated data. Because the payoff functions are input data, it is very simple to change a point or set of points. As a consequence, the user may alter the payoff functions for the year under consideration and/or for future years to determine the effect of possible variations in these functions. In a like manner, actual data can be utilized to revise the form of the payoff functions. This inherent flexibility, coupled with the output options discussed under terminal operation, offers the user added insight into the structure of his problem.

The cost function can be defined in several ways. One possible meaning to be given to the cost scalar is that the highest value represents completion of the task and lower cost values represent partial completion. The highest payoff represents the value at task completion, and lower payoff values represent successful completion of project milestones. Note that full project completion is necessary, since partial payoffs are transitory in that they will disappear in time unless the full project is completed.

A more pertinent meaning to the cost scalar is that, while the highest value represents completion of the task as before, the lower cost values now also represent full completion of a task but to a lesser quality than for full funding. Each point on the cost scalar represents a complete task, and the payoff function represents the final value to be achieved at each cost level. At maximum funding the objective is most closely met; at partial funding it is partially met.

When cash flows accrue in different amounts each year, discounted cash flows may tend to substantiate and provide clearer and more complete justification for proposed investment projects. The payoff functions envisioned increase from some nominal value with an increase in cost. There is concern that projects with low initial payoff requiring moderate initial funding will not be funded when compared to projects with larger initial payoffs and comparable costs, even though the former have a greater long-range payoff. Since a decision now to subsequently fund long-range projects need not entail the expenditure of funds until a later date, it is suggested that a discount factor be applied to such funds.

In the event that the budget to be allocated cannot be considered as a single entity (i.e., is split into various fixed amounts), several approaches may be taken. As a worst case, if the total budget were subdivided into separate funds for contract development, in-house activities, research grants, etc., and each program element had a return for each type of funding, the dynamic programming model would have the same number of state variables as there are separate funds. Thus, if there were three funds, there would be three state variables and, to a rough approximation, the computer storage and run time would be cubed. Although solvable, such a problem is unrealistic in terms of time and storage.

A better approach would be to consider each fund as a separate entity and optimize the allocation for each, thus causing a linear rather than exponential increase in computation time. If only certain program elements (i.e., not all 63) apply to each type of funding, then the problem is further simplified.

To a large extent the mode of solution is dependent upon whether a single (combined funding) return is provided for each program element or whether a separate return is available for each type of funding for each element.

6.1.4 Computer Operations

The dynamic programming model has been designed for implementation on a remote terminal using the FORTRAN II language. The computer program will be written so as to minimize computer storage requirements, although this may result in somewhat greater run time. This design philosophy (i.e., minimization of computer storage requirements at the expense of problem computation time) is necessary if future expansion of the model is contemplated and remote terminal operation required.

The Tymshare FORTRAN II includes the FORTRAN II source language statements (the actual program) and the Conversation Compile System (CCS) command mode which controls the computer operating system.

When the remote terminal is utilized, each FORTRAN II Language statement is compiled and analyzed for errors at the time of entry. Diagnostics are given for any errors allowing for immediate correction. After entering a complete program, it may be listed on the terminal, saved on a disk file, and/or executed. Source programs may be entered from paper tape or a file, or from cards at the computer center, as well as from the terminal.

Provided with a checked-out program, NAPCA need only input a few parameters at the terminal and create a file containing the element return vectors. The program element returns are read into the program from a file named RETURN. This file must be created prior to use of the program, either by entering values through the terminal (on paper tape) or through a card reader at the Tymshare facility. The procedure for physically creating the file will not be discussed here, as the subject is completely described in the Tymshare FORTRAN II manual.

Two program packages are provided. (See Appendix B for program listings). Program Package I gives a single optimal solution for each budget level; Program Package II gives alternate optimal solutions, together with certain sub-optimal solutions at a given budget level. The first package has two options available for output:

(a) The optimal solution for anticipated funding, plus optimal solutions at specified greater and lesser budget levels (including element program decisions for each).

(b) The optimal solution with program element decisions at a fixed level of funding.

The second package outputs as many optimal and sub-optimal solutions (at a fixed budget level) as desired, including element program decisions for each.

The output data will be printed at the terminal. NAPCA may then alter certain input data and return the program to determine the sensitivity of the optimal solution to any element payoff function or other input parameter.

6.1.5 Input/Output Formats

The user must provide inputs both to a file and to the program itself. Input to the file is described in Section 6.1.4. The program is designed so that the terminal prints statements which the user is to complete. These statements are:

- The number of program elements is ______.
- The number of budget levels is ______.
- The budget increment is ______.
- The output option is ______.

- * The lowest budget level of interest is ______.
- * The output increment is ______.

The input/output variables are defined in Table 6-1; the various output formats are then illustrated in terms of these variables in Tables 6-2, 6-3 and 6-4.

TABLE 6-1

Input/Output Variables - Major Goals Model

Δ - Budget increment.

DSTAR - Expenditure on each program element, in units of Δ.

FSTAR - Maximum total return.

JJJ - Decrement to be used in going from highest budget level $KK \cdot \Delta$ to the lowest budget level, $(KK-KM-1) \cdot \Delta$.

KK - Total number of budget levels; total budget will equal $KK \cdot \Delta$.

KM - Total decrement defining lowest budget level to be considered. Lowest level budget will equal $(KK-KM-1) \cdot \Delta$.

LN - Number of sub-optimal solutions desired.

NN - Number of program elements.

6.1.6 Major Goals Program Demonstration

The validity of the results obtained from the resource allocation program depends almost entirely on how well the payoff functions (input by the user) can be estimated for each of the Program Elements which make up the NAPCA Operational Plan.

---

*These inputs are required only if the output option is for multi-budget levels.

TABLE 6-2

MULTIPLE BUDGET LEVEL OUTPUT FORMAT

Budget Level:	KK·Δ	(KK-JJJ)·Δ . . .	(KK-KM-1)·Δ
Maximum Return:	FSTAR(NN,KK)	FSTAR(NN,KK-JJJ) . . .	FSTAR(NN,KK-KM-1)
Program Expenditures:			
NN	DSTAR(NN,KK)·Δ	DSTAR(NN,KK-JJJ)·Δ . . .	DSTAR(NN,KK-KM-1)·Δ
↓	↓	↓	↓
1	DSTAR(1,KK)·Δ	DSTAR(1,KK-JJJ)·Δ . . .	DSTAR(1,KK-KM-1)·Δ

(Delta (Δ) may be omitted. In which case, we would print:

Budget Level (Δ dollars):

- - -

Program Expenditures (Δ dollars):)

TABLE 6-3

SINGLE OPTIMAL RETURN OUTPUT FORMAT

- The total budget is: KN·Δ
- The maximum total return is: FSTAR(NN)
- The program element expenditures which give the maximum return are:

Program Element	Expenditure
NN	DSTAR(NN)·Δ
↓	↓
1	DSTAR(1)·Δ

TABLE 6-4

SUB-OPTIMAL SOLUTION OUTPUT FORMAT

```
Total Budget:                    KN·Δ


Return:                          FSTAR(1,NN)      FSTAR(2,NN) . . .       FSTAR(LN,NN)

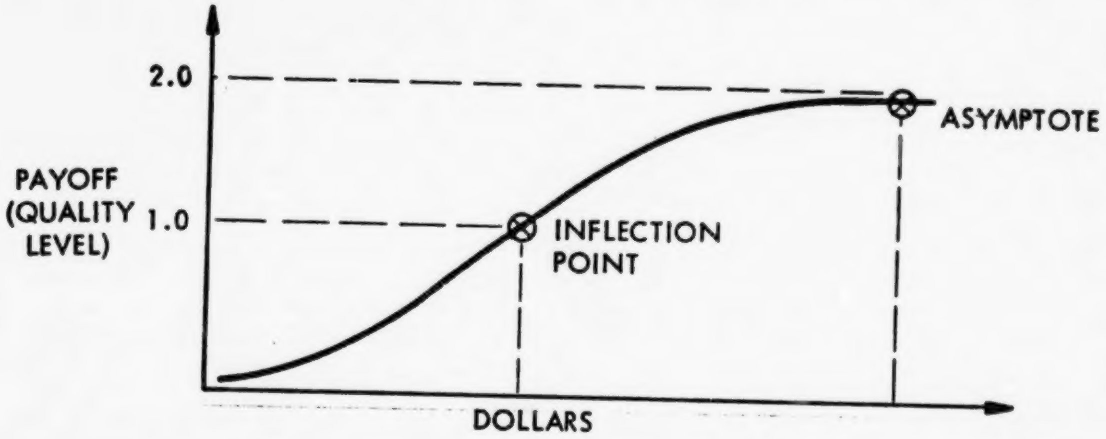

Element Program Expenditures:

              NN                 DSTAR(1,NN)·Δ    DSTAR(2,NN)·Δ . . .     DSTAR(LN,NN)·Δ
              ↓                       ↓                ↓                       ↓
              1                  DSTAR(1,1)·Δ     DSTAR(2,1)·Δ            DSTAR(LN,1)·Δ
```

The payoff function is simply a plot of output from each Program Element as a function of dollars spent. Each point on the curve represents completion of the task, but at various "quality" levels. For this demonstration, only a single year's allocation has been considered. In addition, the budget has been considered as a single resource and is not broken up into various fixed amounts (e.g., grants, contracts, in-house).

The general approach is based on estimates made by managers at two different levels in the organization.

Managers at the Division level or lower estimate dollar amounts required to complete specific Program Element goals at various "quality" levels. It is assumed that all payoff functions have the same S-shape, with only the inflection points and asymptotes varying (see sketch below).



The managers are asked two questions:

(a) At what budget level would they be most efficient in achieving their goals? In other words, at what point is the increase in output (in terms of "quality") for each additional dollar spent at a maximum? This dollar level corresponds to the inflection point on the S-shaped curve.

(b) At what budget level would the incremental return go to zero, i.e., additional dollars spent would produce no increase in the quality of the output? This dollar level corresponds to the asymptote on the S-shaped curve.

Managers at the Bureau or Commissioner's level are then asked to form groups of program elements which are independent of each other (as far as possible), and to weight each of these groupings (called Major Goals) on a scale of 0 to 10 according to the value of each Major Goal in helping NAPCA meets its responsibilities under the Air Quality Act.

Using this general approach, a demonstration case can be constructed through the following steps:

(i) Group the 26 Program Objectives into Major Goals which are independent of each other (recall that goal independence is a restriction in our current model).

(ii) For each of these Major Goals, estimate (a) that level of spending at which the goal is most efficiently completed (i.e., the inflection point, or the point of maximum marginal utility), and (b) that level of spending beyond which no further improvement in the output is possible (i.e., the asymptote).

(iii) Construct S-shaped curves for each of the Major Goals, using the data from (ii) to represent the inflection point and the asymptote. The ordinate values corresponding to these points are 1.0 and 2.0, respectively.

(iv) Establish a relative weighting for each of these Major Goals by ranking them, on a scale of 0 to 10, according to their value in helping NAPCA carry out its responsibilities under the Air Quality Act. Use these weights to scale the ordinates (i.e., the payoff) of each of the S-shaped payoff functions. For example, if the weight for a given Major Goal is 6.0, then the inflection point on its S-shaped curve becomes 6 x 1 = 6, and the asymptote becomes 6 x 2 = 12.

(v) Decide on the total annual budget to be allocated, the alternative budgets (e.g., plus 10% and minus 15%) and the smallest increments to be allocated. Read the payoff functions (S-shaped curves) onto the disk file.

(vi) Find the distribution of resources among the Major Goals which maximizes the payoff by running the resource allocation program. Use the results as an aid in adjusting the input values until an acceptable balance is achieved between the dollar estimates made at the operational levels and the priorities (i.e., weights) established at the top management level.

Two specific examples which were constructed following this procedure are described on the following pages. The funding levels and Major Goal weights were selected by TRW for demonstration purposes only; they have been neither approved nor reviewed by NAPCA.

EXAMPLE #1

Step (i) The 26 Program Objectives were grouped into nine independent Major Goals as follows:

Group	Program Objectives	
I	Criteria Documentation (Program Objective	(# 1)
	Maintain Implementation Schedules	(#18)
	Technical Support to AQCR's	(#20)
	Control Techniques Documents	(#21)
	Federal Abatement Action	(#25)
	Motor Vehicle Control Devices	(# 9)
	Federal MV Emissions Standards	(#10)
	Ensure MV compliance with Federal Standards	(#11)
III	Demonstration - Stationary Control Devices	(#13)
	Research - Stationary Control Devices	(#12)
IV	National Data Bank	(# 2)
	Air Quality and Emission Data	(# 3)
V	Instrumentation	(# 4)
	Physical and Chemical Processes	(# 5)
VI	Federal Regulations - Stationary Sources	(#14)
	Federal Facilities	(#15)
	Feasibility of Additional National Regulatory Programs	(#24)
VII	Assure Adequate Body of Law	(#17)
	Assure Adequate Resources for Enforcement	(#19)
	Emergency Episode Abatement	(#26)
VIII	Technical Information	(#22)
	Manpower Training	(#23)
IX	Health and Economic Effects Surveillance	(# 6)

Group	Program Objectives	
IX	Health and Economic Effects Surveys and Studies	(# 7)
	Health and Economic Lab Studies	(# 8)
	Cost Benefit Studies	(#16)

Step (ii) For each of the nine Major Goals, the following funding levels were established:

Goal	Funding Level at which Goals Achieved Most Efficiently ($M)	Funding Level at which Increment in Output is Zero ($M)
I	50	100
II	20	40
III	50	100
IV	5	10
V	5	10
VI	5	10
VII	5	10
VIII	20	40
IX	5	10

Step (iii) This was done on graph paper.

Step (iv) Weights for each of the Major Goals were established on a scale of 0 to 10 as follows:

Goal	Weight
I	10
II	8
III	6
IV	4
V	4
VI	5
VII	4
VIII	6
IX	4

The ordinates of each of the S-shaped curves were scaled by these factors, and values were read from the scaled curves for various budget levels. These values were then stored on the disk.

Step (v) It was assumed that the total annual budget would be \$100M, divided into 20 increments of \$5M each (i.e., the smallest increment which can be allocated is \$5M). In this simple example, there are no constraints such as maximum or minimum allocations to any of the Major Goals, nor is the budget broken down into categories of fixed size.

The output which resulted from using these values in the resource allocation program is shown below.

```
LØADING CØMPLETE
9469 WØRDS ØF STØRAGE UNUSED
+;G

THE NUMBER ØF PRØGRAM ELEMENTS IS 9,
THE NUMBER ØF BUDGET LEVELS IS 20,
THE BUDGET INCREMENT IS $4.,
THE ØUTPUT ØPTIØN IS 1,


THE TØTAL BUDGET IS $        100.00
THE MAXIMUM TØTAL RETURN IS   61.80
THE PRØGRAM ELEMENT EXPENDITURES WHICH GIVE THE MAXIMUM RETURN ARE

          PRØGRAM ELEMENT          EXPENDITURE          RETURN
                 9                     10.0              61.30
                 8                     25.0              53.30
                 7                     10.0              45.30
                 6                     10.0              37.30
                 5                     10.0              27.30
                 4                     10.0              19.30
                 3                       .0              11.30
                 2                     25.0              11.30
                 1                       .0                .00
*STØP*
```

The funds allocated to each Major Goal (listed by number in the column labeled Program Elements) are shown in millions of dollars. Note that no funds at all are allocated to Goals #1 and #3, two of NAPCA's more important activities. This is because the payoff curves for these high budget goals are just starting to climb when all of the others have leveled off. At this budget level, they don't get a chance to participate. If the budget level were \$300M, most of the money would be allocated to these two goals.

Example #2

As a test of the sensitivity of the allocations to the relative weights assigned to the goals, a second example was constructed by multiplying the weights of goals #1 and #3 by a factor of three. The results of this run are shown below.

```
THE NUMBER ØF PRØGRAM ELEMENTS IS 9,
THE NUMBER ØF BUDGET LEVELS IS 20,
THE BUDGET INCREMENT IS $5.,
THE ØUTPUT ØPTIØN IS 1,

THE TØTAL BUDGET IS $        100.00
THE MAXIMUM TØTAL RETURN IS   75.00
THE PRØGRAM ELEMENT EXPENDITURES WHICH GIVE THE MAXIMUM RETURN ARE

           PRØGRAM ELEMENT          EXPENDITURE          RETURN
                  9                     10.0              75.00
                  8                       .0              67.00
                  7                     10.0              67.00
                  6                     10.0              59.00
                  5                      5.0              49.00
                  4                       .0              45.00
                  3                       .0              45.00
                  2                       .0              45.00
                  1                     65.0              45.00

*STØP*
```

Now the bulk of the total budget is allocated to goal #1, with 2, 3, 4 and 8 receiving no funds at all. These two examples took about two minutes of CPU time and 15 minutes of terminal time to run.

6.2 RESOURCE ALLOCATION MODEL (RAM)

6.2.1 Introduction

The RAM-1 resource allocation model is a computerized, mathematical optimization model that has been developed for the purpose of demonstrating the application of mathematical optimization techniques to the solution of problems encountered in the planning and budgeting of programs of the National Air Pollution Control Administration (NAPCA). The model determines the schedule and budget that minimize the total cost of a project composed of up to fifty tasks that must be completed in a technologically prescribed order within a specified time period. The tasks are assumed to have costs that can be varied, within limits, by varying the lengths of the time periods allotted for performance of the tasks.

NAPCA programs frequently consist of technologically independent projects with goals that must be achieved by specific dates. These projects are composed of individual tasks that must be executed in a technologically prescribed order over a period of several years; the goals of a project are achieved upon completion of all the tasks comprising the project. The costs of the tasks frequently depend on the lengths of the time periods allotted for their completion, so that task costs can be traded off, within limits, against task durations. One of the problems encountered by NAPCA in the process of project planning is determining how these trade-offs can be made so that project goals are achieved by the required dates at minimum cost.

The solution of this cost-time trade-off problem yields the task budgets and schedules that minimize the total project cost given the required goal achievement dates. The problem must be solved before a systematic determination of optimal project and program budgets, funding levels, schedules, and priorities can be made. Solving the cost-time trade-off problem thus aids both NAPCA's yearly budgeting process and any future development of a comprehensive system for optimal planning and budgeting. It was therefore decided to develop a pilot model that would solve this problem for a small project and also schedule and budget the tasks of the project.

The RAM-1 resource allocation model determines the task schedules and budgets that minimize the cost of a small project whose tasks are subject to cost-time trade-offs and whose goals must be achieved by specified dates. The model is computerized and provides information sufficient for scheduling and budgeting the tasks of a project that involves up to fifty tasks. The model has been demonstrated on a portion of NAPCA's program planning network using actual cost data for the tasks. This report describes the model and presents the results of the demonstration. The report specifies the input data the model requires and the output information it produces. The methodology employed by the model is also described.

The RAM-1 model is currently executable on RAC's CDC-6400 computer using input data supplied by NAPCA. The required computer program is listed in Appendix D. While the model has been demonstrated on a portion of NAPCA's planning network and shown to give outputs that are both reasonable and correctly derived from the input data, a full study of the usefulness of the model in actual program planning applications has not been undertaken. The ultimate value of RAM-1 to NAPCA cannot be known until such a study has been completed. Subject to NAPCA's desires, it is intended to undertake a second phase of work on the RAM-1 model that will include a full study of the usefulness of the model in actual practice, expansion of the model to permit treatment of projects composed of more than fifty tasks, and installation of the model on a NAPCA computer so that it can be operated by NAPCA personnel.

6.2.2 Structure and Capabilities of the RAM-1 Model

The resource allocation model RAM-1 has been developed by RAC for NAPCA to provide a pilot scale demonstration of the application of mathematical optimization techniques to problems of planning and budgeting NAPCA programs. RAM-1 is a computerized pilot model that determines the task schedules and budgets that minimize the cost of a project whose tasks are subject to cost-time trade-offs and whose goals must be achieved by specified dates. A project is defined to be a series of tasks that must be performed in a technologically prescribed order and that results, upon completion, in the achievement of one or more final goals. It is assumed that the technological ordering of tasks can be described using a project flow chart or PERT diagram. It is also assumed that there is no significant technological uncertainty as to whether tasks can be completed given sufficient time and money. The RAM-1 model determines the cost minimizing task schedule and budget for a project from the cost-time relationships of the tasks comprising the project and from the dates at which the project goals must be achieved.

This section outlines the structure and capabilities of RAM-1. The model is described in detail in subsequent sections.

DESCRIPTION OF THE RAM-1 MODEL

The structure of the RAM-1 model is illustrated in the flow chart (Figure 6-1). As shown in the figure, the model consists of three main parts: inputs, mathematical programming package, and outputs. A brief description of these parts follows. Each is discussed in detail in later sections of this report.

The inputs required by the model are:

I1. Flow chart or PERT diagram of the project under consideration showing the technologically prescribed ordering of the tasks comprising the project.

I2. Cost-time relationships for the tasks showing the relationship between the duration and estimated cost of each task.

I3. Desired completion dates of the final goals of the project expressed in years from the start date of the project.

I4. Estimates of the uncertainty in the cost-time relationships of input I2 due to possible errors in cost estimation.

The inputs are supplied to a computer-executed mathematical programming package which solves the following mathematical programming problem:

P1. Determine the task durations and costs that minimize the total cost of achieving the project final goals subject to the constraints:

   a. The tasks must be performed in the order specified in input I1.

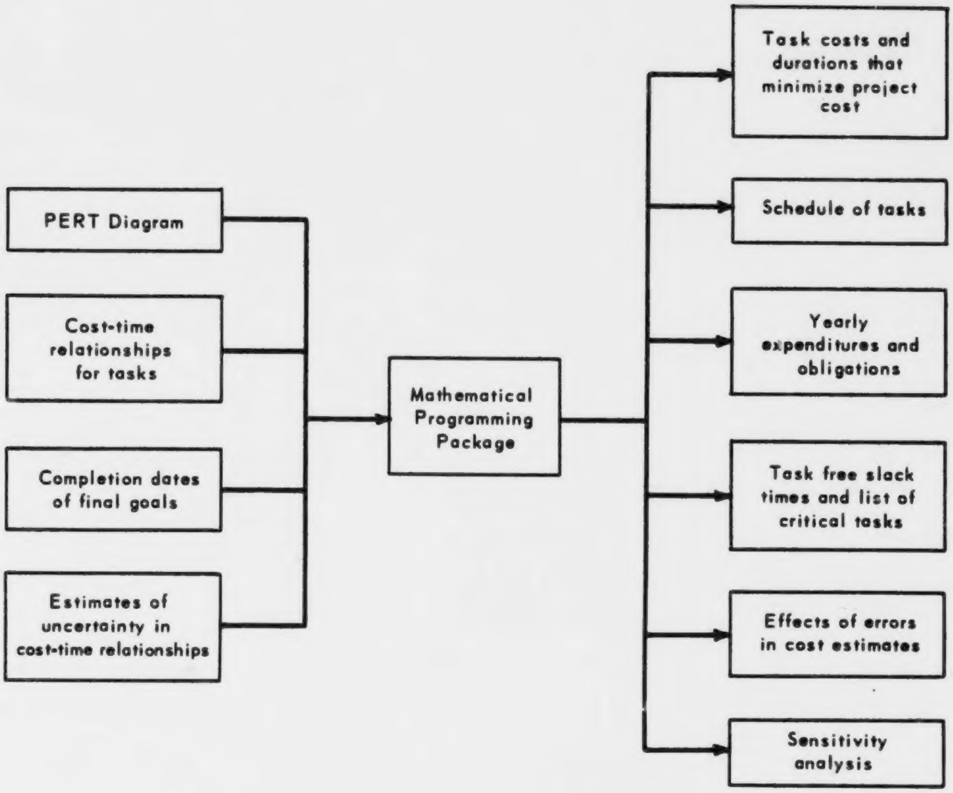

PERT Diagram
Cost-time relationships for tasks
Completion dates of final goals
Estimates of uncertainty in cost-time relationships
Mathematical Programming Package
Task costs and durations that minimize project cost
Schedule of tasks
Yearly expenditures and obligations
Task free slack times and list of critical tasks
Effects of errors in cost estimates
Sensitivity analysis

Figure 6-1. Flow Chart of RAM-1 Model

b. The cost of a task depends on its duration in the manner specified in input I2.

c. The final goals must be achieved by the dates specified in input I3.

Total project cost is considered to be the sum of task costs.

The solution of the mathematical programming problem is passed to a critical path routine and a budget preparation and task scheduling routine, both of which are contained in the mathematical programming package. The outputs obtained from this sequence of routines are:

O1. Task costs and durations that minimize total project cost.

O2. Schedule of tasks.

O3. Yearly project expenditures and obligations.

O4. The amount of time each task can fall behind schedule without delaying other tasks or goal achievement (free slack times) and list of tasks that cannot fall behind schedule without delaying goal achievement (critical tasks).

In addition, the mathematical programming package contains a statistical analysis routine and a sensitivity analysis feature. These yield the following additional outputs:

O5. Effects upon total yearly budgets and total project cost of random errors in the cost estimates included in input I2.

O6. Approximate effects on total project cost of relaxing the goal achievement dates of input I3.

The RAM-1 model supplies sufficient output information to schedule and budget the tasks comprising a project and to monitor the progress of the project. It is to be emphasized, however, that the present version of RAM-1 is a pilot model. Consequently, it is designed to budget only projects consisting of not more than fifty tasks. The model is capable of being expanded in a subsequent phase of work to permit treatment of larger projects.

6.2.3 Inputs

The RAM-1 model has input requirements that are similar to those of a wide variety of budgeting techniques, thus easing problems of input data accumulation. In addition, the model's input requirements are close to the minimum needed to obtain a useful solution of the problem it solves. The inputs required by the model are:

I1. Flow chart or PERT diagram of the project under consideration showing the technologically prescribed ordering of the tasks comprising the project.

I2. Cost-time relationships for the tasks showing the relationship between the duration and estimated cost of each task.

I3. Desired completion dates of the final goals of the project expressed in years from the start date of the project.

I4. Estimates of the uncertainty in the cost-time relationships of input I2 due to possible errors in cost estimation.

Each of these inputs is described in detail in the following discussion.

INPUT I1 - PERT DIAGRAM

A PERT diagram is an arrow diagram showing the technological ordering of the various tasks comprising a project. Each arrow, or arc, of the diagram corresponds to a unique task, and each task corresponds to a unique arc. The junctions of two or more arcs of the diagram are called nodes and correspond to final goals of the project and intermediate milestones, or events, in project execution. Each node is numbered, and the arcs are identified by the numbers of the nodes they connect. Thus, arc (4,7) connects node four to node seven. Of course, each arc label, such as (4,7), also identifies the task corresponding to the arc. The node numbers must be chosen such that if an arc is directed from node number i to node number j, then i is less than j.

A simplified, highly aggregated example of a PERT diagram for the operations involved in designating air quality control regions for cities A and B is shown in Figure 6-2. Table 6-5 gives the correspondence between arcs of the diagram and tasks, and defines the final goals. This diagram is illustrative only and does not necessarily accurately portray the actual process of designating air quality control regions.

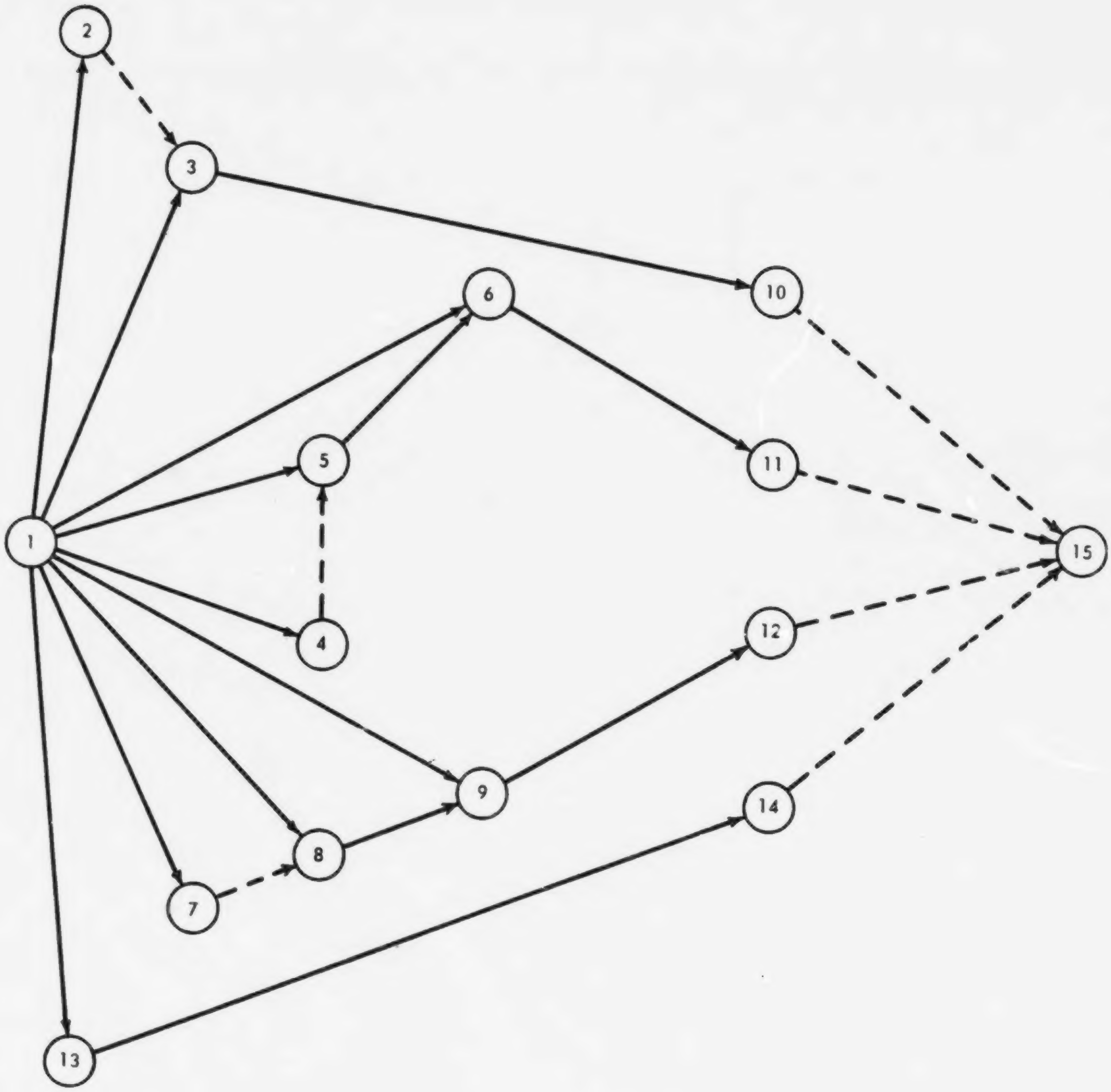

2
3
6
10
5
11
1
15
4
12
9
14
8
7
13

Figure 6-2. Example of PERT Diagram

TABLE 6-5

a. Correspondence between Tasks and Arcs for PERT Diagram of Figure 1

Arc	Task
(1,2)	Epidemiologic Studies
(1,3)	Toxicologic Studies
(2,3)	Dummy
(1,4)	Gather Aerometric Data for Region A
(1,5)	Prepare Emissions Inventory for Region A
(1,6)	Determine Boundaries of Political Jurisdictions in Region A
(4,5)	Dummy
(5,6)	Construct Diffusion Model for Region A
(6,11)	Designate Air Quality Control Region A
(1,7)	Gather Aerometric Data for Region B
(7,8)	Dummy
(1,8)	Prepare Emissions Inventory for Region B
(1,9)	Determine Boundaries of Political Jurisdictions in Region B
(8,9)	Construct Diffusion Model for Region B
(9,12)	Designate Air Quality Control Region B
(1,13)	Develop Control Technology
(3,10)	Publish Criteria Documents
(13,14)	Publish Control Documents
(10,15)	Dummy
(11,15)	Dummy
(12,15)	Dummy
(14,15)	Dummy

b. Final Goals

Node	Goal
10	Criteria Documents Completed
11	Designation of Region A Completed
12	Designation of Region B Completed
14	Control Documents Completed

Node 15 represents a dummy final goal.

The example PERT diagram includes several dummy tasks which are identified in the diagram by dotted lines. The dummies are artificial tasks that are not executed and have zero cost. Their function is to force the formal structure of the PERT diagram to be acceptable to the mathematical programming package under the following circumstances:

1. The mathematical programming package requires that each pair of node numbers identifying an arc apply to a unique arc. If, in the absence of dummies, two or more arcs would have the same initial and final nodes, insertion of a dummy arc is required to force the numbers of the initial and final nodes of the arcs to designate unique arcs. In Figure 6-2, dummies (2,3), (4,5), and (7,8) serve this purpose. These dummies have zero duration as well as zero cost.

2. The mathematical programming package also requires that a project have a unique final goal. If a project has several final goals, as in the case of Figure 6-2 where nodes 10, 11, 12, and 14 are all project final goals, a dummy final node and dummy arcs connecting the real final nodes with the dummy node are inserted. This forces the project to have a unique (dummy) final goal. The dummy arcs correspond to tasks with zero cost. Their durations and the achievement date of the dummy final goal are selected to force the actual final goals to be achieved at their desired achievement dates. The selection of these durations and the dummy goal achievement date are discussed under inputs I2 and I3 in this section.

The insertion of dummies into a PERT diagram is a purely formal device that does not affect the execution, budget, or schedule of a project. Dummies occurring between intermediate nodes in a diagram serve only to indicate task orderings. They are considered to be completed instantaneously. Dummies preceding dummy final goals, such as (10,15), (11,15), (12,15), and (14,15) in Figure 6-2, are never executed, as all work on a project stops when the real final goals are achieved. The construction of PERT diagrams is discussed in further detail in Reference 1.

Inasmuch as the project PERT diagram constitutes part of the input data to the RAM-1 model, the diagram can be changed as desired by the model's user without altering the model.

INPUT I2 - COST-TIME RELATIONSHIPS

The RAM-1 model requires the following input data in order to specify cost-time relationships for each task:

I2a. The minimum amount of time in which each task can be performed and its corresponding cost.

I2b. The minimum cost at which each task can be performed and its corresponding duration.

I2c. The cost of performing each task with a duration equal to the average of the durations of I2a and I2b.

All costs are expressed in dollars, and all durations are expressed in years to two decimal places.

The above data are used to define three points on a cost-time curve for each task. The computer completes this curve by connecting the three points with straight lines as in Figure 6-3. This results in a cost-time curve that makes possible the determination of the cost of a task for any duration within the limits set by inputs I2a and I2b. The figure shows a convex (bent downward) cost-time curve. The model also accepts input data that result in concave (bent upward) and linear curves as well as data from tasks whose durations are not variable. In the latter case the points obtained from I2a, I2b, and I2c coincide, and the cost-time relationship consists of a single point. Dummy tasks are considered to have zero costs. If they correspond to arcs connecting intermediate nodes of the PERT diagram, they are also considered to have zero duration. Their cost-time curves then consist of a single point. A dummy task that connects a real final goal with a dummy final goal is considered to have a duration that lies between $T - T_G$ and T, where T is the achievement data assigned to the dummy final goal (see the subsequent discussion of input I3) and T_G is the desired achievement date of the real final goal from which the dummy arc emanates. The optimal duration, within these limits, is determined by the mathematical programming package.

The input data I2a, I2b, and I2c require that estimates of task costs be made prior to performance of tasks. These cost estimates may be subject to some uncertainty, and an informed individual may find that it is desirable to specify a range of possible costs for a task of given

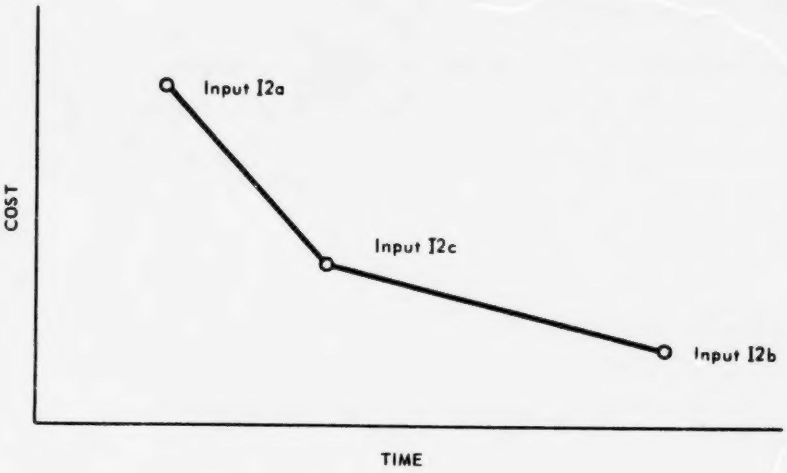


Figure 6-3. Example of Cost-Time Curve

duration. When this occurs, the input cost data should consist of the average estimates of the cost of each task for the appropriate durations. Thus, I2a should specify the average estimate of the cost of each task when performed as rapidly as possible; I2b should specify the average estimate of the cost of each task when performed in the minimum cost manner, and so forth. The costs supplied to the model should not be the upper limits of ranges of costs, as this will result in project budgets far in excess of those required to cover cost overruns arising from normal uncertainties in cost estimates.

INPUT I3 - DESIRED COMPLETION DATES OF FINAL GOALS

The desired completion dates of the final goals should be specified in years from the project start date. The RAM-1 model accepts dates that are less than ten years and are specified to two decimal places. A project with more than one final goal must be assigned a dummy final goal as previously described. The completion date of the dummy final goal can be any date that exceeds the completion date of the last completed final goal. If a dummy final goal is used, the completion dates of the real final goals are incorporated into the durations of dummy tasks connecting the real and dummy final goals as described in the discussion of input I2.

INPUT I4 - ESTIMATES IN UNCERTAINTY IN COST-TIME RELATIONSHIPS

As previously mentioned, cost estimates for tasks are frequently subject to uncertainty. In order to compute the effects of this uncertainty on project budgets, the RAM-1 model requires input data specifying the degree of uncertainty in the task cost estimates of input I2. These data consist of one two-place decimal number for each task giving the fractional increase in the estimated costs specified for the task in input I2 in order to have a probability of 90 percent that the task will not show a cost overrun. For example, if the cost estimates for a given task must be increased by 50 percent to give a 90 percent probability that the task will not show a cost overrun, the appropriate number is 0.50; if a one hundred percent increase is required, the appropriate number is 1.00. The number supplied for each task should be applicable to all three cost estimates included in input I2. If desired, the user can suppress the statistical analysis feature of the RAM-1 model by setting all cost uncertainty estimates equal to zero.

INPUT DATA TABLE

All input data required by the RAM-1 model are displayed in Table 6-6 in a format that facilitates their transferral to a computer. The structure of the project PERT diagram is contained in the assignment of node numbers i and j to tasks. In addition to the data previously described, the model requires the total number of nodes and the total number of arcs (including dummies) in the project PERT diagram to be supplied as inputs. The model permits the user to identify each task by a short name or number in addition to its node numbers. The user may supply this additional identification or not as desired.

6.2.4 Mathematical Programming Package

The mathematical programming package determines the task costs and durations that minimize project cost. It also performs all subsequent manipulations of costs and durations required to budget and schedule the project under consideration.

The mathematical programming package, whose structure is illustrated in Figure 6-4, consists of four computational phases:

MP1. Mathematical programming algorithm.
   a. Branch-and-bound algorithm.
   b. Fulkerson network optimization algorithm.
MP2. Critical path algorithm.
MP3. Budget preparation and task scheduling routine.
MP4. Statistical analysis routine.

The mathematical programming algorithm, phase MP1, determines the task costs that minimize total project cost and the corresponding task durations. The task durations are supplied to the critical path algorithm, phase MP2, which determines task start dates, finish dates, and slack times, and identifies critical tasks. The budget preparation and task scheduling routine, phase MP3, then prepares complete schedule and budget reports. Finally, the statistical analysis routine, phase MP4, computes

TABLE 6-6

RAM-1 INPUT REQUIREMENTS

Task Name	i	j	d_1	d_2	d_3	c_1	c_2	c_3	s

T = ________
Arcs = ________
Nodes = ________

Symbols:

i = Number of node at which task arc originates.
j = Number of node at which task arc terminates.
d_1 = Minimum duration of task.
d_3 = Task duration when task is performed at minimum cost.
d_2 = Average of d_1 and d_3.
c_1, c_2, c_3 = Task costs corresponding to durations d_1, d_2, and d_3 respectively.
T = Final goal completion date. This final goal is a dummy if necessary.
s = Fractional increase in c_1, c_2, and c_3 required to give 90 percent probability that the indicated task will not have a cost overrun.

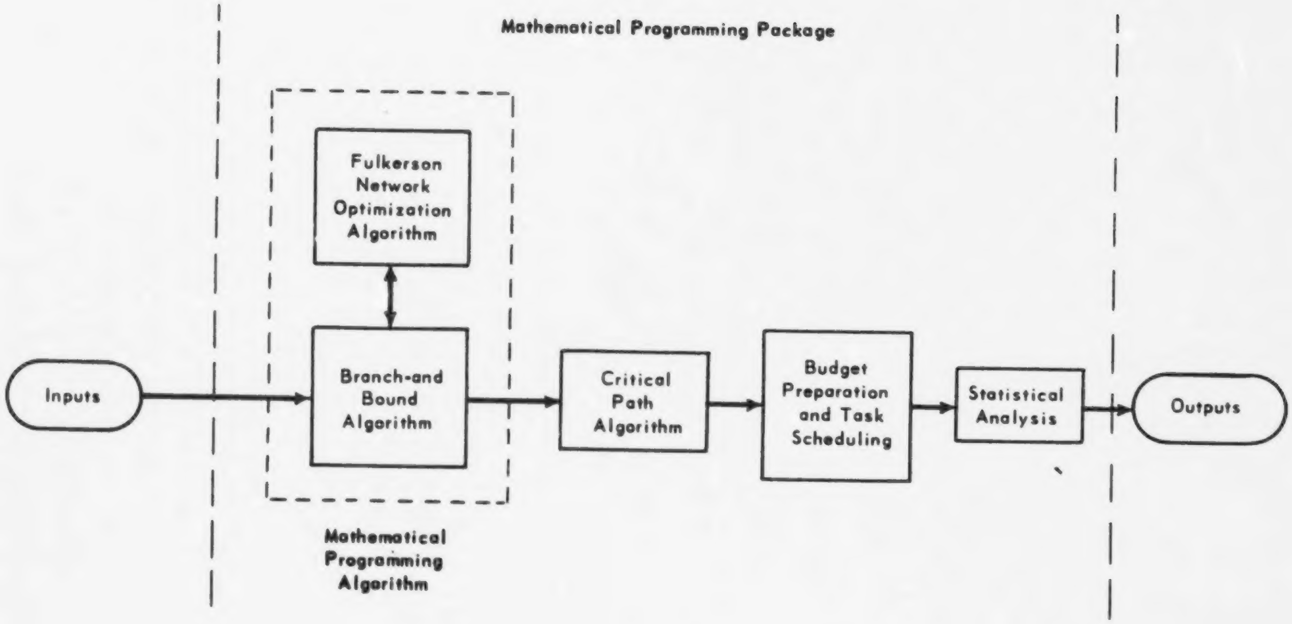

Mathematical Programming Package
Inputs
Fulkerson Network Optimization Algorithm
Branch-and Bound Algorithm
Mathematical Programming Algorithm
Critical Path Algorithm
Budget Preparation and Task Scheduling
Statistical Analysis
Outputs

Figure 6-4. Flow Chart of Mathematical Programming Package

the effects on the project budget of uncertainties in cost estimates. Each of these computational phases is briefly described below. They are described in greater detail in Appendix C.

PHASE MP1 - MATHEMATICAL PROGRAMMING ALGORITHM

The mathematical programming algorithm solves the mathematical programming P1 of Section 6.2.2. Restated in terms of the results of the input procedures described in Section 6.2.3, this problem is:

P1. Determine the task durations and costs that minimize total project cost subject to the constraints:

a. All tasks must be performed in the order specified in the project PERT diagram.

b. The cost of a task depends on its duration as specified by the cost-time curves described in Section 6.2.3.

c. The final goal must be achieved by the date specified in input I3 if it is unique. If there are multiple real final goals, a dummy unique final goal must be achieved by the date specified in the Section 6.2.3 discussion of input I3.

The solution of this problem yields the minimum total cost of the input project and the corresponding costs and durations of its component tasks.

Before solving problem P1, the mathematical programming algorithm classifies task cost-time curves according to whether they are convex (bent downward), concave (bent upward), or linear. The algorithm then restructures the input PERT diagram in a manner that makes it possible for each task with a convex cost-time curve to be represented by two sequential tasks with linear cost-time curves. This restructuring is a mathematical manipulation that does not affect the final results of the model.

Tasks with concave cost-time curves require special treatment to prevent the mathematical programming algorithm from yielding spurious budget allocations. The branch-and-bound algorithm of Falk and Soland (Reference 2) included in the mathematical programming algorithm permits accurate treatment of concave cost-time curves. It does this by generating a finite series of linear approximations to the concave cost-time curves. This, together with the above described manipulation of convex cost-time curves, results in a finite series of linear programming problems in which cost-time curves that were found to be linear or convex from the original input data are represented exactly by linear curves, and cost-time curves that were originally found to be concave are approximated by linear curves. The exact solution of the original problem P1 is obtained upon completion of the solution of this finite series of linear programming problems. The branch-and-bound algorithm is described in detail in Appendix C.

The linear programming problems in the series generated by the branch-and-bound algorithm are solved using a network optimization algorithm due to D. R. Fulkerson (Reference 3). This algorithm solves the problems by taking advantage of special characteristics of their structure. The algorithm solves linear programming problems of the type encountered in RAM-1 much more efficiently than conventional linear programming algorithms such as simplex. The Fulkerson algorithm is described in detail in Appendix C.

The mathematical programming algorithm automatically performs a sensitivity analysis as it solves problem P1. Thus, upon termination of this algorithm, the approximate effect on total project cost of changes in goal achievement dates is known.

PHASE MP2 - CRITICAL PATH ALGORITHM

The task durations obtained from the mathematical programming algorithm are passed to the critical path algorithm. This algorithm determines task start dates, finish dates, and slack times, and identifies critical tasks.

The critical path algorithm schedules tasks to start and finish at the earliest dates permitted by the task ordering requirements specified in the project PERT diagram and the task durations obtained from the mathematical programming algorithm. This type of scheduling is called early start scheduling. Early start scheduling is not necessarily required in order to minimize project cost because there may be tasks that can be started within a range of dates without affecting project cost or final goal achievement dates. However, it is possible that unexpected budgetary or technological difficulties may arise in the course of a project and cause some tasks to be delayed beyond their scheduled start dates or to require more time than originally planned for execution. The RAM-1 model therefore uses early start scheduling in order to provide a schedule with the greatest possible flexibility for dealing with unexpected delays in task performance without delaying final goal achievement or increasing project cost.

As an additional aid for dealing with unexpected project delays, the critical path algorithm computes the free slack of each task. Free slack is defined as the maximum amount of time the start and/or completion dates of a task can fall behind schedule without delaying either goal achievement or the start dates of other tasks. Free slack thus measures the amount of flexibility in a project's schedule.

The critical path algorithm computes task start and finish dates by noting that the earliest date at which a task can finish (early finish date) is equal to the task's early start date plus its duration. The early start date of a given task is equal to the latest of the early finish dates of all tasks that precede the given task in the technological task ordering defined in the project PERT diagram. Tasks with no predecessors start at the project start date. All task start dates are expressed in years from the project start date. The free slack of a task is computed as the difference between the earliest start date of all technological successors to the task and the task's early finish date. All slacks are expressed in years.

Critical tasks are tasks that cannot fall behind schedule without delaying final goal achievement. They are identified as tasks with the property that the latest date at which they can start without delaying goal achievement (late start date) is equal to their early start date. The late start date of a given task is computed as the minimum difference between the late start dates of all technological successors of the given task and the duration of the given task.

PHASE MP3 - BUDGET PREPARATION AND TASK SCHEDULING ROUTINE

The budget preparation and task scheduling routine is an accounting and sorting routine that puts the information obtained from the previous computational phases into a form suitable for use by project planners. The routine performs the following tasks:

1. Identifies the tasks that are scheduled to start in each year of the project planning period. Years are counted from the project start date.

2. Identifies the tasks that are scheduled to finish in each year of the planning period.

3. Identifies task active in each year of the planning period and tasks whose activity is continued from a previous year.

4. Determines new obligations for each year of the planning period.

5. Determines the yearly cost of each task for each year of the planning period.

6. Determines total yearly costs of the project.

The term "new obligations for a given year" is defined as the costs that will be incurred over the full durations of the tasks started in the given year.

Computations of yearly costs are based on the following two assumptions:

1. Each task is budgeted for a continuous, uninterrupted period of time equal to the duration obtained for the task from the mathematical programming algorithm.

2. Each task incurs costs at a uniform rate over its duration.

While no data are presently available that invalidate these assumptions, they can be altered at a later date without major restructuring of the RAM-1 model if they prove to be unrealistic.

The budget preparation and task scheduling routine identifies tasks that start, finish, are active, and/or continue in a given year directly from the schedule and task durations obtained from the mathematical programming and critical path algorithms. New obligations for a given year are computed as the full-duration costs of the task started in that year. Yearly costs for a given year and task are computed by multiplying the full-duration task cost by the fraction of the given year during which the task is active. The required fraction is obtained from the duration and start date of the task and the assumption that tasks proceed from start to finish without interruption. Total yearly costs are computed as the sums of the yearly costs of individual tasks.

PHASE MF4 - STATISTICAL ANALYSIS ROUTINE

The statistical analysis routine determines the effects on a project's budget of random errors in task cost estimates. It does this by computing the following quantities:

S1. An upper limit on total project cost such that the probability that actual project cost will not exceed this limit is 95 percent.

S2. Upper limits on yearly project expenditures such that the probability that all yearly expenditures will fall within these limits is 95 percent.

These upper limits consider only the effects of random errors in cost estimation and apply only to projects that stay on schedule.

Due to possible random errors in cost estimation, task cost estimates can be considered to specify only probability distributions for actual task costs and not exact actual task costs. It is assumed in RAM-1 that these probability distributions are gamma distributions. The justification for this assumption is presented in Appendix C. The mean of the cost distribution for a given task and duration is considered to be the cost of performing the task in the given duration as specified by the task's cost-time curve; the 90th percentile of the distribution is computed from its mean using input I4. The mean and 90th percentile of a gamma distribution determine the variance of the distribution. The computations required to evaluate the variances of task cost distributions are performed by the statistical analysis routine.

Random errors in task cost estimates produce uncertainties in the total and yearly costs of a project. The total and yearly costs therefore have probability distributions whose mean values are respectively the total and yearly project costs computed by the mathematical programming algorithm and the budget preparation and task scheduling routine. The statistical analysis routine computes the variances of these distributions from the variances of the task cost distributions. In the course of these computations, the assumption is made that the variance of the distribution of the yearly cost of a task in a given year is proportional to the estimated yearly cost of the task obtained for the given year from the budget preparation and task scheduling routine. This assumption is made in the absence of data on yearly task cost variances. It can be changed if computational experience with the RAM-1 model indicates this is necessary.

The total and yearly project cost distributions can be considered normal due to the central limit theorem of statistics. The upper bound in S1 is the 95th percentile of the total project cost distribution. It is computed from the mean and variance of this distribution using the properties of the normal distribution. The upper bounds in S2 are the $(.95)^{1/K}$ percentiles of the yearly cost distributions, where K is the last year of the project. These percentiles are computed from the means and variances of the yearly cost distributions using the properties of the normal distribution.

It is to be emphasized that the statistical analysis routine computes only the effects on project budgets of random errors in cost estimates. Random errors are those that cause cost estimates to be high sometimes and low sometimes but correct on the average. Systematic errors in cost estimates, i.e., errors that cause cost estimates to be consistently high or consistently low, are not treated by the statistical analysis routine. Errors of this type will cause the RAM-1 model to yield consistently high or consistently low budgets for the tasks and years affected by these errors as well as for the total project. As previously noted, the results of the statistical analysis routine apply only to projects that stay on schedule.

6.2.5 Outputs

The outputs of the RAM-1 model convey the following information:

O1. Task costs and durations that minimize total project cost.

O2. Schedule of tasks.

O3. Yearly project expenditures and obligations.

O4. Task free slack times and list of critical tasks.

O5. Effects upon total yearly budgets and total project cost of random errors in the cost estimates included in input I2.

O6. Approximate effect on total project cost of relaxing the goal achievement dates included in input I3.

These outputs are obtained from the computations described in Section 6.2.4. They supply sufficient information to schedule and budget the tasks comprising a project.

The specific output data elements generated by the RAM-1 model are:

1. All input data. These are included for user reference and to enable the user to check the correctness of the data received by the computer.
2. Durations of individual tasks.
3. Total cost of each task.
4. Total project cost.
5. Yearly project costs.
6. Yearly costs of individual tasks.
7. Task start dates.
8. Task finish dates.
9. Lists of tasks that start, finish, continue, and are active in each year of the project.
10. Task free slack times.
11. List of critical tasks.
12. Yearly obligations.
13. Upper limit on total project cost (as defined in Section 6.2.4).
14. Upper limits on yearly project costs (as defined in Section 6.2.4).
15. Final goal achievement dates.
16. Approximate change in project cost due to small, intentional increases in individual final goal achievement dates.
17. Approximate project savings due to uniform, intentional increases in all final goal achievement dates.*

---

*The approximations of 16 and 17 are exact if there are no concave cost-time curves.

Costs are expressed in dollars, durations in years, and dates in years from the project start date. Tables 6-7 through 6-15 show the formats used by the RAM-1 model to present output data.

6.2.6 Demonstration of Model

The RAM-1 model has been demonstrated on a project drawn from NAPCA's program planning network. The objective of the demonstration project is to perform the NAPCA activities required to prepare two regions, Region A and Region B, to begin to discharge their responsibilities with respect to one specific air pollutant under the Air Quality Act of 1967. The specific goals of the project are:

(a) Publish air quality criteria for the pollutant under consideration.

(b) Designate an Air Quality Control Region for Region A.

(c) Designate an Air Quality Control Region for Region B.

(d) Publish reports on available control techniques for the pollutant under consideration.

The objective of the demonstration project is considered achieved upon achievement of all of the above goals. The tasks required to achieve the goals are listed in Table 6-16 and diagrammed in Figure 6-5. The dates at which the goals are to be achieved are two years, ten months, and one year from project initiation for goals one through four respectively. The input cost-time data for the demonstration project are shown in Table 6-17. These data are based on data obtained from past experience with projects similar to the demonstration project. The project budget information yielded by RAM-1 is shown in Tables 6-18 through 6-25. All costs are expressed in units of one thousand dollars.

The demonstration project and associated input data have been prepared by RAC in consultation with the TRW Systems Group. The project is presented as an example for demonstration purposes and does not necessarily duplicate any project actually undertaken by NAPCA.

TABLE 6-7

INPUT DATA

Task Name	i	j	d_1	d_2	d_3	c_1	c_2	c_3	s

See Table 2 for symbol definitions.

TABLE 6-8

TASK COSTS AND SCHEDULES

Task Name	Arc I.D.		Cost	Duration	Start Date	Finish Date	Slack
	i	j					

Total Project Cost __________

Upper Limit on Total Project Cost is ____________ with Probability 95%.

TABLE 6-9

FINAL GOAL ACHIEVEMENT DATES

Goal Name	Node Number	Achievement Date

TABLE 6-10

TOTAL YEARLY COST OF PROJECT

Year	Project Cost	Upper Bound on Cost with Probability 95%

TABLE 6-11

YEARLY TASK COSTS

Task Name	Arc	I.D.	Year			
	i	j	1	2	3	- - - -
			Year 1 costs ↓	Year 2 costs ↓	Year 3 costs ↓	

TABLE 6-12

OBLIGATIONS AND NON-DUMMY TASKS STARTING, FINISHING,
ACTIVE AND CONTINUING IN EACH YEAR
(ONE TABLE FOR EACH YEAR)

Year ______

Active			Start				Finish			Continue		
Task	Arc I.D.		Task	Arc I.D.		Obligation	Task	Arc I.D.		Task	Arc I.D.	
	i	j		i	j			i	j		i	j

TABLE 6-13

CRITICAL TASKS

Task Name	Arc	I.D.
	i	j

TABLE 6-14

SENSITIVITY ANALYSIS I*

Node No.	Sensitivity

*Each entry in the column labeled "sensitivity" is the approximate change in project cost per year of intentional increase in the achievement date of the final goal corresponding to the indicated node. The data in this table apply to small increases in the completion dates of the final goals of projects with multiple final goals. The approximations are exact if there are no concave cost-time curves.

TABLE 6-15

SENSITIVITY ANALYSIS II*

Saving	Delay

*Entries in the above table give the approximate project savings resulting when the achievement dates of all final goals are extended beyond the input dates by the specified amounts. The data in this table apply to projects with single or multiple final goals. The approximations are exact if there are no concave cost-time curves.

TABLE 6-16

a. Correspondence between Tasks and Arcs for PERT Diagram of Demonstration Project

Arc	Task	Abbreviation
(1,2)	Epidemiologic Studies	EPI STUD
(1,3)	Toxicologic Studies	TOX STUD
(2,3)	Dummy	D1
(1,4)	Gather Aerometric Data for Region A	AERO A
(1,5)	Prepare Emissions Inventory for Region A	EM A
(1,6)	Urban Factors Analysis for Region A	URB FAC A
(4,5)	Dummy	D2
(5,6)	Diffusion Modeling for Region A	DIF MOD A
(6,11)	Designate Air Quality Control Region A	DESIG A
(1,7)	Gather Aerometric Data for Region B	AERO B
(7,8)	Dummy	D3
(1,8)	Prepare Emissions Inventory for Region B	EM B
(1,9)	Urban Factors Analysis for Region B	URB FAC B
(8,9)	Diffusion Modeling for Region B	DIF MOD B
(9,12)	Designate Air Quality Control Region B	DESIG B
(1,13)	Develop and Publish Control Techniques	D&P TECH
(3,10)	Develop and Publish Criteria Documents	D&P CRIT
(10,14)	Dummy	D4
(11,14)	Dummy	D5
(12,14)	Dummy	D6
(13,14)	Dummy	D7

b. Final Goals

Node	Goal	Abbreviation
10	Air Quality Criteria Published	CRIT
11	Designation of Region A Completed	AQCR A
12	Designation of Region B Completed	AQCR B
13	Reports on Control Techniques Published	CONT

Node 14 represents a dummy final goal.

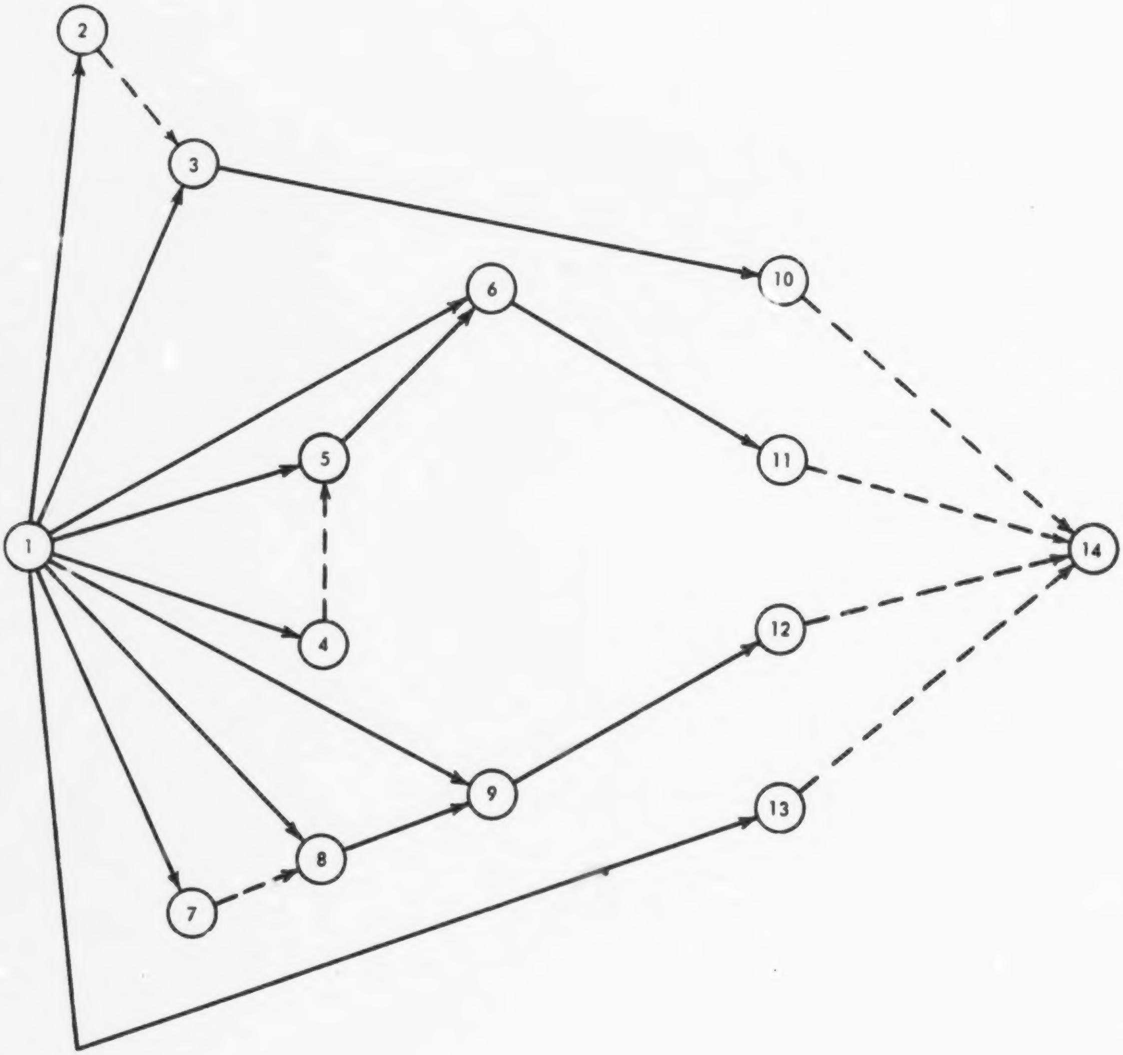

1
2
3
4
5
6
7
8
9
10
11
12
13
14

Figure 6-5. PERT Diagram for Demonstration Project

TABLE 6-17

INPUT DATA FOR DEMONSTRATION PROJECT

TASK NAME	I	J	D(1)	D(2)	D(3)	C(1)	C(2)	C(3)	S(I,J)
EPT STUD	1	2	1.00	1.50	2.00	85.00	75.00	70.00	.20
TOY STUD	1	3	1.00	1.50	2.00	225.00	150.00	125.00	.20
AERO A	1	4	.50	.50	.50	30.00	30.00	30.00	.20
EM A	1	5	.13	.31	.50	23.00	17.00	15.00	.20
URB FAC A	1	6	.17	.33	.50	15.00	13.50	12.00	.20
AERO B	1	7	.50	.50	.50	35.00	35.00	35.00	.20
EM B	1	8	.13	.31	.50	30.00	22.00	20.00	.20
URB FAC B	1	9	.17	.33	.50	15.00	13.50	12.00	.20
D+D TECH	1	13	.75	1.00	1.25	175.00	150.00	125.00	.20
D 1	2	3	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00
D+D CRIT	3	10	.50	.75	1.00	150.00	125.00	115.00	.20
D 2	4	5	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00
DIF MOD A	5	6	.08	.10	.12	10.00	9.50	9.00	.20
DESIG A	6	11	.17	.25	.33	10.00	10.00	8.00	.20
D3	7	8	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00
DIF MOD B	8	9	.08	.10	.12	12.50	12.00	11.50	.20
DESIG B	9	12	.17	.25	.33	11.00	11.00	9.00	.20
D4	10	14	.30	.65	1.00	-0.00	-0.00	-0.00	-0.00
D5	11	14	1.47	1.70	2.00	-0.00	-0.00	-0.00	-0.00
D6	12	14	1.47	1.70	2.00	-0.00	-0.00	-0.00	-0.00
D7	13	14	1.30	1.70	2.00	-0.00	-0.00	-0.00	-0.00

Output Tables for Demonstration Project

TABLE 6-18

TASK COSTS AND SCHEDULES

```
TASK NAME      I    J        COST  DURATION  INIT-DATE  END-DATE     SLACK

 EPT STUD      1    2       75.00      1.50       0.00      1.50      0.00
 TOX STUD      1    3      150.00      1.50       0.00      1.50      0.00
   AERO A      1    4       30.00       .50       0.00       .50      0.00
     EM A      1    5       15.00       .50       0.00       .50       .00
URB FAC A      1    6       12.00       .50       0.00       .50       .12
   AERO B      1    7       35.00       .50       0.00       .50      0.00
     EM B      1    8       20.00       .50       0.00       .50       .00
URB FAC B      1    9       12.00       .50       0.00       .50       .12
 D+P TECH      1   13      150.00      1.00       0.00      1.00      0.00
      D 1      2    3        0.00      0.00       1.50      1.50      0.00
 D+P CRIT      3   10      150.00       .50       1.50      2.00      0.00
      D 2      4    5        0.00      0.00        .50       .50      0.00
DIF MOD A      5    6        9.00       .12        .50       .62      0.00
  DESIG A      6   11       10.00       .21        .62       .83      0.00
       D3      7    8        0.00      0.00        .50       .50      0.00
DIF MOD B      8    9       11.50       .12        .50       .62      0.00
  DESIG B      9   12       11.00       .21        .62       .83      0.00
       D4     10   14        0.00       .30       2.00      2.30      0.00
       D5     11   14        0.00      1.47        .83      2.30       .00
       D6     12   14        0.00      1.47        .83      2.30       .00
       D7     13   14        0.00      1.30       1.00      2.30      0.00
TOTAL PROJECT COST     690.50
UPPER LIMIT ON TOTAL PROJECT COST IS    760.75
   WITH PROBABILITY 95 PERCENT.
```

TABLE 6-19

FINAL GOAL ACHIEVEMENT DATES

GOAL NAME	NODE NO.	ACHIEVEMENT DATE
CRIT	10	2.00
ADCR A	11	.83
ADCR B	12	.83
CONT	13	1.00

TABLE 6-20

TOTAL YEARLY COSTS OF PROJECT

YEAR	PROJECT COST	UP. BND.-COST-.95 PROB.
1	465.50	534.97
2	225.00	283.65
3	0.00	0.00

TABLE 6-21

YEARLY TASK COSTS

```
 TASK NAME    I    J      YR1        YR2        YR3

  EPI STUD    1    2     50.00      25.00       0.00
  TOX STUD    1    3    100.00      50.00       0.00
    AERO A    1    4     30.00       0.00       0.00
      EM A    1    5     15.00       0.00       0.00
 URB FAC A    1    6     12.00       0.00       0.00
    AERO B    1    7     35.00       0.00       0.00
      EM B    1    8     20.00       0.00       0.00
 URB FAC B    1    9     12.00       0.00       0.00
  D+P TECH    1   13    150.00       0.00       0.00
       D 1    2    3      0.00       0.00       0.00
  D+P CRIT    3   10      0.00     150.00       0.00
       D 2    4    5      0.00       0.00       0.00
 DIF MOD A    5    6      9.00       0.00       0.00
   DESIG A    6   11     10.00       0.00       0.00
        D3    7    8      0.00       0.00       0.00
 DIF MOD B    8    9     11.50       0.00       0.00
   DESIG B    9   12     11.00       0.00       0.00
        D4   10   14      0.00       0.00       0.00
        D5   11   14      0.00       0.00       0.00
        D6   12   14      0.00       0.00       0.00
        D7   13   14      0.00       0.00       0.00
```

TABLE 6-22

OBLIGATIONS AND NON-DUMMY TASKS STARTING,
FINISHING, CONTINUING, AND ACTIVE IN EACH YEAR

YEAR 1

ACTIVE			START				FINISH			CONTINUE		
TASK	I	J	TASK	I	J	OBLGTN	TASK	I	J	TASK	I	J
EPI STUD	1	2	EPI STUD	1	2	75.00		0	0		0	0
TOX STUD	1	3	TOX STUD	1	3	150.00		0	0		0	0
AERO A	1	4	AERO A	1	4	30.00	AERO A	1	4		0	0
EM A	1	5	EM A	1	5	15.00	EM A	1	5		0	0
URB FAC A	1	6	URB FAC A	1	6	12.00	URB FAC A	1	6		0	0
AERO B	1	7	AERO B	1	7	35.00	AERO B	1	7		0	0
EM B	1	8	EM B	1	8	20.00	EM B	1	8		0	0
URB FAC B	1	9	URB FAC B	1	9	12.00	URB FAC B	1	9		0	0
D+P TECH	1	13	D+P TECH	1	13	150.00		0	0		0	0
DIF MOD A	5	6	DIF MOD A	5	6	9.00	DIF MOD A	5	6		0	0
DESIG A	6	11	DESIG A	6	11	10.00	DESIG A	6	11		0	0
DIF MOD B	8	9	DIF MOD B	8	9	11.50	DIF MOD B	8	9		0	0
DESIG B	9	12	DESIG B	9	12	11.00	DESIG B	9	12		0	0

TABLE 6-22 (Continued)

YEAR 2

ACTIVE			START				FINISH			CONTINUE		
TASK	I	J	TASK	I	J	OBLGTN	TASK	I	J	TASK	I	J
EPI STUD	1	2		0	0	0.00	EPI STUD	1	2	EPI STUD	1	2
TOX STUD	1	3		0	0	0.00	TOX STUD	1	3	TOX STUD	1	3
	0	0		0	0	0.00	D+P TECH	1	13		0	0
D+P CRIT	3	10	D+P CRIT	3	10	150.00		0	0		0	0

YEAR 3

ACTIVE			START				FINISH			CONTINUE		
TASK	I	J	TASK	I	J	OBLGTN	TASK	I	J	TASK	I	J
	0	0		0	0	0.00	D+P CRIT	3	10		0	0

TABLE 6-23

CRITICAL TASKS

TASK NAME	I	J
EPI STUD	1	2
TOX STUD	1	3
AERO A	1	4
FM A	1	5
AERO B	1	7
FM B	1	8
D+P TECH	1	13
D 1	2	3
D+P CRIT	3	14
D 2	4	5
DIF MOD A	5	6
DESIG A	6	11
D3	7	8
DIF MOD B	8	9
DESIG B	9	12
D4	10	14
D5	11	14
D6	12	14
D7	13	14

TABLE 6-24

SENSITIVITY ANALYSIS I

NODE NO.	SENSITIVITY
10	100.00
11	12.50
12	12.50
13	100.00

TABLE 6-25

SENSITIVITY ANALYSIS II

SAVING	DELAY
93.00	1.70
93.00	1.00
91.00	.95
83.00	.75
77.00	.65
53.00	.25
27.00	.12

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APPENDIX A

GENERAL STRUCTURE OF THE REGIONAL ECONOMIC MODEL

The variables of the model developed in this section can be specified and defined as follows:

Y_i = regional income of i^{th} SMSA,

C_i = regional consumption expenditure of i^{th} SMSA,

I_{ij} = investment expenditure by industry j of i^{th} SMSA,

Π_{ij} = capital share or gross profit by industry j of i^{th} SMSA,

K_{ij} = capital stock by industry j of i^{th} SMSA,

X_{ij} = value added by industry j of i^{th} SMSA,

N_{ij} = employment by industry j of i^{th} SMSA,

W_{ij} = average wage by industry j of i^{th} SMSA,

u_i = unemployment rate of i^{th} SMSA,

U_i = unemployment of i^{th} SMSA,

$\overline{N}_i$ = employment by industries other than manufacturing industries of i^{th} SMSA,

N_i^T = $(\sum_j N_{ij} + \overline{N}_i)$ total regional employment of i^{th} SMSA,

L_i = regional labor force of i^{th} SMSA,

P_i = population of i^{th} SMSA,

Q_{ij} = electric power demand by industry of i^{th} SMSA,

M_i = migration of i^{th} SMSA,

G_i = government expenditure of i^{th} SMSA,

T_i = government revenue of i^{th} SMSA.

Integration of the Keynsian system and economic base theory can be best explained by the income determination block of this model:

(1) $Y_i = f(C_i, \sum_j X_{ij}, G_i)$

(2) $C_i = f(Y_i, C_i(t-1))$

(3) $G_i = f(T_i)$

This model is a cross-sectional regional growth model. Thus, the national influence upon a local economy can be measured by the regional market share of the output in each manufacturing industry. Because manufacturing is also regarded as export-oriented, the value added, $\sum_j X_{ij}$, also reflects the role of economic base theory of local demand, namely, local consumption and government expenditures. In this model, therefore, government expenditure is a simple function of government revenue.

The sectoral structure of the export-oriented industries has been based on 13 sector.

(4) $X_{ij}(t) = f(Y_i(t), Y_i(t-1), X_{ij}(t-1))$ $j=1, \ldots, 13$

(5) $I_{ij}(t) = f(\Pi_{ij}(t), K_{ij}(t-1))$ $j=1, \ldots, 13$

(6) $K_{ij}(t) = K_{ij}(t-1) + I_{ij}(t) - d_j K_{ij}(t)$ $j=1, \ldots, 13$

(7) $X_{ij}(t) = A_{ij} N_{ij}^{a}(t) K_{ij}^{1-a}(t)$ $j=1, \ldots, 13$

(8) $W_{ij}(t) = f(u_i, M_i, W_{ij}(t-1))$ $j=1, \ldots, 13$

(9) $Q_{ij}(t) = f(X_{ij}(t))$ $j=1, \ldots, 13$

Equation (4) is the demand function for the products in terms of the value added by industry j in region i, where Y_i (t) and Y_i (t-1) includes both local and external market demand of present and previous years, and X_{ij} (t-1) measures market share from the previous period. Equation (5) is a typical investment function, and equations (6) and (7) are familiar production structures developed in the St. Louis Model. Since output level (value added) and capital stock have been determined by the three previous equations ((4), (5) and (6)), equation (7) becomes an implicit demand function for labor requirements by industry j (N_{ij}). Wages (equation 8) are a function of the local unemployment rate (U_i), migration (M_i) and lagged wages ($W_{ij}(t-1)$). This relationship not only includes labor productivity but also shows, in part, the union bargaining power based on the last period wage level. Finally, equation (9), electric power demand by industry, is expressed in terms of a single function, current industrial output levels.

The equations for total industrial employment, other than for manufacturing industries, and therefore a functional description of the labor market in the regional economic model, is as follows:

(10) $\overline{N}_i = f(Y_i - \Sigma X_{ij})$

(11) $N_i^T = \sum_j N_{ij} + \overline{N}_i$

(12) $L_i = f(u_i, P_i, \sum_j W_{ij} N_{ij} / \sum_j N_{ij})$

(13) $U_i = L_i - N_i^T$

(14) $u_i = \frac{L_i - N_i^T}{L_i} \times 100$

Equation (10) is a labor demand function for the industries other than manufacturing industry. Equation (11) is derived from equations (7) and (10). Equation (12) represents the supply side of the labor market. The labor force (L_i) is expressed as a function of regional unemployment (u_i), the weighted average wage rate ($\sum_j W_{ij} N_{ij} / \sum_j N_{ij}$) and size of the population (P_i). Unemployment in equation (13) is the result of the difference between the size of the labor force and total regional employment. The amount of unemployment is expressed as a rate of unemployment in equation (14).

5. Model Estimation

The Model has been estimated on data from 31 largest SMSA's:

A. Regional income equation

(1) $$Y_i = \underset{(194.4)}{64.78} + \underset{(.137)}{.702}\, C_i + \underset{(.100)}{.481} \sum_{j=1}^{13} X_{ij} + \underset{(.591)}{3.02}\, G_i \qquad R^2 = .994$$

B. Regional consumption function

(2) $$C_i = \underset{(40.9)}{15.11} + \underset{(.030)}{.012}\, Y_i + \underset{(.048)}{1.061}\, C_{it-1} \qquad R^2 = .999$$

C. Demand function of export-oriented industry

(3) $$X_{ijt} = a_j + b_j Y_{it} + c_j Y_{it-1} + d_j X_{ijt-1} \qquad j = 1, 2, \ldots, 13$$

The results of estimation are:

Equation Number	Industry Code	a_j	b_j	c_j	d_j	R_2
(3. 1)	1	-441. 9	. 103	-. 155	1. 135	. 998
		(532. 0)	(. 65)	(. 698)	(. 009)	
(3. 2)	2	224. 3	. 116	-. 160	1. 085	. 999
		(242. 3)	(. 218)	(. 232)	(. 006)	
(3. 3)	3	-529. 3	. 672	-. 942	1. 122	. 998
		(1138. 7)	(1. 38)	(1. 47)	(. 013)	
(3. 4)	4	-97. 52	. 252	-. 220	1. 090	. 999
		(927. 9)	(1. 11)	(1. 18)	(. 005)	
(3. 5)	5	-449. 9	. 313	-. 234	1. 097	. 999
		(927. 8)	(1. 08)	(1. 15)	(. 006)	
(3. 6)	6	-695. 9	. 635	-. 864	1. 135	. 998
		(946. 7)	(1. 20)	(1. 29)	(. 012)	
(3. 7)	7	-57. 36	1. 899	-2. 028	1. 061	. 999
		(117. 7)	(. 717)	(. 767)	(. 003)	
(3. 8)	8	-1745. 2	. 738	-. 890	1. 131	. 997
		(1089. 7)	(1. 248)	(1. 34)	(. 013)	
(3. 9)	9	-6063. 1	3. 783	-3. 580	1. 095	. 998
		(5835. 1)	(6. 07)	(7. 04)	(. 008)	
(3. 10)	10	-412. 4	9. 380	10. 127	1. 121	. 999
		(710. 5)	(2. 59)	(2. 77)	(. 011)	
(3. 11)	11	4680. 7	1. 309		1. 047	. 999
		(6835. 9)	(. 814)		(. 006)	
(3. 12)	12	-3165. 2	60. 785	-66. 56	1. 159	. 999
		(3635. 3)	(23. 32)	(24. 89)	(. 008)	
(3. 13)	13	23296. 6	7. 337	-26. 331	1. 154	. 997
		(29916. 3)	(41. 2)	(42. 7)	(. 030)	

D. Production functions

(4) $$X_{ij} = A_j N_{ij}^{\alpha j} K_{ij}^{1-\alpha j} \qquad j = 1, \ldots, 13$$

Equation Number	Industry Code	A_j	α_j	$1-\alpha_j$	R_2
(4.1)	1	4.505 (1.002)	.5217	.4783	.974
(4.2)	2	2.6375 (.659)	.5208	.4792	.993
(4.3)	3	3.1312 (.418)	.5315	.4685	.993
(4.4)	4	4.708 (.919)	.5269	.4731	.973
(4.5)	5	2.924 (.811)	.5293	.4707	.983
(4.6)	6	4.000 (.366)	.5215	.4785	.997
(4.7)	7	3.548 (.531)	.5085	.4915	.986
(4.8)	8	3.579 (.525)	.5269	.4731	.995
(4.9)	9	3.296 (.423)	.5301	.4699	.998
(4.10)	10	3.559 (.525)	.5292	.4708	.997
(4.11)	11	4.517 (1.101)	.5252	.4748	.998
(4.12)	12	6.204 (.358)	.5231	.4769	.996
(4.13)	13	4.656 (.358)	.5269	.4731	.998

E. Capital share (or gross profit) relation

Given the Cobb-Douglas production function, capital share or gross profit can be derived as follows:

(5) $\pi_{ij} = (1 - \alpha_j) X_{ij}$ $\quad j = 1, \ldots, 13$

Capital share coefficients, α_j, can be obtained as in the preceding table for the production function. Therefore, there are 13 equations (5.1) through (5.13).

F. Investment function

(6) $I_{ijt} = a_j + b_j \pi_{ijt} + c_j K_{ijt-1}$ $\quad j = 1, \ldots, 13$

Equation Number	Industry Code	a_j	b_j	c_j	R_2
(6.1)	1	-420.6	.168	.0788	.963
		(326.1)	(.067)	(.0516)	
(6.2)	2	-40.24	.4707	-.0809	.986
		(183.7)	(.116)	(.0526)	
(6.3)	3	-903.6	.6033	-0.0852	.963
		(949.3)	(.099)	(.0411)	
(6.4)	4	-123.1	-.067	.2032	.971
		(824.1)	(.047)	(.0304)	
(6.5)	5	-624.5	-.4419	.2792	.974
		(1668.6)	(.133)	(.034)	
(6.6)	6	-782.2	.4573	-.1403	.967
		(504.4)	(.0942)	(.0682)	
(6.7)	7	87.10	.0686	.1004	.989
		(108.2)	(.147)	(.079)	

Equation Number	Industry Code	a_j	b_j	c_j	R_2
(6.8)	8	-2046.9	.0686	.1004	.989
		(1078.2)	(.126)	(.0589)	
(6.9)	9	-2923.3	.6747	-.0979	.992
		(2378)	(.0595)	(.023)	
(6.10)	10	-62.79	.6177	-.1568	.982
		(414.1)	(.118)	(.062)	
(6.11)	11	5832.6	.2157	.0034	.986
		(2637.3)	(.0398)	(.025)	
(6.12)	12	1459.8	.2174	-.1571	.979
		(997.9)	(.044)	(.078)	
(6.13)	13	(11708.3)	.3586	-.1348	.964
		(11516)	(.048)	(.0412)	

G. Capital stock indentity

(7) $K_{ijt} = K_{ijt-1} + I_{ijt} - d_j K_{ijt}$ $j = 1, \ldots, 13$

where d_j is depreciation rate in industry j.

Equation Number	Industry Code	Depreciation Rate d_j
(7.1)	1	.0675
(7.2)	2	.0480
(7.3)	3	.0534
(7.4)	4	.0643
(7.5)	5	.0614

Equation Number	Industry Code	Depreciation Rate d_j
(7.6)	6	.0618
(7.7)	7	.0625
(7.8)	8	.0624
(7.9)	9	.0564
(7.10)	10	.0529
(7.11)	11	.0806
(7.12)	12	.0736
(7.13)	13	.0714

G. Wage functions

(8) $$W_{ijt} = a_j + b_j W_{ijt-1} + c_j u_i + d_j M_i \qquad j = 1, \ldots, 13$$

Equation Number	Industry Code	a_j	b_j	c_j	d_j	R_2
(8.1)	1	.3388 (.402)	.985 (.031)	-.0516 (.0969)	-.0183 (.022)	.981
(8.2)	2	-.658 (.458)	1.009 (.063)	.2109 (.106)	-.0069 (.0195)	.949
(8.3)	3	-2.103 (.482)	1.2906 (.066)	.1455 (.112)	.0384 (.0263)	.944
(8.4)	4	.6105 (.432)	.9961 (.032)	-.1556 (.146)	-.0135 (.0298)	.979
(8.5)	5	.0244 (.454)	.9782 (.023)	.0098 (.135)	.0473 (.028)	.987

Equation Number	Industry Code	a_j	b_j	c_j	d_j	R_2
(8.6)	6	.4938 (.399)	.9593 (.059)	-.0909 (.076)	.0025 (.018)	.911
(8.7)	7	.0817 (.171)	1.0179 (.018)	-.0214 (.044)	.0062 (.010)	.996
(8.8)	8	.1974 (.571)	1.048 (.055)	-.1399 (.127)	-.0275 (.032)	.934
(8.9)	9	.2051 (.522)	1.0629 (.085)	-.1701 (.175)	-.0233 (.039)	.917
(8.10)	10	.2925 (.120)	.8941 (.018)	.0419 (.025)	.0206 (.006)	.998
(8.11)	11	-.2676 (1.34)	.9721 (.064)	.2279 (.490)	.0057 (.064)	.938
(8.12)	12	.2202 (.416)	.9936 (.029)	.0906 (.157)	-.0050 (.0195)	.984
(8.13)	13	.8552 (.90)	1.0008 (.078)	-.2850 (.278)	-.0443 (.035)	.847

H. Employment by industries other than manufacturing industries

(9) $$\overline{N}_i = \underset{(31.96)}{121.73} + \underset{(.004)}{.1262}\,(Y_i - \sum_{j=1}^{13} X_{ij}) \qquad R^2 = .968$$

I. Total employment

(10) $$N_i^T = \overline{N}_i + \sum_{j=1}^{13} N_{ij}$$

J. Labor force supply function

(11) $$L_i = \underset{(145.9)}{312.735} + \underset{(.005)}{.4662}\, P_i - \underset{(18.62)}{39.749}\left(\sum_{j=1}^{13} W_{ij} N_{ij} / \sum_{j=1}^{13} N_{ij}\right)$$

$$- \underset{(17.1)}{31.319}\, u_i \qquad R^2 = .995$$

K. Regional unemployment

(12) $$U_i = L_i - N_i^T$$

L. Regional unemployment rate

(13) $$u_i = \frac{U_i}{L_i} \times 100$$

M. Electric power demand function by industry

(14) $$Q_{ijt} = a_j + b_j V_{ijt}$$

Equation Number	Industry Code	a_j	b_j	R^2
(14.1)	1	4.6152 (2.81028)	.79804 (.042681)	.933
(14.2)	2	29.0536 (11.39258)	.205259 (.27310)	.799
(14.3)	3	2.49808 (.00002362)	-.0049799 (.0031956)	.999
(14.4)	4	-22.75882 (32.82634)	11.98117 (.20547)	.991
(14.5)	5	-.0051536 (.0052080)	3.76464 (.00003890)	.999

Equation Number	Industry Code	a_j	b_j	R^2
(14.6)	6	-0.0054016 (.0017908)	1.50925 (.00001555)	.999
(14.7)	7	-5.84172 (4.16915)	1.83096 (.14389)	.915
(14.8)	8	0.028437 (3.85147)	2.38909 (.033961)	.994
(14.9)	9	2.01174 (253.2238)	5.15529 (.39807)	.879
(14.10)	10	-5.53004 (2.19407)	1.76572 (.05378)	.994
(14.11)	11	-11.33485 (10.35392)	.75855 (.010675)	.996
(14.12)	12	-0.005127 (.007177)	.75402 (.00001392)	.999
(14.13)	13	-.004861 (.0020049)	.10026 (.00000058)	.999

N. Government Expenditure function

$$G_{ij} = 55.2434 + .92961T \qquad R^2 = .998 \tag{15}$$

$$(10.1457) \; (.007116)$$

STANDARD METROPOLITAN STATISTICAL AREA (SMSA) CODE NUMBER

Code Number	Standard Metropolitan Statistical Area
01	New York, New York
02	Chicago, Illinois
03	Los Angeles/Long Beach, California
04	Philadelphia, Pennsylvania - New Jersey
05	Detroit, Michigan
06	San Francisco/Oakland, California
07	Pittsburgh, Pennsylvania
08	St. Louis, Missouri - Illinois
09	Washington, D. C. - Maryland - Virginia
10	Cleveland, Ohio
11	Baltimore, Maryland
12	Newark, New Jersey
13	Minneapolis/St. Paul, Minnesota
14	Houston, Texas
15	Buffalo, New York
16	Milwaukee, Wisconsin
17	Cincinnati, Ohio - Indiana - Kentucky
18	Paterson/Clifton/Passaic, New Jersey
19	Dallas, Texas

Code Number	Standard Metropolitan Statistical Area
20	Seattle/Everett, Washington
21	Kansas City, Kansas
22	San Diego, California
23	Atlanta, Georgia
24	Indianapolis, Indiana
25	Miami, Florida
26	Denver, Colorado
27	New Orleans, Louisiana
28	Portland, Oregon - Washington
29	Columbus, Ohio
30	Rochester, New York
31	Dayton, Ohio

Capital Stock by Sector (per million) K_{ijt} (t = 1966)

SMSA Code No.	Ind. 1 K_{i1}	Ind. 2 K_{i2}	Ind. 3 K_{i3}	Ind. 4 K_{i4}	Ind. 5 K_{i5}	Ind. 6 K_{i6}	Ind. 7 K_{i7}
01	32165.9	63636.5	367201.3	44209.5	46500.2	109990.5	18537.5
02	47462.9	0.0	495707.7	139167.5	327552.0	253039.3	0.0
03	146195.8	0.0	258557.4	121714.5	607794.6	228640.2	10610.3
04	107191.2	95930.8	426807.5	224913.0	654113.2	155807.0	10709.6
05	8134.3	0.0	91695.2	170250.4	174414.5	117239.7	38726.8
06	105485.3	0.0	141116.5	148451.0	643722.1	24925.2	1936.1
07	3672.1	0.0	52221.3	81173.4	27916.0	20368.8	77434.8
08	47241.1	0.0	109140.8	251341.1	432778.6	26532.2	2176.4
09	0.0	0.0	17767.3	5691.9	0.0	0.0	0.0
10	17627.9	34694.0	90305.9	352933.5	123179.6	53748.8	3363.3
11	11182.4	13168.2	95616.8	80966.4	33715.6	31718.8	15593.5
12	7230.7	46276.7	45539.3	161671.4	0.0	64322.7	0.0
13	34011.7	10353.0	97851.6	13963.3	122791.3	0.0	0.0
14	21851.3	6303.8	70311.1	624010.6	1075774.2	16582.1	781.5
15	11099.4	19130.9	132235.8	148593.0	63393.8	47564.9	0.0
16	6805.4	15016.0	66402.7	6400.5	7205.7	12920.4	0.0
17	37241.5	0.0	110221.4	117319.4	0.0	45672.9	1207.3
18	4783.8	117328.2	141298.8	71051.0	15310.6	73224.7	6388.2
19	17917.7	5597.8	58289.5	23045.7	17615.8	11930.5	0.0
20	22819.0	1427.4	99064.3	3294.0	4629.4	1725.3	0.0
21	21793.2	10291.6	0.0	115948.1	199648.4	12239.8	1004.0
22	2264.3	0.0	1020.0	10330.0	959.7	1121.3	0.0
23	7877.1	1405.2	92468.2	20177.9	5766.1	5838.7	0.0
24	19120.2	0.0	48245.0	3556.1	6324.2	44788.0	0.0
25	11604.7	6477.2	20588.0	15733.5	2885.7	6512.0	1547.5
26	30024.1	0.0	0.0	3515.2	29345.3	44267.4	0.0
27	11799.2	0.0	35626.2	30441.8	70653.4	1243.5	0.0
28	18468.4	0.0	182742.3	37917.7	6092.6	5700.7	0.0
29	6733.0	0.0	27501.9	13819.5	0.0	18343.9	836.0
30	111607.6	0.0	42412.2	2980.3	0.0	25541.3	1031.4
31	3052.3	0.0	93205.4	4530.9	14242.6	98506.9	0.0

Capital Stock by Sector (per million) K_{ijt} (t = 1966)
(continued)

SMSA Code No.	Ind. 8 K_{i8}	Ind. 9 K_{i9}	Ind. 10 K_{i10}	Ind. 11 K_{i11}	Ind. 12 K_{i12}	Ind. 13 K_{i13}	
01	143771.5	161174.9	91860.5	0.0	254768.6	6438420.0	
02	349789.8	2921994.0	149719.1	0.0	198809.0	5373637.0	
03	332216.2	296511.3	29828.0	358853.7	521942.6	4201971.0	
04	202528.6	703106.8	34942.2	0.0	138937.0	2905063.5	
05	177695.3	1177470.7	9833.1	2751829.5	39940.8	2088720.2	
06	123386.6	250621.7	7127.0	87758.9	28049.5	1063755.7	
07	157073.9	2243596.0	0.0	0.0	33177.5	955292.3	
08	181547.3	196047.3	7936.6	0.0	298801.5	1239772.2	
09	2818.1	0.0	0.0	0.0	12660.8	317935.5	
10	75563.5	749344.6	11590.0	533735.6	27597.1	1348834.2	
11	78325.1	42710.0	0.0	279188.2	0.0	1133338.7	
12	55578.6	76783.2	52621.3	206798.7	0.0	1778995.2	
13	169960.5	32673.3	0.0	0.0	47811.7	1163082.2	
14	98273.2	177877.5	0.0	8402.2	7320.6	615085.6	
15	119994.3	610344.1	0.0	391140.8	0.0	639730.7	
16	43346.4	166947.3	0.0	167282.8	0.0	1402495.2	
17	46674.4	13863.8	0.0	367207.5	0.0	976590.7	
18	39228.5	24656.6	0.0	155300.4	0.0	967524.8	
19	70677.5	0.0	0.0	178691.0	0.0	668293.3	
20	45366.9	22703.9	0.0	750338.6	0.0	251557.7	
21	83751.0	59848.4	0.0	333996.8	1912.4	745717.6	
22	15057.2	1862.7	0.0	616.4	45255.6	290649.2	
23	65252.8	0.0	0.0	471607.5	0.0	454371.5	
24	39232.9	31130.5	0.0	359111.3	0.0	642854.1	
25	34748.9	0.0	0.0	1958.5	6416.1	231094.1	
26	41973.8	0.0	0.0	0.0	66208.4	414897.0	
27	78891.6	0.0	0.0	0.0	40944.9	297830.3	
28	39797.6	0.0	0.0	17418.2	12155.3	414432.7	
29	72435.1	36700.8	0.0	0.0	59720.7	480911.2	
30	30711.5	9139.6	0.0	0.0	1234.8	1372604.2	
31	49260.4	26764.3	0.0	0.0	42093.8	759646.1	

Quantity of Electric Power by Sector (millions KWH) Q_{ijt} -(t = 1966)

SMSA Code No.	Ind 1 Q_{i1}	Ind. 2 Q_{i2}	Ind. 3 Q_{i3}	Ind. 4 Q_{i4}	Ind. 5 Q_{i5}	Ind. 6 Q_{i6}	Ind. 7 Q_{i7}
01	31.38	134.70	732.81	590.36	71.67	229.30	20.91
02	84.31	0.00	989.27	1858.39	504.86	527.53	0.00
03	183.55	0.00	515.99	1135.97	936.80	476.67	11.97
04	104.59	225.40	851.77	2452.16	1008.19	324.82	12.08
05	10.63	0.00	182.99	1798.19	268.82	244.42	43.69
06	102.93	0.00	281.62	1443.92	992.17	51.96	2.18
07	6.52	0.00	104.21	886.42	43.02	42.46	196.45
08	81.39	0.00	217.81	3186.21	667.04	55.31	2.45
09	0.00	0.00	35.45	76.00	0.00	0.00	0.00
10	17.20	146.05	180.22	3964.75	189.85	112.05	3.79
11	10.91	55.43	190.82	940.62	51.96	66.12	43.47
12	8.45	194.81	90.88	2123.87	0.00	134.10	0.00
13	60.42	43.58	195.28	172.90	189.26	0.00	0.00
14	38.81	26.53	140.31	7435.35	1658.10	34.57	0.88
15	10.83	80.53	263.90	1984.26	97.70	99.16	0.00
16	12.08	63.21	132.51	85.46	11.10	26.93	0.00
17	56.47	0.00	219.96	1168.78	0.00	95.21	1.36
18	8.49	195.36	281.98	948.79	23.59	152.65	7.20
19	24.52	23.56	116.32	215.48	27.15	24.87	0.00
20	27.29	6.00	197.70	43.98	7.13	3.59	0.00
21	37.55	43.32	0.00	1469.85	307.72	25.51	1.13
22	4.02	0.00	2.03	137.94	1.47	2.33	0.00
23	7.75	5.91	184.53	141.21	8.88	12.17	0.00
24	33.96	0.00	96.28	7.69	9.74	93.37	0.00
25	11.32	27.26	41.08	210.10	4.44	13.57	1.74
26	29.29	0.00	0.00	46.94	45.23	92.28	0.00
27	12.86	0.00	71.09	372.58	108.89	2.59	0.00
28	18.02	0.00	364.69	338.10	9.39	11.88	0.00
29	11.96	0.00	54.88	93.47	0.00	38.24	0.94
30	108.90	0.00	84.64	6.44	0.00	53.24	1.16
31	5.42	0.00	186.00	4.70	21.95	205.36	0.00

Quantity of Electric Power by Sector (millions KWH) Q_{ijt} (t = 1966)
(continued)

SMSA Code No.	Ind. 8 Q_{i8}	Ind. 9 Q_{i9}	Ind. 10 Q_{i10}	Ind. 11 Q_{i11}	Ind. 12 Q_{i12}	Ind. 13 Q_{i13}	
01	326.69	328.72	157.20	0.00	638.02	1119.06	
02	713.70	14543.56	0.00	0.00	497.88	933.99	
03	696.93	1793.01	38.61	370.89	1307.10	730.34	
04	432.50	1824.61	51.63	0.00	347.94	504.93	
05	432.23	3028.42	13.47	2844.13	100.02	363.04	
06	255.23	1972.97	9.86	90.70	70.24	184.89	
07	350.70	6049.58	0.00	0.00	83.08	166.03	
08	374.34	1859.34	10.98	0.00	748.29	215.48	
09	5.74	0.00	0.00	0.00	31.70	55.26	
10	160.22	2600.31	15.00	551.63	69.11	234.44	
11	161.98	921.46	0.00	288.55	0.00	196.98	
12	113.40	1126.81	87.36	213.73	0.00	309.20	
13	346.78	435.69	0.00	0.00	119.73	202.15	
14	201.92	709.58	0.00	0.00	18.33	106.90	
15	244.83	1623.11	0.00	404.26	0.00	111.19	
16	88.44	301.72	0.00	172.89	0.00	243.76	
17	97.40	21.73	0.00	379.52	0.00	169.74	
18	91.53	56.37	0.00	0.00	0.00	168.16	
19	144.20	0.00	0.00	184.68	0.00	116.15	
20	92.56	63.02	0.00	775.50	0.00	43.72	
21	172.69	93.29	0.00	345.20	4.78	129.61	
22	30.72	5.17	0.00	0.63	113.33	50.51	
23	133.14	0.00	0.00	487.42	0.00	78.97	
24	80.05	48.79	0.00	371.15	0.00	111.73	
25	73.68	0.00	0.00	2.02	16.06	40.16	
26	85.64	0.00	0.00	0.00	165.80	72.11	
27	160.96	0.00	0.00	0.00	102.53	51.76	
28	81.20	0.00	0.00	18.00	30.44	72.03	
29	149.29	57.52	0.00	0.00	149.55	83.58	
30	64.51	14.32	0.00	0.00	3.09	238.57	
31	100.51	41.95	0.00	0.00	105.41	132.03	

Value Added by Sector ($1000) X_{ijt} (t = 1966)

SMSA Code No.	Ind. 1 X_{ijt}	Ind. 2 X_{ijt}	Ind. 3 X_{ijt}	Ind. 4 X_{ijt}	Ind. 5 X_{ijt}	Ind. 6 X_{ijt}	Ind. 7 X_{ijt}
01	44140.7	57967.8	293352.0	43622.0	19038.0	151935.8	23472.7
02	65132.4	0.0	396014.0	137317.9	134105.9	349536.6	0.0
03	200621.7	0.0	206557.9	120096.8	24[illegible]842.4	313832.8	13435.1
04	147096.4	87385.4	340970.6	221923.7	[illegible]806.3	215224.5	13560.8
05	11162.6	0.0	74354.0	167987.[illegible]	[illegible]1408.6	161949.4	49036.9
06	144755.5	0.0	112736.0	[illegible]	263552.0	34430.5	2451.6
07	5039.2	0.0	41718.9	[illegible]	11429.3	28136.6	98050.1
08	64828.1	0.0	87191.[illegible]	[illegible]	177187.7	36650.4	2755.8
09	0.0	0.0	14194.[illegible]	[illegible].2	0.0	0.0	0.0
10	24190.5	31603.5	72144.2	348242.6	50432.0	74246.1	4258.7
11	15345.5	11995.2	76386.9	79890.2	13803.8	43814.8	19745.0
12	9922.6	42154.3	36380.7	159522.7	0.0	88852.3	0.0
13	46673.6	9430.8	78172.3	13777.7	50273.0	0.0	0.0
14	29986.2	5742.3	56170.5	615717.1	440442.2	22905.8	989.6
15	15231.5	17426.7	105641.4	146618.1	25954.6	65703.9	0.0
16	9338.9	13678.4	53048.2	6315.4	2950.1	17847.7	0.0
17	51105.8	0.0	88054.3	115760.1	0.0	63090.5	1528.8
18	6564.8	106876.6	112881.6	70166.7	6268.4	101149.1	8089.0
19	24588.2	5099.1	46566.7	22739.4	7212.2	16480.2	0.0
20	31314.1	1300.2	79141.1	3250.2	1895.3	2383.3	0.0
21	29906.3	9374.8	0.0	114407.0	81739.8	16907.4	1271.3
22	3107.2	0.0	814.9	10192.7	392.9	1548.9	0.0
23	10809.6	1280.0	73871.6	19909.7	2360.7	8065.4	0.0
24	26238.2	0.0	38542.2	3508.8	2589.2	61868.1	0.0
25	15924.9	5900.2	16147.4	15524.4	1181.4	8995.3	1959.5
26	41201.5	0.0	0.0	3468.5	12014.5	61149.0	0.0
27	16191.9	0.0	28461.2	30037.2	28926.8	1717.7	0.0
28	25343.8	0.0	145990.3	37413.7	2494.4	7874.7	0.0
29	9239.6	0.0	21970.9	13635.8	0.0	25339.5	1058.6
30	153156.9	0.0	33882.5	2940.7	0.0	35281.6	1306.0
31	4188.7	0.0	74460.5	4470.7	5831.1	136072.8	0.0

Value Added by Sector ($1000) X_{ijt} (t = 1966)
(continued)

SMSA Code No.	Ind. 8 X_{ijt}	Ind. 9 X_{ijt}	Ind. 10 X_{ijt}	Ind. 11 X_{ijt}	Ind. 12 X_{ijt}	Ind. 13 X_{ijt}
01	126894.5	122991.0	90684.3	0.0	846154.7	846154.7
02	308728.7	2229746.0	147802.0	0.0	660298.1	660298.1
03	293218.0	226264.9	29446.1	491883.0	1733511.2	1733511.2
04	178754.2	536534.1	34494.7	0.0	461446.8	461446.8
05	156835.9	898516.3	9797.2	3771949.5	132654.4	132654.4
06	108902.5	191247.0	7035.7	120291.7	93159.9	93159.9
07	138635.3	1712066.2	0.0	0.0	110191.3	110191.3
08	160235.8	149601.8	7834.9	0.0	992399.6	992399.6
09	2487.2	0.0	0.0	0.0	42050.0	42050.0
10	66693.2	571817.7	11441.6	731594.7	91657.6	91657.6
11	69130.7	32591.6	0.0	382685.0	0.0	0.0
12	49054.3	58592.5	51947.5	283460.4	0.0	0.0
13	150009.2	24932.7	0.0	0.0	158795.4	158795.4
14	86737.1	135736.6	0.0	11517.0	24313.8	24313.8
15	105908.4	465747.6	0.0	536139.3	0.0	0.0
16	38258.0	127395.9	0.0	229295.5	0.0	0.0
17	41195.4	10579.3	0.0	503333.6	0.0	0.0
18	34623.5	18814.7	0.0	212871.2	0.0	0.0
19	62380.8	0.0	0.0	244932.9	0.0	0.0
20	40041.3	17325.1	0.0	1028493.7	0.0	0.0
21	73919.6	45669.7	0.0	457811.5	6351.7	6351.7
22	13289.7	1421.4	0.0	844.9	150306.0	150306.0
23	57592.9	0.0	0.0	646435.3	0.0	0.0
24	34627.5	23755.4	0.0	492236.1	0.0	0.0
25	30669.8	0.0	0.0	2684.5	21309.8	21309.8
26	37046.6	0.0	0.0	0.0	219895.9	219895.9
27	69630.7	0.0	0.0	0.0	135989.1	135989.1
28	35125.8	0.0	0.0	23875.2	40371.1	40371.1
29	63932.1	28006.0	0.0	0.0	198348.5	198348.5
30	27106.3	6974.3	0.0	0.0	4101.1	4101.1
31	43477.8	20423.5	0.0	0.0	139804.8	139804.8

Employment by Sector N_{ijt} (t = 1966)

SMSA Code No.	Ind. 1 N_{ijt}	Ind. 2 N_{ijt}	Ind. 3 N_{ijt}	Ind. 4 N_{ijt}	Ind. 5 N_{ijt}	Ind. 6 N_{ijt}	Ind. 7 N_{ijt}
01	3791.0	6201.8	30842.5	2600.3	1212.2	17980.2	2971.5
02	6834.0	0.0	37080.4	6429.3	7274.7	31834.3	0.0
03	12955.2	0.0	17387.2	5457.5	11576.5	26957.2	1363.7
04	8852.1	12115.4	26053.9	11551.6	16546.5	15540.0	1429.1
05	1080.2	0.0	6206.0	9441.3	1824.9	13840.0	2245.0
06	9860.3	0.0	8913.6	6937.7	7717.1	2651.2	241.4
07	342.4	0.0	4316.6	2685.4	858.7	2148.9	10031.3
08	3047.2	0.0	9260.6	12997.7	6811.8	3626.1	284.8
09	0.0	0.0	1547.3	364.8	0.0	0.0	0.0
10	1487.5	4731.9	6935.8	8195.9	2249.6	8061.3	418.2
11	1968.5	1691.2	7862.5	4946.0	714.4	5581.1	2434.4
12	1062.2	3892.6	8538.7	6944.7	0.0	9101.2	0.0
13	2252.8	1802.3	7110.2	695.3	1613.4	0.0	0.0
14	14553.3	693.0	4198.2	15520.9	13664.7	1901.5	130.8
15	1448.0	1628.5	8847.5	9501.9	1069.4	5743.3	0.0
16	522.4	1765.1	5618.8	344.3	242.2	1843.6	0.0
17	2069.1	0.0	8978.1	6443.4	0.0	4264.4	204.6
18	472.7	10957.4	10065.9	3481.1	688.0	10111.8	702.1
19	2736.4	1042.5	4726.7	973.1	791.5	1803.4	0.0
20	3033.0	172.7	3997.8	211.1	155.5	283.9	0.0
21	1405.7	1280.3	0.0	5996.1	3142.4	1672.7	131.4
22	181.4	0.0	125.4	434.4	54.9	184.5	0.0
23	1277.3	5138.4	8051.4	1292.6	193.8	669.3	0.0
24	1050.9	0.0	4364.0	331.0	461.1	5711.9	0.0
25	2345.7	712.0	1793.0	1279.1	165.0	1043.6	213.9
26	3257.0	0.0	0.0	264.6	785.6	6639.3	0.0
27	2402.7	0.0	2868.3	1379.3	1656.2	204.6	0.0
28	2009.8	0.0	9582.2	1206.7	444.0	696.4	0.0
29	544.2	0.0	1955.7	1222.6	0.0	1727.1	145.3
30	10436.0	0.0	3752.8	202.6	0.0	3231.9	180.1
31	205.3	0.0	7857.4	700.3	477.7	13306.8	0.0

Employment by Sector
(continued)

N_{ijt}

(t = 1966)

SMSA Code No.	Ind. 8 N_{ijt}	Ind. 9 N_{ijt}	Ind. 10 N_{ijt}	Ind. 11 N_{ijt}	Ind. 12 N_{ijt}	Ind. 13 N_{ijt}	
01	9154.1	9538.9	8162.1	0.0	62280.9	1147389.7	
02	25768.2	147310.5	10947.5	0.0	41646.6	834303.7	
03	23332.1	18825.5	2676.5	21717.2	146401.4	638727.1	
04	14396.0	35622.5	2963.2	0.0	45008.6	479845.0	
05	10115.1	50654.6	2093.6	195779.7	12781.6	367613.8	
06	8196.4	11433.4	459.0	7485.2	8208.3	162377.2	
07	12301.5	131792.2	0.0	0.0	8889.5	193568.5	
08	12422.8	13515.2	591.9	0.0	62767.5	214006.6	
09	1768.3	0.0	0.0	0.0	4903.7	38944.6	
10	5192.0	39884.4	1132.0	48866.0	10601.1	228624.5	
11	4623.9	2304.4	0.0	27049.6	0.0	160009.5	
12	3482.6	3901.6	4503.8	14394.4	0.0	264810.7	
13	7805.6	1942.9	0.0	0.0	9025.8	181084.6	
14	6212.5	10822.7	0.0	27422.7	2835.5	70122.8	
15	8371.1	32659.5	0.0	36803.1	0.0	98211.8	
16	3470.0	14153.7	0.0	19878.6	0.0	205990.1	
17	3274.7	1316.6	0.0	28152.5	0.0	137401.8	
18	2919.0	1722.9	0.0	17033.0	0.0	149410.4	
19	4352.9	0.0	0.0	10508.4	0.0	110165.0	
20	2955.2	1308.3	0.0	71431.3	0.0	39262.4	
21	5730.8	4483.0	0.0	28955.7	737.2	125770.5	
22	811.8	187.6	0.0	106.7	13417.2	46989.8	
23	5028.3	0.0	0.0	34839.9	0.0	76950.5	
24	3724.3	1904.8	0.0	39724.6	0.0	94874.5	
25	2707.5	0.0	0.0	348.8	2413.2	49059.6	
26	3782.7	0.0	0.0	0.0	19063.8	48358.2	
27	4624.9	0.0	0.0	0.0	13152.3	39333.9	
28	2433.6	0.0	0.0	1919.4	4392.2	65409.9	
29	4889.4	3036.8	0.0	0.0	18501.7	67678.4	
30	1874.9	761.0	0.0	0.0	532.9	161183.6	
31	3409.6	3073.8	0.0	0.0	13776.9	138793.6	

Investment by Sector ($1000) I_{ijt} (t = 1966)

SMSA Code No.	Ind. 1 I_{ijt}	Ind. 2 I_{ijt}	Ind. 3 I_{ijt}	Ind. 4 I_{ijt}	Ind. 5 I_{ijt}	Ind. 6 I_{ijt}	Ind. 7 I_{ijt}
01	3804.8	6283.0	38266.4	5086.6	5216.1	12384.5	2284.9
02	8506.1	0.0	81861.2	24491.5	56700.6	43908.9	0.0
03	29247.4	0.0	48086.4	23956.5	117877.9	44439.8	2175.4
04	16937.5	13282.4	61432.7	34812.4	99359.3	23732.7	1745.4
05	961.1	0.0	9543.4	19565.8	19541.7	13185.2	4768.2
06	15988.7	0.0	19402.9	22021.5	93635.9	3636.1	303.0
07	470.1	0.0	5950.1	10129.3	3403.0	2491.6	10297.9
08	8786.5	0.0	18763.0	45935.7	77848.4	4783.8	415.6
09	0.0	0.0	1960.5	689.8	0.0	0.0	0.0
10	2895.0	5019.4	13559.7	56822.1	19476.8	8521.3	569.0
11	1381.5	1369.4	10467.2	9741.6	3959.4	3738.2	2004.0
12	1071.1	5950.6	6105.2	23428.1	0.0	9162.8	0.0
13	5536.9	1482.9	14552.2	2228.0	19239.1	0.0	0.0
14	3146.9	784.5	0136.0	87850.3	148350.6	2293.6	116.4
15	1305.4	1875.9	13691.0	16996.2	7068.3	5323.5	0.0
16	1126.2	2191.3	10054.1	1038.5	1148.4	2064.6	0.0
17	5565.4	0.0	14920.1	17153.4	0.0	6565.5	186.4
18	659.4	13878.6	17488.0	9564.3	2016.8	9676.8	912.3
19	3525.2	991.8	10747.7	4459.7	3358.1	2279.3	0.0
20	1854.6	88.1	6656.9	257.0	347.9	130.4	0.0
21	3782.0	1584.7	0.0	19747.4	33427.2	2054.4	179.2
22	363.4	0.0	149.3	1624.5	148.1	173.5	0.0
23	1381.6	218.9	14916.7	3473.8	976.1	990.8	0.0
24	3256.5	0.0	7537.8	594.1	1038.4	7373.3	0.0
25	2210.5	1107.1	3631.8	2946.1	532.0	1203.3	302.4
26	7518.5	0.0	0.0	868.9	7169.1	10833.4	0.0
27	1309.0	0.0	3451.0	3279.0	7406.7	130.8	0.0
28	3339.3	0.0	30469.8	6733.5	1064.3	998.3	0.0
29	926.0	0.0	3395.5	1856.1	0.0	2418.7	119.1
30	21993.8	0.0	7760.9	577.6	0.0	4887.9	208.3
31	493.5	0.0	13757.7	717.9	2215.7	15366.3	0.0

Investment by Sector ($1000) I_{ijt} (t = 1966)
(continued)

SMSA Code No.	Ind. 8 I_{ijt}	Ind. 9 I_{ijt}	Ind. 10 I_{ijt}	Ind. 11 I_{ijt}	Ind. 12 I_{ijt}	Ind. 13 I_{ijt}	
01	16267.0	17272.9	9520.0	0.0	31689.4	786710.3	
02	60889.4	491181.3	24638.5	0.0	36842.1	984015.6	
03	64753.5	56022.2	5530.2	76497.1	107600.4	857026.6	
04	30960.4	103281.7	5009.3	0.0	22800.8	470370.6	
05	20071.6	126031.4	1017.7	361229.1	4962.7	254941.6	
06	18067.5	35200.8	975.8	14452.7	4422.5	165387.1	
07	19300.3	262272.5	0.0	0.0	4449.5	126021.3	
08	32832.9	34283.6	1359.8	0.0	57396.9	235426.7	
09	336.1	0.0	0.0	0.0	1652.5	40799.4	
10	12021.2	114733.2	1733.6	94655.1	4700.5	226782.6	
11	9274.1	4801.8	0.0	38154.4	0.0	144444.1	
12	7947.7	10521.1	7024.4	33347.7	0.0	270486.6	
13	26794.4	4955.7	0.0	0.0	8074.9	193881.0	
14	13647.3	23639.0	0.0	1320.2	1098.9	90980.9	
15	13495.7	64997.6	0.0	51132.4	0.0	77736.4	
16	6950.4	25771.7	0.0	29877.2	0.0	237570.0	
17	6735.1	1917.6	0.0	59691.9	0.0	149754.5	
18	5205.6	3124.5	0.0	23443.7	0.0	137141.7	
19	13542.1	0.0	0.0	37500.3	0.0	134092.3	
20	3453.8	1592.8	0.0	70823.8	0.0	21426.9	
21	14103.7	9720.8	0.0	62343.0	343.5	132323.9	
22	2339.2	278.2	0.0	107.0	7539.3	47782.5	
23	11109.3	0.0	0.0	88901.9	0.0	81466.9	
24	6480.3	4955.9	0.0	65872.4	0.0	111997.7	
25	6440.4	0.0	0.0	398.7	1261.3	44921.6	
26	10295.0	0.0	0.0	0.0	16983.3	105516.0	
27	8346.8	0.0	0.0	0.0	4792.2	34204.8	
28	6991.2	0.0	0.0	3377.8	2271.9	76552.0	
29	9590.4	4639.9	0.0	0.0	8578.3	68022.8	
30	5894.1	1699.4	0.0	0.0	250.8	275846.6	
31	7711.2	4029.7	0.0	0.0	7062.5	125786.4	

Capital Stock by Sector at t-1 ($1000) K_{ijt-1} (t = 1966)

SMSA Code No.	Ind. 1 K_{ijt-1}	Ind. 2 K_{ijt-1}	Ind. 3 K_{ijt-1}	Ind. 4 K_{ijt-1}	Ind. 5 K_{ijt-1}	Ind. 6 K_{ijt-1}	Ind. 7 K_{ijt-1}
01	30532.5	60405.0	348554.5	41964.5	44138.9	104405.1	17596.1
02	42160.8	0.0	440332.1	123621.1	290961.0	224772.2	0.0
03	126817.6	0.0	224285.7	105581.2	527231.6	198334.0	9203.9
04	97489.7	87248.6	388179.1	204557.1	594912.6	141705.6	9740.3
05	7722.3	0.0	87051.0	161627.6	165580.8	111301.8	36765.3
06	96617.6	0.0	129253.3	135971.3	589606.8	22829.8	1775.4
07	3449.9	0.0	49061.4	76261.5	26226.8	19136.3	72749.2
08	41643.7	0.0	96209.1	221560.6	381500.1	23388.5	1918.5
09	0.0	0.0	16756.0	5367.9	0.0	0.0	0.0
10	15922.9	31338.3	81571.2	318796.5	111265.2	48550.0	3038.0
11	10555.8	12430.2	90258.3	76428.9	31826.1	29941.2	14719.7
12	6647.6	42545.1	41867.2	148634.9	0.0	59136.0	0.0
13	30770.8	9366.5	88527.6	12632.7	111090.8	0.0	0.0
14	20179.5	5821.5	64931.7	576269.2	993469.6	15313.4	721.7
15	10543.2	18172.3	125610.1	141147.7	60217.4	45181.6	0.0
16	6138.6	13544.7	59896.4	5773.3	6499.7	11654.4	0.0
17	34190.1	0.0	101190.3	107706.7	0.0	41930.7	1108.4
18	4447.4	109075.7	131360.3	66053.6	14233.7	68074.3	5938.9
19	15602.0	4874.3	50756.2	20067.2	15339.1	10388.6	0.0
20	22504.8	1407.7	97700.4	3248.6	4565.6	1701.6	0.0
21	19482.3	9200.3	0.0	103653.4	178478.4	10941.9	897.5
22	2053.7	0.0	925.2	9369.4	870.5	1017.0	0.0
23	7027.3	1253.6	82492.1	18000.9	5144.0	5208.8	0.0
24	17154.4	0.0	43284.9	3190.5	5674.0	40183.3	0.0
25	10177.5	5680.6	18056.1	13798.6	2530.8	5711.1	1357.2
26	24532.4	0.0	0.0	2872.2	23977.8	36170.5	0.0
27	11286.7	0.0	34078.6	29119.5	67584.4	1189.5	0.0
28	16375.8	0.0	162036.4	33621.3	5402.2	5054.8	0.0
29	6261.5	0.0	25575.8	12851.6	0.0	17059.2	777.5
30	97148.0	0.0	36917.4	2594.2	0.0	22232.3	897.7
31	2764.9	0.0	84427.6	4104.2	12901.2	89229.8	0.0

Capital Stock by Sector at t-1 ($1000) K_{ijt-1} (t = 1966)
(continued)

SMSA Code No.	Ind. 8 K_{ijt-1}	Ind. 9 K_{ijt-1}	Ind. 10 K_{ijt-1}	Ind. 11 K_{ijt-1}	Ind. 12 K_{ijt-1}	Ind. 13 K_{ijt-1}	
01	136470.6	152990.3	87195.7	0.0	241831.2	6111469.0	
02	310714.7	2595578.0	132994.0	0.0	176600.0	4773346.0	
03	288181.0	257208.8	25874.3	311287.7	452759.3	3645001.0	
04	184198.7	639471.8	31779.7	0.0	126362.4	2642139.5	
05	168695.4	1117834.2	9335.1	2612455.5	37917.9	1982931.7	
06	113014.0	229552.9	6527.8	80381.4	25691.5	974329.6	
07	147569.3	2107835.0	0.0	0.0	31169.9	897487.2	
08	160036.4	172818.4	6996.2	0.0	263397.6	1092876.2	
09	2657.7	0.0	0.0	0.0	11940.2	299839.4	
10	68254.7	676865.2	10469.0	482110.7	24927.8	1218370.2	
11	73935.7	40316.5	0.0	263542.1	0.0	1069824.7	
12	51096.9	70591.7	48378.2	190123.4	0.0	1635544.2	
13	153765.5	29560.0	0.0	0.0	43255.8	1052255.2	
14	90754.5	164268.5	0.0	7759.4	6760.5	568027.1	
15	113981.9	579762.6	0.0	371542.6	0.0	697676.7	
16	39099.2	150589.4	0.0	150892.0	0.0	1265075.7	
17	42850.1	12727.8	0.0	337120.1	0.0	896573.2	
18	36469.3	22921.8	0.0	144377.1	0.0	899472.8	
19	61543.1	0.0	0.0	155596.9	0.0	581923.1	
20	44742.3	22391.3	0.0	740007.7	0.0	248094.2	
21	74870.3	53502.3	0.0	298580.9	1709.6	666644.2	
22	13657.0	1689.5	0.0	559.1	41047.2	263621.5	
23	58212.9	0.0	0.0	420727.0	0.0	405350.6	
24	35199.4	27929.9	0.0	322190.7	0.0	576761.6	
25	30475.5	0.0	0.0	1717.6	5627.1	202674.5	
26	34296.4	0.0	0.0	0.0	54098.2	339008.3	
27	75464.7	0.0	0.0	0.0	39166.3	284893.2	
28	35288.2	0.0	0.0	15444.6	10778.0	367474.8	
29	67362.0	34130.4	0.0	0.0	55538.0	447229.6	
30	26732.6	7955.5	0.0	0.0	1074.8	1194773.7	
31	44621.2	24243.7	0.0	0.0	38129.5	688105.1	

Wage by Sector ($1000) W_{ijt} (t = 1966)

SMSA Code No.	Ind. 1 W_{ijt}	Ind. 2 W_{ijt}	Ind. 3 W_{ijt}	Ind. 4 W_{ijt}	Ind. 5 W_{ijt}	Ind. 6 W_{ijt}	Ind. 7 W_{ijt}
01	6.7732	5.4373	5.5329	9.7587	9.1361	4.9156	4.5951
02	4.8304	0.0000	5.4129	10.8250	9.3432	5.5649	0.0000
03	8.4542	0.0000	6.4856	12.0137	11.7351	6.3962	5.3785
04	8.7560	3.8006	6.8960	10.1231	8.5284	7.2978	5.0000
05	5.8600	0.0000	6.6936	10.0890	22.1897	6.6356	12.3865
06	7.3485	0.0000	6.3309	10.5685	17.0949	6.5006	5.0832
07	8.8297	0.0000	5.7983	17.8940	7.9853	7.8554	5.8641
08	10.7960	0.0000	4.7778	9.6825	13.2000	5.1291	4.9101
09	0.0000	0.0000	6.2612	10.5079	0.0000	0.0000	0.0000
10	8.8293	3.6260	5.6473	23.0688	12.1714	5.0004	5.5286
11	4.0437	3.6791	5.0395	8.3787	10.0228	4.0722	4.2072
12	4.5172	5.2366	2.0603	11.1076	0.0000	4.7208	0.0000
13	10.9611	2.7684	5.8167	10.4836	16.4854	0.0000	0.0000
14	0.7179	2.8871	4.6619	13.8224	11.2307	4.1972	2.6360
15	5.4463	5.5406	6.1822	7.9893	12.5662	5.9233	0.0000
16	9.5959	4.1597	5.0678	9.8460	6.5379	5.1965	0.0000
17	11.4982	0.0000	4.5657	8.3634	0.0000	6.8873	3.4784
18	7.4842	5.2564	6.0435	10.8532	4.9100	5.3907	6.2088
19	4.7428	2.5816	5.2000	12.3342	4.8096	4.8235	0.0000
20	7.6646	5.5894	14.6963	11.4301	9.0482	6.2321	0.0000
21	9.4104	3.2388	0.0000	8.4396	11.5056	4.4709	4.2792
22	11.4233	0.0000	4.3333	15.6477	4.7723	5.5983	0.0000
23	3.9389	0.1159	4.2704	7.1690	5.6697	5.6086	0.0000
24	13.7801	0.0000	4.8745	5.8507	3.0991	5.9781	0.0000
25	3.4547	4.2168	4.6679	6.1761	3.6436	4.3862	4.6615
26	6.0658	0.0000	0.0000	6.2853	7.3332	4.4163	0.0000
27	2.9644	0.0000	4.3648	9.5794	7.6830	3.6930	0.0000
28	6.3921	0.0000	7.7230	15.7167	2.8477	5.7319	0.0000
29	8.8191	0.0000	5.8355	5.7933	0.0000	7.6209	3.7845
30	6.1844	0.0000	3.8046	6.1164	0.0000	4.6002	3.0555
31	11.4388	0.0000	5.3131	3.5793	6.8438	5.7332	0.0000

Wage by Sector ($1000) W_{ijt} (t = 1966)
(continued)

SMSA Code No.	Ind. 8 W_{ijt}	Ind. 9 W_{ijt}	Ind. 10 W_{ijt}	Ind. 11 W_{ijt}	Ind. 12 W_{ijt}	Ind. 13 W_{ijt}	
01	8.0638	7.5004	6.4631	0.0000	7.9033	5.6588	
02	6.0723	7.6716	6.8427	0.0000	8.0357	5.6592	
03	6.8608	6.5616	6.0062	12.3651	6.4643	6.2262	
04	6.5428	7.9364	6.1340	0.0000	5.4023	5.5303	
05	8.7925	10.0588	2.6292	10.9254	5.8854	5.5857	
06	6.6507	8.3729	7.6727	8.0443	5.6811	5.6848	
07	6.7613	7.7937	0.0000	0.0000	7.4368	5.1329	
08	6.5455	5.6171	6.7171	0.0000	8.0233	5.0964	
09	0.9600	0.0000	0.0000	0.0000	5.8529	9.6598	
10	6.9740	7.7838	5.4875	8.1283	4.6941	5.5529	
11	7.7553	7.3364	0.0000	7.3386	0.0000	6.3694	
12	6.8112	7.2619	5.5775	9.5225	0.0000	5.6317	
13	10.1676	6.7893	0.0000	0.0000	9.3080	5.8909	
14	[illegible].8647	4.3700	0.0000	0.1463	2.9877	5.2984	
15	6.5506	7.3837	0.0000	7.5427	0.0000	5.8467	
16	5.9181	4.8314	0.0000	6.1916	0.0000	6.3357	
17	5.8562	3.7406	0.0000	8.3230	0.0000	5.7360	
18	6.3922	5.8851	0.0000	6.7351	0.0000	6.0499	
19	7.5642	0.0000	0.0000	6.5965	0.0000	5.5509	
20	10.0538	9.8300	0.0000	10.6890	0.0000	8.2458	
21	5.7053	4.5060	0.0000	6.9934	3.8110	4.5465	
22	10.9172	5.0527	0.0000	5.2811	7.4707	7.1509	
23	5.3310	0.0000	0.0000	8.6359	0.0000	4.7644	
24	5.1316	6.8832	0.0000	6.8390	0.0000	6.4832	
25	5.7643	0.0000	0.0000	3.9162	4.4935	4.1554	
26	4.6961	0.0000	0.0000	0.0000	5.5310	7.1320	
27	6.6228	0.0000	0.0000	0.0000	4.5482	5.7742	
28	7.3166	0.0000	0.0000	6.3054	4.6593	5.5678	
29	6.7919	4.7903	0.0000	0.0000	5.5686	6.3987	
30	6.0923	3.8620	0.0000	0.0000	3.2430	6.2211	
31	7.1494	3.7252	0.0000	0.0000	5.6895	5.3198	

Wage by Sector at t-1 ($1000) W_{ijt-1} (t = 1966)

SMSA Code No.	Ind. 1 W_{ijt-1}	Ind. 2 W_{ijt-1}	Ind. 3 W_{ijt-1}	Ind. 4 W_{ijt-1}	Ind. 5 W_{ijt-1}	Ind. 6 W_{ijt-1}	Ind. 7 W_{ijt-1}
01	6.9457	5.5757	5.6738	10.0070	9.3691	5.0408	4.7121
02	4.9797	0.0000	5.5802	11.1595	9.6319	5.7369	0.0000
03	8.9383	0.0000	6.8570	12.7017	12.4072	6.7625	5.6862
04	8.8498	3.8413	6.9699	10.2316	8.6197	7.3760	5.0538
05	5.7556	0.0000	6.5745	9.9103	21.7944	6.5175	12.1660
06	7.5971	0.0000	6.5459	10.9259	17.6734	6.7206	5.2555
07	8.8489	0.0000	5.8112	17.9340	8.0034	7.8731	5.8772
08	11.2192	0.0000	4.9651	10.0618	13.7171	5.3301	5.1015
09	0.0000	0.0000	6.0825	10.2090	0.0000	0.0000	0.0000
10	9.0113	3.7008	5.7636	23.5439	12.4221	5.1034	5.6418
11	3.9220	3.5685	4.8879	8.1267	9.7207	3.9498	4.0806
12	4.5678	5.2951	2.0833	11.2316	0.0000	4.7736	0.0000
13	10.6115	2.6802	5.6313	10.1486	15.9597	0.0000	0.0000
14	0.7331	2.9485	4.7604	14.1145	11.4682	4.2860	2.6912
15	5.5045	5.5996	6.2480	8.0743	12.7002	5.9863	0.0000
16	9.8123	4.2535	5.1820	10.0666	6.6846	5.3134	0.0000
17	11.4636	0.0000	4.5520	8.3384	0.0000	6.8666	3.4667
18	7.2932	5.1228	5.8899	10.5773	4.7853	5.2537	6.0511
19	4.6407	2.5263	5.0882	12.0695	4.7061	4.7198	0.0000
20	5.8670	4.2789	11.2491	8.7488	6.9243	4.7720	0.0000
21	9.3948	3.2334	0.0000	8.4254	11.4862	4.4632	4.2742
22	11.0917	0.0000	4.2084	15.1908	4.6325	5.4363	0.0000
23	3.9304	0.1156	4.2607	7.1531	5.6564	5.5960	0.0000
24	13.3776	0.0000	4.7324	5.6794	3.0094	5.8038	0.0000
25	3.3970	4.1467	4.5900	6.0730	3.5825	4.3127	4.5861
26	6.6589	0.0000	0.0000	6.9000	8.0503	4.8480	0.0000
27	2.9787	0.0000	4.3859	9.6255	7.7198	3.7107	0.0000
28	6.3835	0.0000	7.7127	15.6968	2.8437	5.7242	0.0000
29	8.7555	0.0000	5.7933	5.7516	0.0000	7.5665	3.7579
30	6.4199	0.0000	3.9495	6.3475	0.0000	4.7754	3.1709
31	11.3189	0.0000	5.2556	3.5401	6.7700	5.6712	0.0000

Wage by Sector at t-1 ($1000) W_{ijt-1} (t = 1966)
(continued)

SMSA Code No.	Ind. 8 W_{ijt-1}	Ind. 9 W_{ijt-1}	Ind. 10 W_{ijt-1}	Ind. 11 W_{ijt-1}	Ind. 12 W_{ijt-1}	Ind. 13 W_{ijt-1}
01	8.2692	7.6914	6.6277	0.0000	8.1045	5.8029
02	6.2600	7.9087	7.0542	0.0000	8.2841	5.8341
03	7.2537	6.9373	6.3499	13.0732	6.8345	6.5828
04	6.6129	8.0215	6.1997	0.0000	5.4602	5.5896
05	8.6360	9.8798	2.5824	10.7310	5.7806	5.4862
06	6.8757	8.6461	7.9324	8.3164	5.8732	5.8771
07	6.7763	7.8111	0.0000	0.0000	7.4534	5.1443
08	6.8020	5.8372	6.9798	0.0000	8.3376	5.2961
09	0.9326	0.0000	0.0000	0.0000	5.6857	9.3840
10	7.1177	7.9442	5.6006	8.2958	4.7908	5.6673
11	7.5221	7.1156	0.0000	7.1179	0.0000	6.1778
12	6.8872	7.3430	5.6398	9.6288	0.0000	5.6946
13	9.8436	6.5728	0.0000	0.0000	9.0115	5.7032
14	4.9675	4.4624	0.0000	0.1494	3.0509	5.4104
15	6.6203	7.4622	0.0000	7.6229	0.0000	5.9089
16	6.0514	4.9403	0.0000	6.3310	0.0000	6.4784
17	5.8387	3.7295	0.0000	8.2981	0.0000	5.7188
18	6.2300	5.7356	0.0000	6.5639	0.0000	5.8961
19	7.4014	0.0000	0.0000	6.4547	0.0000	5.4315
20	7.6995	7.5249	0.0000	8.1819	0.0000	6.3117
21	5.6957	4.4985	0.0000	6.9816	3.8047	4.5389
22	10.5985	4.9036	0.0000	5.1250	7.2525	6.9420
23	5.3190	0.0000	0.0000	8.6165	0.0000	4.7536
24	4.9819	6.6823	0.0000	6.6395	0.0000	6.2941
25	5.6681	0.0000	0.0000	3.8509	4.4186	4.0861
26	5.1552	0.0000	0.0000	0.0000	6.0717	7.8292
27	6.6547	0.0000	0.0000	0.0000	4.5701	5.8020
28	7.3066	0.0000	0.0000	6.2970	4.6531	5.5604
29	6.7431	4.7559	0.0000	0.0000	5.5287	6.3528
30	6.3243	4.0090	0.0000	0.0000	3.3669	6.4580
31	7.0720	3.6848	0.0000	0.0000	5.6280	5.2622

Capital Share or Gross Profit by Sector ($1000) π_{ijt} (t = 1966)

SMSA Code No.	Ind. 1 π_{ijt}	Ind. 2 π_{ijt}	Ind. 3 π_{ijt}	Ind. 4 π_{ijt}	Ind. 5 π_{ijt}	Ind. 6 π_{ijt}	Ind. 7 π_{ijt}
01	18463.1	24246.7	122702.9	18246.2	7963.2	63551.6	9818.2
02	32121.1	0.0	195300.4	67720.3	66136.3	172379.3	0.0
03	91094.8	0.0	93790.2	54531.5	112990.0	143407.7	6100.4
04	69586.8	41339.3	161302.8	104985.4	126690.0	101816.2	6415.2
05	4832.6	0.0	31713.4	72725.9	30914.5	70111.7	21229.2
06	72296.6	0.0	56304.8	73156.9	131628.3	17196.0	1224.5
07	2015.8	0.0	16689.6	32041.7	4572.2	11256.0	39224.8
08	31930.3	0.0	42944.9	122149.6	87271.6	18051.7	1357.4
09	0.0	0.0	4505.9	1782.8	0.0	0.0	0.0
10	11056.9	14445.2	32975.2	159172.5	23051.2	33936.9	1946.5
11	7385.4	5773.0	36763.1	38449.1	6643.5	21086.9	9502.8
12	5124.4	21770.0	18788.3	82383.2	0.0	45886.5	0.0
13	21980.3	4441.2	36814.2	6488.4	23675.4	0.0	0.0
14	19538.0	3741.5	36598.8	401179.8	286976.9	14924.6	644.8
15	7345.2	8403.7	50943.7	70703.9	12516.2	31684.4	0.0
16	4326.0	6336.0	24573.0	2925.3	1366.5	8267.4	0.0
17	27314.7	0.0	47062.8	61870.8	0.0	33720.3	817.0
18	3027.0	29279.4	52048.2	2325.2	2890.3	46638.5	3729.8
19	11609.9	2407.7	21987.5	10736.9	3405.3	7781.5	0.0
20	8067.0	334.9	20388.0	837.2	488.2	614.0	0.0
21	16678.1	5228.1	0.0	63802.1	45584.4	9428.9	709.0
22	1035.0	0.0	271.5	3395.3	130.8	516.0	0.0
23	5778.4	684.2	39488.9	10643.0	1261.9	4311.5	0.0
24	11756.6	0.0	17269.6	1572.2	1160.1	27721.3	0.0
25	7821.2	2897.7	8077.7	7624.5	580.2	4417.7	0.0
26	21445.0	0.0	0.0	1805.4	6253.5	31827.5	962.4
27	9069.3	0.0	15941.4	16824.2	16202.1	962.1	0.0
28	12496.7	0.0	71986.1	18448.3	1230.0	3883.0	0.0
29	4440.2	0.0	10558.4	6552.9	0.0	12177.3	0.0
30	88616.3	0.0	19604.4	1701.5	0.0	20413.8	508.7
31	1840.2	0.0	32712.9	1964.0	2561.8	59781.1	755.7
							0.0

Capital Share or Gross Profit by Sector (\$1000) π_{ijt} (t = 1966)
(continued)

SMSA Code No.	Ind. 8 π_{ijt}	Ind. 9 π_{ijt}	Ind. 10 π_{ijt}	Ind. 11 π_{ijt}	Ind. 12 π_{ijt}	Ind. 13 π_{ijt}	
01	53077.3	51444.5	37931.3	0.0	353928.6	4668672.0	
02	152254.3	1099633.7	72890.8	0.0	325636.1	4594192.0	
03	133139.3	102738.3	13370.4	223345.7	787122.5	3307630.0	
04	84563.2	253817.9	16318.4	0.0	218296.4	2382477.0	
05	67897.8	388988.8	4202.5	1632965.7	57429.1	1567616.2	
06	54390.2	95516.3	3513.8	60078.3	46527.7	921027.6	
07	55460.7	684908.6	0.0	0.0	44081.8	662516.8	
08	78922.2	73684.5	3859.0	0.0	488794.0	1058594.7	
09	789.5	0.0	0.0	0.0	13349.0	174971.5	
10	30483.6	261362.0	5229.7	334392.6	41894.3	1068792.2	
11	33270.8	15685.5	0.0	184176.5	0.0	945585.3	
12	25333.4	30259.3	26827.5	146389.0	0.0	1592718.7	
13	70644.7	11741.7	0.0	0.0	74782.3	949555.1	
14	56514.8	88441.3	0.0	7504.1	15842.0	694771.3	
15	51072.3	224598.3	0.0	258543.5	0.0	534812.5	
16	17721.9	59012.3	0.0	106214.4	0.0	1126257.2	
17	22017.9	5654.4	0.0	269018.8	0.0	904873.0	
18	15964.5	8675.2	0.0	98152.1	0.0	773380.1	
19	29454.4	0.0	0.0	115650.3	0.0	547035.3	
20	10315.2	4463.2	0.0	264957.9	0.0	112346.8	
21	41223.3	25469.0	0.0	255310.8	3542.1	720949.8	
22	4427.1	473.5	0.0	281.3	50069.3	167846.9	
23	30786.9	0.0	0.0	345558.6	0.0	421071.6	
24	15515.6	10644.1	0.0	220556.7	0.0	499352.4	
25	15062.7	0.0	0.0	1318.5	10465.9	196757.2	
26	19282.3	0.0	0.0	0.0	114453.7	374370.5	
27	39000.8	0.0	0.0	0.0	76168.9	289194.8	
28	17320.1	0.0	0.0	11772.6	19906.5	354264.0	
29	30723.5	13458.7	0.0	0.0	95319.1	400649.0	
30	15683.6	4035.3	0.0	0.0	2372.8	1376800.5	
31	19101.2	8972.7	0.0	0.0	61420.6	578564.6	

(t = 1966)

SMSA	Y_{it} (millions of dollars)	Y_{it-1} (millions of dollars)	C_{it-1} (millions of dollars)	C_{it} (millions of dollars)	M_{it} (percentage)	G_{it} (millions of dollars)
01	46347.00	43424.0	28640.93	31005.88	-0.4	5977.00
02	26229.00	24249.0	16529.03	17885.18	-5.0	1941.00
03	26609.00	24598.0	17986.88	19438.81	10.4	2867.00
04	15211.00	13959.0	10640.89	11271.97	3.6	1280.00
05	15013.00	13872.0	9154.10	9738.41	3.6	1459.00
06	12228.00	11380.0	8600.99	9284.51	3.3	1441.00
07	7541.00	7045.0	5024.11	5215.88	3.3	600.00
08	7601.00	7047.0	4313.77	4584.80	3.6	624.00
09	9294.00	8531.0	6181.30	7080.91	1.0	1055.00
10	7375.00	6837.0	4471.55	4845.26	3.6	613.00
11	6292.00	5796.0	4121.19	4450.55	3.3	645.00
12	7160.00	6689.0	4510.72	4663.34	3.6	669.00
13	5971.00	5467.0	3174.55	3691.55	3.3	643.00
14	5113.00	4687.0	3773.89	4035.26	3.6	483.00
15	4130.00	3937.0	2952.28	2907.79	3.3	502.00
16	7825.00	4475.0	2688.25	2874.64	1.0	535.00
17	4229.00	3882.0	2622.93	2812.46	3.6	400.00
18	4339.00	4084.0	3573.11	3709.91	3.6	358.00
19	4350.00	3981.0	3164.74	3550.18	3.3	337.00
20	4611.00	3928.0	2738.23	3179.12	1.0	431.00
21	3961.00	3665.0	2808.61	3312.05	3.6	320.00
22	3694.00	3253.0	3183.59	3596.05	3.3	463.00
23	4123.00	3727.0	2804.48	3222.22	1.0	359.00
24	3589.00	3265.0	2158.49	2423.21	3.6	297.00
25	3453.00	3160.0	2533.97	2578.27	3.6	324.00
26	3617.00	3236.0	2374.38	2599.97	3.3	334.00
27	3020.00	2777.0	2127.72	2395.84	1.0	276.00
28	3021.00	2777.0	1870.46	2023.27	3.6	282.00
29	2594.00	2405.0	1843.00	1952.76	3.6	246.00
30	2914.00	2659.0	1677.35	1726.87	3.3	358.00
31	2753.00	2494.0	1840.15	1974.02	1.0	217.00

(t = 1966)

SMSA	P_{it} (thousands)	L_{it} (thousands)	N^T_{it} (thousands)	$\overline{N}_{it}$ (thousands)	U_{it} (thousands)	u_{it} (percentage)
01	11511.80	5421.95	5199.65	4050.56	222.30	4.10
02	6780.60	3120.00	3042.00	2053.30	78.00	2.50
03	6986.60	3091.11	2952.01	2139.61	139.10	4.50
04	4753.90	1966.66	1901.76	1330.76	64.90	3.30
05	4040.50	1587.87	1535.47	938.30	52.40	3.30
06	3045.80	1329.54	1271.04	1070.95	58.50	4.40
07	2399.10	920.00	892.40	593.68	27.60	3.00
08	2282.70	1006.25	974.05	686.93	32.20	3.20
09	2486.80	1112.50	1085.80	1028.44	26.70	2.40
10	2078.80	888.46	865,92	557.92	23.10	2.60
11	1882.00	803.44	780.14	573.65	23.30	2.90
12	1870.50	860.97	825.67	565.55	35.30	4.10
13	1648.90	763.63	746.83	560.75	16.80	2.20
14	1774.60	716.66	699.46	580.26	17.20	2.40
15	1350.20	541.02	519.92	339.49	21.10	3.90
16	1318.10	591.30	577.70	366.52	13.60	2.30
17	1371.60	530.00	514.10	350.14	15.90	3.00
18	1341.50	525.58	502.98	312.30	22.60	4.30
19	1369.30	600.00	585.60	449.28	14.40	2.40
20	1237.40	563.33	546.43	400.80	16.90	3.00
21	1234.00	560.00	537.60	401.99	22.40	4.00
22	1214.80	359.61	340.91	284.45	18.70	5.20
23	1236.70	578.57	562.37	447.31	16.20	2.80
24	1005.90	461.90	452.20	316.98	9.70	2.10
25	1124.20	465.71	449.41	396.01	16.30	3.50
26	1118.70	462.49	447.69	375.15	14.80	3.20
27	1054.60	409.09	395.59	338.58	13.50	3.30
28	908.40	402.94	389.24	312.34	13.70	3.40
29	871.20	372.00	362.70	275.54	9.30	2.50
30	821.50	365.21	356.81	217.20	8.40	2.30
31	817.30	337.50	329.40	206.82	8.10	2.40

(t = 1966)

SMSA	T_{it} (millions of dollars)	G_{it} (millions of dollars)
01	6416.00	5944.00
02	1940.00	1941.00
03	2980.00	2867.00
04	1233.00	1280.00
05	1392.00	1459.00
06	1441.00	1345.00
07	599.00	600.00
08	582.00	624.00
09	967.00	1055.00
10	592.00	613.00
11	652.00	645.00
12	624.00	669.00
13	579.00	643.00
14	421.00	483.00
15	505.00	502.00
16	523.00	535.00
17	394.00	400.00
18	358.00	358.00
19	301.00	337.00
20	405.00	431.00
21	323.00	320.00
22	466.00	463.00
23	341.00	359.00
24	287.00	297.00
25	312.00	324.00
26	344.00	334.00
27	241.00	276.00
28	277.00	282.00
29	239.00	235.00
30	332.00	358.00
31	214.00	217.00

The list of data and the data sources are as follows:

Sources

Annual Survey of Manufacturers, 1954-1966
Census of Manufacturers, 1963
County City Data Book, 1967
Individual Income Tax Returns, 1959-1966
Current Population Reports
County Business Patterns, 1967
Employment and Earnings Statistics for States and Areas, 1958-1967
Manpower Report of President, 1969
Census of Government, 1962
Consumer Expenditures and Income, 1960-1961, for SMSA
Sales Management Magazine, 1956-1968
Survey of Current Business (monthly publication)
Statistical Abstract of the United States, 1967, 1968

Data

Land Area (1960: square miles: F10. 0)*
Population Estimates (1000; F10. 0)
Total Percent Change on Population (1960, 1965; tenths)
Natural Increase in Population (1960-1965; tenths)
Net Migration (1960-1965; tenths)
Number of Employees (1000's; F10. 2)
Taxable Payrolls ($1, 000; F10. 2)
Total Number of Non-Agricultural Employees (1, 000; F10. 0)
Percent Non-Agricultural Employment in Contract Construction (tenths)
Percent Non-Agricultural Employment in Manufacturing (tenths)
Percent Non-Agricultural Employment in Transportation (tenths)
Percent Non-Agricultural Employment in Wholesale and
 Retail Trade (tenths)
Percent Non-Agricultural Employment in Finance, Insurance,
 Real Estate (tenths)

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*The F statements refer to the computer format in the CONSAD program.

Data (continued)

Percent Non-Agricultural Employment in Service and Miscellaneous (tenths)
Percent Non-Agricultural Employment in Government (tenths)
Average Hour Earning of Manufacturing
Production Worker (dollar; F10.2)
Unemployment (1000)
Total Industry Income Tax (million dollar; F10.0)
Salaries and Wages (Percent)
Percent of Families with Income under $3,000 (1959; tenths)
Percent of Families with Income $10,000 and Over (1959; tenths)
Exports of Manufacturing Products: Value (1966; million dollars)
Exports of Manufacturing Products: Value (1963; million dollars)
Manufacturing Employment, Annual Average (1,000; F10.0)
Manufacturing Payroll, Entire Year (million dollar; F10.0)
Manufacturing Value Added (million dollar; F10.0)
Manufacturing Capital Expenditure (million dollar; F10.0)
Median Income for all Families (1959; dollar; F10.0)

```
      DIMENSION F(2,0/100),R(0/100),ID(0/100),FSTAR(63),IOSTAR(100)
     ,DELT(10),OSTAR(100),X(0/100),Y(0/100),D(101)
C     THIS PROGRAM UTILIZES THE DYNAMIC PROGRAMMING
C     METHODOLOGY TO DETERMINE THE MAXIMUM RETURN
C     AND OPTIMAL DECISIONS FOR AN N STAGE
C     BUDGET OPTIMIZATION
      WRITE 1,600
      READ 0,601,NN
      WRITE 1,610
      READ 0,601,KN
      WRITE 1,620
      READ 0,621,DELTA
      WRITE 1,630
      READ 0,601,IOUT
      IF(IOUT-2) 670,660,660
660   WRITE 1,640
      READ 0,601,KM
      WRITE 1,650
      READ 0,601,MM
600   FORMAT(/$ THE NUMBER OF PROGRAM ELEMENTS IS $)
601   FORMAT(I5)
610   FORMAT(/$ THE NUMBER OF BUDGET LEVELS IS $)
620   FORMAT(/$ THE BUDGET INCREMENT IS$,24 $)
621   FORMAT(F9.2)
630   FORMAT(/$ THE OUTPUT OPTION IS $)
640   FORMAT(/$ THE LOWEST BUDGET LEVEL OF INTEREST IS $)
650   FORMAT(/$ THE OUTPUT INCREMENT IS $)
670   CONTINUE
      WRITE 1,680
680   FORMAT(//)
      J=1
      LN=0
C     READ RETURN FUNCTIONS FROM DISK
      DO 110 L=1,NN
      OPEN(3,RANDIN,SYMBOLIC,/RETURN DATA/)
      I=(L-1)*(2*LN+1)*5+1
      POSITION(3,I)
      READ(3,2) LN, (X(J),Y(J),J=1,LN)
      DO 700 J=1,KN
      DO 699 N=1,LN
      D(J)=DELTA*J
      IF(D(J)-X(N)) 697,698,699
697   IF(D(J)-X(N-1)) 699,698,698
698   R(J)=((Y(N)-Y(N-1))/(X(N)-X(N-1)))*(D(J)-X(N-1))+Y(N+1)
      GO TO 700
699   CONTINUE
700   CONTINUE
2     FORMAT(I5,202F5.0)
      CLOSE
C     COMPUTE OPTIMAL RETURNS AND DECISIONS AND WRITE ON DISK
      DO 130 K=0,KN
      F(N,K)=0.
      DO 140 J=0,KN
      IF(K-J) 150,200,200
200   IF(N-1) 210,160,210
```

```
210       Q=R(J)+F(1,K-J)
          GO TO 170
160       Q=R(J)
170       IF(Q-F(N,K)) 140,220,220
220       F(N,K)=Q
          ID(K)=J
140       CONTINUE
150       OPEN(3,RANDOUT,BINARY,/OPTIMAL/)
          I=(L-1)*(KN+1)*3+(3*K+1)
          POSITION(3,I)
          WRITE(3) F(N,K),ID(K)
          CLOSE
130       CONTINUE
C         PUSHDOWN OPTIMAL RETURNS FOR USE WITH NEXT STAGE
          IF(N-2) 240,230,240
230       DO 180 K=0,KN
          F(1,K)=F(2,K)
180       CONTINUE
240       N=2
110       CONTINUE
          L=NN
C         SELECT PRINT OPTION AND PRINT RESULTS
          IF(IOUT-1) 250,250,260
C         THIS SECTION OUTPUTS SINGLE OPTIMAL SOLUTION
250       DELTE=KN*DELTA
          TYPE 4,DELTE,F(2,KN)
4         FORMAT(/$ THE TOTAL BUDGET IS$,3H $ ,F11.2,
     1    /$ THE MAXIMUM TOTAL RETURN IS$ F6.2,
     2    /$ THE PROGRAM ELEMENT EXPENDITURES WHICH GIVE$,
     3    $ THE MAXIMUM RETURN ARE$,
     4    //10X,$PROGRAM ELEMENTS$,10X,$EXPENDITURES$,10X,$RETURNS$//)
          OPEN(3,RANDIN,BINARY,/OPTIMAL/)
          LN=KN
300       I=(L-1)*(KN+1)*3+(3*LN+1)
930       POSITION(3,I)
          READ(3) FSTAR(L),IDSTAR(L)
          IF(L-1) 310,320,310
310       LN=LN-IDSTAR(L)
          L=L-1
          GO TO 300
320       CLOSE
          L=NN
330       X2L=IDSTAR(L)*DELTA
          TYPE 6,L,X2L,FSTAR(L)
6         FORMAT(17X,I2,18X,F8.1,12X,F6.2/)
          IF(L-1) 550,550,340
340       L=L-1
          GO TO 330
C         THIS SECTION OUTPUTS MULTIPLE OPTIMAL RETURNS
260       JJJ=0
          KK=KN
          DO 500 I=KM,KN,MM
          DELT(I)=I*DELTA
500       CONTINUE
          TYPE 7,(DELT(JN),JN=KM,KN,MM)
7         FORMAT(/$ BUDGET LEVELS $,5(F9.2,2X))
```

```
           TYPE 3,(F(2,K),K=KM,KN,MM)
   3       FORMAT(/$ MAX. RETURNS  $,5(F9.2,2X))
           TYPE 9
   9       FORMAT(/$ PROGRAM EXPENDITURES $)
           OPEN(3,RANDIN,BINARY,/OPTIMAL/)
           JN=1
           JNN=1
           KJ=KN
 460       CONTINUE
           I=(L-1)*(KN+1)*3+(3*KK+3)
 950       POSITION(3,I)
           READ(3) IOSTAR(JN)
           IF(L-1) 420,420,410
 410       KK=KK-IOSTAR(JN)
           OSTAR(JN)=IOSTAR(JN)*DELTA
           L=L-1
           JN=JN+(KN-KM+1)
           GO TO 460
 420       L=NN
           JNN=JNN+1
           JN=JNN
           JJJ=JJJ+MM
           KK=KN-JJJ
           IF(KK-KM+1) 350,350,460
 350       CONTINUE
           KB=KN-KM+1
           DO 510 I=1,L
           J=L+1-I
           KC=(I-1)*KB+1
           KD=I*KB
 510       TYPE 10,J,(OSTAR(K),K=KD,KC,-1)
 10        FORMAT(/6X,I3,5X,5(F9.2,2X))
           CLOSE
 550       CONTINUE
           END
```

```
        DIMENSION F(20,2,100),ID(20,100),FSTAR(20,63),IDSTAR(20,100)
       ,DELTA(10),DSTAR(20,100)
C       THIS PROGRAM UTILIZES THE DYNAMIC PROGRAMMING METHODOLOGY TO
        DETERMINE L OPTIMAL AND SUB-OPTIMAL RETURNS (TOGETHER WITH
        THE STAGE DECISIONS) FOR AN N STAGE BUDGET ALLOCATION
        PROBLEM.
        WRITE 1,600
600     FORMAT(/$ THE NUMBER OF PROGRAM ELEMENTS IS $)
        READ 0,601,NN
601     FORMAT(I5)
        WRITE 1,610
610     FORMAT(/$ THE NUMBER OF BUDGET LEVELS IS $)
        READ 0,601,KN
        WRITE 1,620
        READ 0,621,DELTA
        WRITE 1,630
        READ 0,601,LN
620     FORMAT(/$ THE BUDGET INCREMENT IS $,2H $)
621     FORMAT(F9.2)
630     FORMAT(/$ THE NUMBER OF "SUB" - OPTIMAL SOLUTIONS IS $)
        WRITE 1,680
680     FORMAT(//)
        M=1
        DO 110 L=1,NN
        F(M,1)=0
C       READ RETURN FUNCTIONS FROM DISK
        OPEN(3,RANDIN,SYMBOLIC,/RETURN/)
        I=(L-1)*KN*4+1
        POSITION(3,1)
        DO 120,J=1,KN
        READ(3,2) R(J)
2       FORMAT(100F4.2)
120     CONTINUE
        CLOSE
C       COMPUTE OPTIMAL AND SUB-OPTIMAL RETURNS AND DECISIONS AND
        WRITE ON DISK.
        DO 130 K=2,KN+1
        DO 135 L=1,LN
        F(L,M,K)=0
135     ID(L,K)=0
        DO 140 J=1,JN
200     IF ((K-1)-J) 150,150,200
        IF (N-1) 210,160,210
210     Q=R(J)+F(1,1,K-J)
        GO TO 170
160     Q=R(J)
170     DO 600 L=LN,1,-1
        IF (Q-F(L,M,K)) 610,610,600
600     CONTINUE
        DO 620 L=LN,1,-1
        F(L,M,K)=F(L-1,M,K)
620     ID(L,K)=ID(L-1,K)
        GO TO 220
```

```
610      IF (L-(LN-2)) 630,630,640
640      IF (L-LN+1) 650,641,650
641      L=L+1
         GO TO 220
630      LL=L+2
         DO 635 L=LN,LL,-1
         F(L,M,K)=F(L-1,M,K)
635      ID(L,K)=D(L-1,K)
220      F(L,M,K)=Q
         ID(L,K)=J
650      CONTINUE
140      CONTINUE
150      OPEN(3,RANDOUT,BINARY,/OPTIMAL/)
         DO 1001 L=1,LN
         I=(L-1)*KN*3+(3*K-5)
         POSITION(3,I)
1001     WRITE(3) F(L,M,K) ID(L,K)
130      CONTINUE
         IF(M-2) 111,980,111
980      DO 981 K=2,KN
         DO 982 L=1,LD
         F(L,1,K)=F(L,2,K)
982      CONTINUE
981      CONTINUE
         GO TO 110
111      M=2
110      CONTINUE
         DELTE=KN*DELTA
         OPEN(3,RANDIN,BINARY,/OPTIMAL/)
         LN=KN
         DO 991 L=1,LN
         DO 991 N=NN,1,-1
         I=(L-1)*KN*3+(3*LN-2)
         POSITION(3,I)
         READ(3) FSTAR(L,N),IDSTAR(L,N)
         DSTAR(L,N)=IDSTAR(L,N)*DELTA
         XSTAR(L,N)=KN*DELTA
         LN=LN-IDSTAR(L,N)
991      CONTINUE
         TYPE 800,DELTE
800      FORMAT(/$ THE TOTAL BUDGET IS $,3H $,
       1 F11.2,//$ RETURN:$)
         TYPE 801,(FSTAR(L,NN),L=1,LN)
801      FORMAT(20X,(5F8.2/))
         TYPE 802
802      FORMAT(/$ PROGRAM EXPENDITURES:$ /)
         DO 990 L=NN,1,-1
         TYPE 803,L,(DSTAR(N,L),N=1,LN)
803      FORMAT(10X,I2,8X,(58.2/))
990      CONTINUE
         CLOSE
         END
```

MATHEMATICAL PROGRAMMING ALGORITHM

The task durations and costs that minimize total project cost subject to project completion by a specified date are given by the solution to the following mathematical programming problem:

(A1) $$\text{minimize} \sum_{ij} c_{ij}(d_{ij})$$

subject to:

$$\left.\begin{array}{l} t_i + d_{ij} \le t_j \\ a_{ij} \le d_{ij} \le b_{ij} \end{array}\right\} \text{for } ij \text{ pairs corresponding to arcs}$$

$$t_G \le T_G$$

$$t_i \ge 0$$

where $c_{ij}(d_{ij})$ is the cost of task ij when it has duration d_{ij}; t_i is the occurrence time of node i; t_G is the occurrence time of the final goal (which may be a dummy); T_G is the desired final goal occurrence time; a_{ij} and b_{ij} are respectively the lower and upper limits on d_{ij}, and the sum is over all ij pairs corresponding to arcs in the project PERT chart. It is assumed that nodes are numbered such that $i < j$ for all arcs.

In the RAM-1 model, the functions $c_{ij}(d_{ij})$ are all piecewise linear functions defined by the three cost-time data points obtained for each task from input I 2. Labeling the data points of inputs I 2a, I 2b, and I 2c by (c_{ij1}, d_{ij1}), (c_{ij3}, d_{ij3}), and (c_{ij2}, d_{ij2}) respectively, the functions $c_{ij}(d_{ij})$ are given by

$$\text{(A2)}\quad c_{ij}(d_{ij}) = \begin{cases} c_{ij1} + \dfrac{c_{ij2} - c_{ij1}}{d_{ij2} - d_{ij1}}(d_{ij} - d_{ij1}); \; d_{ij1} \leq d_{ij} \leq d_{ij2} \\[2ex] c_{ij2} + \dfrac{c_{ij3} - c_{ij2}}{d_{ij3} - d_{ij2}}(d_{ij} - d_{ij2}); \; d_{ij2} \leq d_{ij} \leq d_{ij3} \end{cases}$$

Each function $c_{ij}(d_{ij})$ is either linear, strictly convex, or strictly concave. The RAM-1 mathematical programming algorithm, consisting of the branch-and-bound algorithm and the Fulkerson network optimization algorithm, solves the problem A1 with objective function components of the form A2.

BRANCH-AND-BOUND-ALGORITHM

The branch-and-bound algorithm of Falk and Soland (Reference 2) converts problem A2 into a finite series of linear programming problems that yields the exact solution to problem A1. The algorithm is diagrammed in Figure A1.

The initial step in the algorithm consists of a substitution that enables any convex functions $c_{ij}(d_{ij})$ to be represented by linear functions. This is done by defining two new variables, $d_{ij}^{(1)}$ and $d_{ij}^{(2)}$, and two new functions, $c_{ij}^{(1)}$ and $c_{ij}^{(2)}$, as follows:

$$c_{ij}^{(1)} = \frac{c_{ij2} - c_{ij1}}{d_{ij2} - d_{ij1}}(d_{ij}^{(1)} - d_{ij2}), \; d_{ij1} \leq d_{ij}^{(1)} \leq d_{ij2}$$

$$c_{ij}^{(2)} = c_{ij2} + \frac{c_{ij3} - c_{ij2}}{d_{ij3} - d_{ij2}} d_{ij}^{(2)}, \; 0 \leq d_{ij}^{(2)} \leq d_{ij3} - d_{ij2} \; .$$

These new functions and variables are then substituted into problem A1 in place of the old convex functions and variables to yield a modified problem, A1′. The substitution is:

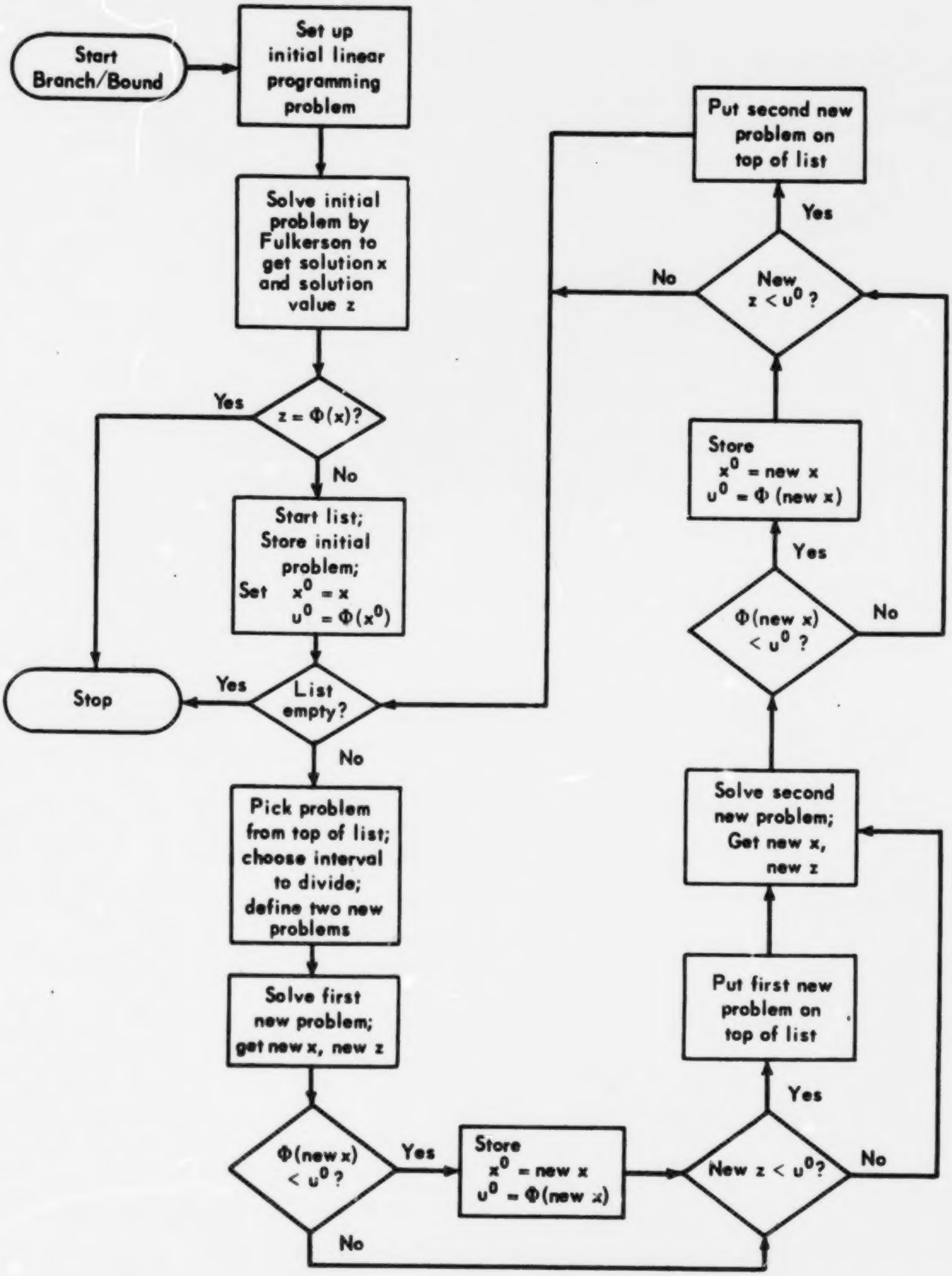
Start Branch/Bound
Set up initial linear programming problem
Solve initial problem by Fulkerson to get solution x and solution value z
z = Φ(x)?
Yes
No
Start list; Store initial problem; Set x⁰ = x u⁰ = Φ(x⁰)
Stop
Yes
List empty?
No
Pick problem from top of list; choose interval to divide; define two new problems
Solve first new problem; get new x, new z
Φ(new x) < u⁰?
Yes
No
Store x⁰ = new x u⁰ = Φ(new x)
New z < u⁰?
Yes
No
Put first new problem on top of list
Solve second new problem; Get new x, new z
Φ(new x) < u⁰ ?
Yes
No
Store x⁰ = new x u⁰ = Φ (new x)
New z < u⁰ ?
Yes
No
Put second new problem on top of list

Fig. A1—Flow Chart for Branch and Bound Algorithm
Φ = sum of task cost functions.

If $c_{ij}(d_{ij})$ is convex, substitute

$c_{ij}^{(1)}(d_{ij}^{(1)}) + c_{ij}^{(2)}(d_{ij}^{(2)})$ for $c_{ij}(d_{ij})$

in the objective function of A1.

Replace the constraint

$$t_i + d_{ij} \le t_j$$

with the constraints

$$t_i + d_{ij}^{(1)} \le t_i^{(1)}$$

$$t_i^{(1)} + d_{ij}^{(2)} \le t_j \ ,$$

and set

$$d_{ij} = d_{ij}^{(1)} + d_{ij}^{(2)} \ .$$

The variable $t_i^{(1)}$ serves as a pseudo event occurrence time and is a variable of the mathematical programming problem A1′. The values of c_{ij} and d_{ij} obtained from the solution of problem A1′ are the same as those obtained from the solution of problem A1. In the following discussion, the objective function of problem A1′ will be denoted by Φ.

Upon completion of the substitution step, the branch-and-bound algorithm proceeds to solve problem A1′ as follows:

A. Start. Linearize all concave functions $c_{ij}(d_{ij})$ by fitting them with functions $y_{ij}(d_{ij})$ using the formula

$$y_{ij}(d_{ij}) = \frac{c_{ij3} - c_{ij1}}{d_{ij3} - d_{ij1}}(d_{ij} - d_{ij1}) + c_{ij1} \ .$$

Solve problem A1′ using the functions y_{ij} in place of the corresponding concave functions c_{ij}. This is a linear programming problem that is solved using the Fulkerson network optimization algorithm. Obtain solutions d^1 and t^1, where d^1 and t^1 are vectors whose components are respectively the solution values of d_{ij} and t_i. Set the solution value of the linearized objective function equal to z^1. Evaluate Φ at d^1. Call this value u^1. If $z^1 = u^1$, stop; d^1 and t^1 are the desired solution. Otherwise, start a list of problems by storing the parameters of problem A1′ together with its solution. Set $d^b = d^1$, $t^b = t^1$, and $u^b = u^1$.

B. Iterative Step

1. Select a Branching Variable: Suppose problem k is the first problem on the list, where problem k is the same as problem A1′ except that d_{ij} may be restricted to an interval $\ell^k_{ij} \le d_{ij} \le L^k_{ij}$ that is different from the interval $a_{ij} \le d_{ij} \le b_{ij}$. Let (d^k, t^k) be the solution of this problem. Choose a branching variable by selecting that ij pair corresponding to a concave c_{ij} function for which $c_{ij}(d^k_{ij}) - y^k_{ij}(d^k_{ij})$ is maximal, where

(A3)
$$y^k_{ij}(d_{ij}) = \frac{c_{ij}(L^k_{ij}) - c_{ij}(\ell^k_{ij})}{L^k_{ij} - \ell^k_{ij}}(d_{ij} - \ell^k_{ij}) + c_{ij}(\ell^k_{ij}) .$$

2. Solve the First Subproblem: Replace y^k_{ij} of the k^{th} problem by $\bar{y}^k_{ij}$, where $\bar{y}^k_{ij}$ is formed by replacing L^k_{ij} with d^k_{ij} in A3. Also replace the interval (ℓ^k_{ij}, L^k_{ij}) by (ℓ^k_{ij}, d^k_{ij}). Solve the resulting linear programming problem with the Fulkerson algorithm to obtain the solution point $(\bar{d}^k, \bar{t}^k)$ and values $\bar{z}^k$ and $\bar{u}^k$.

3. Compare Values: If $u^b > \bar{u}^k$, set $d^b = \bar{d}^k$, $t^b = \bar{t}^k$ and $u^b = \bar{u}^j$. If $\bar{z}^k < u^b$, put the first subproblem on top of the list by storing its parameters and solution values.

4. Solve the Second Subproblem: Replace y^k_{ij} of the k^{th} problem by $\bar{\bar{y}}^k_{ij}$ which is formed by replacing ℓ^k_{ij} with d^k_{ij} in A3. Also, replace

the interval (ℓ_{ij}^k, L_{ij}^k) by (d_{ij}^k, L_{ij}^k). Solve the resulting subproblem using the Fulkerson algorithm to obtain the solution point $(\bar{\bar{d}}^k, \bar{\bar{t}}^k)$ and values $\bar{\bar{z}}^k$ and $\bar{\bar{u}}^k$.

5. Compare Values: If $u^b > \bar{\bar{u}}^k$, set $d^b = \bar{\bar{d}}^k$, $t^b = \bar{\bar{t}}^k$, and $u^b = \bar{\bar{u}}^k$. If $\bar{\bar{z}}^k < u^b$, put the second subproblem on top of the list by storing the parameters and solution values of the problem.

6. Check for Empty List: If the list is empty, stop. The solution to problem A1′ is (d^b, t^b) with objective function value u^b. Otherwise, go to B.

It is proved in Reference 2 that the branch-and-bound algorithm converges in a finite number of steps.

THE FULKERSON NETWORK OPTIMIZATION ALGORITHM

The Fulkerson network optimization algorithm (Reference 3) provides an efficient technique for solving mathematical programming problems of the form A1 with linear functions $c_{ij}(d_{ij})$. All linear programming problems generated by the branch-and-bound algorithm are of this type. The Fulkerson algorithm is diagrammed in Figure A2. The following description of the algorithm is from Fulkerson (Reference 3).

Start: Define

$$a_{ijk} = \begin{cases} b_{ij}, & k = 1 \\ a_{ij}, & k = 2 \end{cases}$$

$$c_{ijk} = \begin{cases} \text{slope of } c_{ij}(d_{ij}), & k = 1 \\ \infty, & k = 2 \end{cases}$$

$$\bar{a}_{ijk} = a_{ijk} + t_i - t_j \ .$$

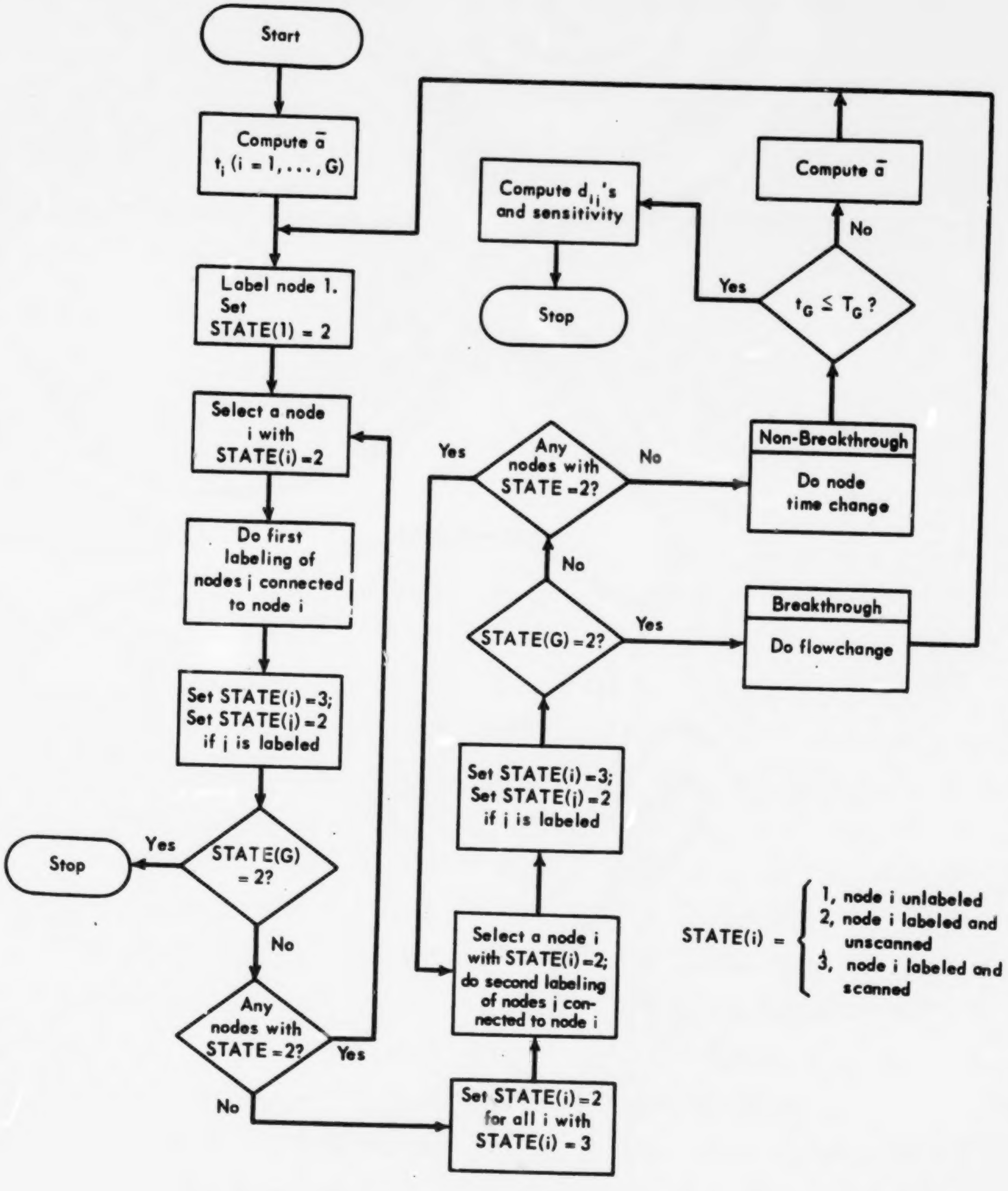
Start
Compute ā
t_i (i = 1, . . . , G)
Label node 1.
Set
STATE(1) = 2
Select a node
i with
STATE(i) = 2
Do first
labeling of
nodes j connected
to node i
Set STATE(i) = 3;
Set STATE(j) = 2
if j is labeled
STATE(G)
= 2?
Yes
Stop
No
Any
nodes with
STATE = 2?
Yes
No
Set STATE(i) = 2
for all i with
STATE(i) = 3
Select a node i
with STATE(i) = 2;
do second labeling
of nodes j connected to node i
Set STATE(i) = 3;
Set STATE(j) = 2
if j is labeled
STATE(G) = 2?
Yes
No
Breakthrough
Do flowchange
Any
nodes with
STATE = 2?
Yes
No
Non-Breakthrough
Do node
time change
$t_G \leq T_G$?
Yes
No
Compute ā
Compute d_{ij}'s
and sensitivity
Stop
STATE(i) = { 1, node i unlabeled
2, node i labeled and unscanned
3, node i labeled and scanned

Fig. A2—Flow Chart for Fulkerson Network Optimization Algorithm

Also define flow variables f_{ijk} and set them equal to zero. Successively compute $t_1, t_2, \ldots, t_G$ by the recursion

$$t_1 = 0$$

$$t_j = \max_i \, (t_i + b_{ij}) \; .$$

Iterative Procedure. Enter with a flow $f = f_{ijk}$ and node times t_i satisfying

$$t_1 = 0$$

$$a_{ijk} + t_i - t_j < 0 \Rightarrow f_{ijk} = 0$$

$$a_{ijk} + t_i - t_j > 0 \Rightarrow f_{ijk} = c_{ijk} \; .$$

Initially, these are the ones generated by the start routine.

A. Labeling Process: During this routine, a node is considered to be in one of three states: unlabeled, labeled and unscanned, or labeled and scanned. Initially all nodes are unlabeled.

1. First Labeling. Assign node 1 the label $[-,-,-,\epsilon(1) = \infty]$. This node is now labeled and unscanned. All other nodes are unlabeled. In general, select any labeled, unscanned node i and search for all unlabeled nodes j such that (i,j) is an arc and $\bar{a}_{ij2} = 0$. Label such nodes j with $[i,2,+,\epsilon(j) = \infty]$. Such nodes j are now labeled and unscanned, and node i is labeled and scanned. Repeat the general step until either the final node is labeled and unscanned or no more nodes can be labeled and the final node is unlabeled. In the former case, terminate. In the latter case, go to the second labeling.

2. Second Labeling. Nodes labeled above retain their labels and the labeling process continues as follows. All nodes revert to the unscanned state. The general step is to select any labeled, unscanned

node i and scan it for all unlabeled nodes j such that either ij is an arc with $\bar{a}_{ijk} = 0$ and $f_{ijk} < c_{ijk}$, or ji is an arc with $\bar{a}_{jik} = 0$ and $f_{jik} > 0$. If the former is the case, assign j the label $[i,k,+,\epsilon(j)]$, where

$$\epsilon(j) = \min[\epsilon(i),\ c_{ijk} - f_{ijk}] .$$

If the latter is the case, assign j the label $[i,k,-,\epsilon(j)]$ where

$$\epsilon(j) = \min[\epsilon(i), f_{jik}] .$$

Such nodes j are now labeled and unscanned, and node i is labeled and scanned. Repeat the general step until either the final node is labeled and unscanned (breakthrough), or until no more labels can be assigned and the final node is unlabeled (non-breakthrough). In the case of breakthrough, go to routine B. If non-breakthrough, go to routine C.

B. Flow Change. The labeling process has resulted in breakthrough. Change the flow f as follows. The last node, node G, will be labeled $[j,k,+,\epsilon(G)]$. Add $\epsilon(G)$ to f_{jGk}. Then go to node j and its label. The general step: if node p is labeled $[i,k,+,\epsilon(p)]$, add $\epsilon(G)$ to f_{ipk}. If node p is labeled $[i,k,-,\epsilon(p)]$, subtract $\epsilon(G)$ from f_{pik}. In either case, go on to node i and its label. Repeat the general step until node 1 is reached. Then discard the labels and go back to A.

C. Node Time Change. The labeling process has resulted in non-breakthrough. Define the following sets:

$$A_1 = \{(i,j,k) \mid i \text{ labeled, } j \text{ unlabeled, } \bar{a}_{ijk} < 0\}$$

$$A_2 = \{(i,j,k) \mid i \text{ unlabeled, } j \text{ labeled, } \bar{a}_{ijk} > 0\}$$

Define

$$\delta_1 = \min_{A_1}(-\bar{a}_{ijk})$$

$$\delta_2 = \min_{A_2}(\bar{a}_{ijk})$$

$$\delta = \min(\delta_1, \delta_2) \ .$$

Change the node times t_i by subtracting δ from all t_i corresponding to unlabeled i. If $t_G \leq T_G$, stop. Otherwise, discard the labels and go back to A.

Upon termination, set

$$d^*_{ij} = \max(b_{ij}, t_j - t_i) \ .$$

If $t_G = T_G$, d^*_{ij} is the desired solution value of d_{ij}. If $t_G < T_G$, the desired value of d_{ij} is

$$d_{ij} = d^{**}_{ij} + \frac{d^*_{ij} - d^{**}_{ij}}{t_G - t^{**}_G}(T_G - t^{**}_G) \ ,$$

where d^{**}_{ij} and t^{**}_G are respectively the task durations and node times that would have been obtained if the algorithm had been stopped immediately after the next to the last node time change.

The Fulkerson algorithm is proved to converge in a finite number of iterations in Reference 3.

The change in total project cost per unit increase in the achievement date of real final goal i, as given by duality theory, is approximately

$$\text{Sens } (i) = \max(0, f_{iG1} + f_{iG2} - c_{iG1}) \ ,$$

for small increases in the achievement date of final goal i, where f_{iG1} and f_{iG2} are evaluated at the termination of the algorithm, and it is assumed that there are multiple real final goals i and a dummy final goal G. If there is one real final goal and no dummy, the sensitivity is approximately

$$\text{Sens} = \frac{\text{cost}(d_{ij}) - \text{cost}(d_{ij}^{**})}{T_G - t_G^{**}}$$

where $\text{cost}(d_{ij})$ is the total project cost with task durations d_{ij}. The latter equation also approximates the cost sensitivity to uniform changes in the achievement dates of multiple real final goals. All of the approximations are exact if there are no concave cost-time curves.

CRITICAL PATH ALGORITHM

Define the following symbols:

ES(ij) = Early start time of task ij.

EF(ij) = Early finish time of task ij.

LF(ij) = Latest date task ij can finish without delaying goal achievement.

LS(ij) = Latest date task ij can start without delaying goal achievement.

d_{ij} = Duration of task ij.

T_G = Final goal achievement date.

FS(ij) = Free slack of task ij.

G = Final goal node number.

The critical path algorithm computes task start and finish dates, task free slack times, and critical tasks using the following equations (Reference 1):

$$ES(ij) = \begin{cases} 0 \quad , & i = 1 \\ \max[EF(\text{all predecessors of } ij)] & i \neq 1 \end{cases}$$

$$EF(ij) = ES(ij) + d_{ij}$$

$$LF(ij) = \begin{cases} T_G \quad , & \text{if } j = G \\ \min[LS(\text{all successors of } ij)] \; , & j \neq G \end{cases}$$

$$LS(ij) = LF(ij) - d_{ij}$$

$$FS(ij) = \min[ES(\text{all immediate successors of } ij)] - EF(ij)$$

A task ij is critical if

$$LS(ij) - ES(ij) = 0 \; .$$

BUDGET PREPARATION AND TASK SCHEDULING ROUTINE

Define

$$A_{ijk} = \begin{cases} 1, & k - 1 \leq t_i < k \\ 0, & \text{otherwise} \end{cases}$$

$$B_{ijk} = \begin{cases} 1, & k - 1 \leq t_i + d_{ij} < k \\ 0, & \text{otherwise} \end{cases}$$

$$C_{ijk} = \begin{cases} 1, & t_i < k - 1 \text{ and } k \leq t_i + d_{ij} \\ 0, & \text{otherwise} \end{cases}$$

Then A_{ijk} labels tasks ij starting in year k; B_{ijk} labels tasks ij finishing in year k, and C_{ijk} labels tasks ij active in year k but not starting or finishing in year k.

Assuming a uniform rate of expenditure for each task, the cost of task ij in year k is

$$\text{Cost }(ijk) = \frac{c_{ij}}{d_{ij}}[A_{ijk}(k - t_i)$$

$$+ B_{ijk}(t_i + d_{ij} + 1 - k) + C_{ijk}$$

$$- A_{ijk}B_{ijk}] , \quad d_{ij} \neq 0$$

$$\text{Cost }(ijk) = 0 , \quad \text{if } d_{ij} = 0 .$$

The project budget for year k is

$$\text{Cost }(k) = \sum_{ij} \text{Cost }(ijk) .$$

A non-dummy task ij is active in year k if Cost (ijk) $\neq 0$. It starts in year k if $(A_{ijk})\cdot$Cost (ijk) $\neq 0$. It finishes in year k if $B_{ijk} \cdot c_{ij} \neq 0$. It continues in a year k from a previous year if it is active in year k and $A_{ijk} = 0$.

STATISTICAL ANALYSIS ROUTINE

Actual task costs are assumed to be related to estimated task costs through the gamma distribution. The density function of this distribution is

$$P(x) = \frac{1}{\Gamma(\alpha + 1)\beta^{\alpha+1}} x^{\alpha} e^{-x/\beta}$$

where Γ is the gamma function and α and β are constant parameters. The mean of the gamma distribution is

$$\bar{x} = \beta(\alpha + 1) ,$$

and the variance is

$$\sigma^2 = \beta^2(\alpha + 1) .$$

The mean of the gamma cost distribution for a task is set equal to the cost estimate obtained from the cost-time curve for the task.

The variance of the distribution is computed from its mean and 90th percentile. The latter is obtained from input I 4 of the RAM-1 model. The cost distribution for a task is considered to give the probability distribution of actual task cost.

The gamma distribution has the following properties that make it useful as a cost distribution:

1. The gamma distribution has the shape that cost distributions are normally expected to have (see Figure A3).
2. It is a two parameter distribution and thus can be specified using a minimum of input data.
3. The distribution places an absolute lower bound on task costs, but no absolute upper bound. However, costs far in excess of estimated costs are extremely unlikely.
4. The median and mean of a gamma distribution whose standard deviation, σ, does not exceed 50 percent of the mean are equal to within approximately 7 percent. It is therefore not of significance to the statistical analysis routine whether cost estimates are considered to be distribution means or medians provided the estimates are accurate to within approximately 50 percent.
5. The 90th percentile of the gamma distribution is the mean plus 1.3σ provided σ does not exceed 50 percent of the mean. Thus, distributions resulting from cost estimates whose accuracy is within approximately 50 percent are computationally easy to use.

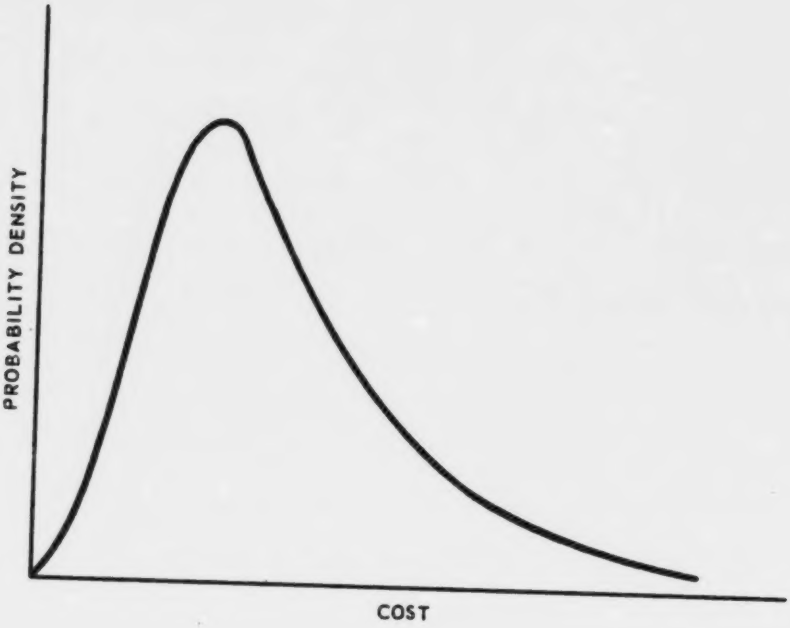

Fig. A3—The Gamma Distribution

The mean of the cost distribution for task ij is c_{ij}, the task cost obtained from the mathematical programming algorithm. The 90th percentile of the distribution is $s_{ij}c_{ij}$, where s_{ij} is obtained from input I 4 of the RAM-1 model. The variance V of the distribution of total project cost due to random errors in cost estimation is then

$$V = \sum_{ij} \left(\frac{s_{ij}c_{ij}}{1.3}\right)^2 .$$

The sum is over all tasks ij. Due to the central limit theorem of statistics (Reference 4), the total project cost distribution can be considered normal with mean equal to the total project cost obtained from the budget preparation and task scheduling routine. It follows that with probability 95 percent, random errors in cost estimation will not cause total project cost to exceed the bound B, where

$$B = \text{Total Cost} + 1.65V^{\frac{1}{2}} ,$$

and Total Cost is the mean of the total project cost distribution.

It is assumed that the variance v_{ijk} of the distribution of the cost of task ij in year k is related to the variance v_{ij} in the full-duration task cost distribution by

$$v_{ijk} = v_{ij} \frac{\text{cost}_{ijk}}{c_{ij}} ,$$

where, as previously indicated,

$$v_{ij} = \left(\frac{s_{ij}c_{ij}}{1.3}\right)^2 .$$

The variance V_k of the distribution of the cost of a project in year k due to random errors in cost estimates is then

$$V_k = \sum_{ij} v_{ijk} .$$

Again using the central limit theorem, yearly project costs can be considered normally distributed. It follows that for a project that terminates in its K^{th} year, there is 95 percent probability that project cost will not exceed B_1 in year 1, B_2 in year 2,..., and B_k in year k ($k \leq K$), where

$$B_k = \text{Cost}(k) + \alpha(K)V_k^{\frac{1}{2}}, \; k = 1,2,\ldots,K$$

and $\alpha(K)$ is the number of standard deviations between the mean and the $(.95)^{1/K}$ percentile of the normal distribution.

A detailed discussion of the properties of the gamma distribution is given in References 4 and 5.

MAIN PROGRAM

```
      PROGRAM MAINP(INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT)
C......
      COMMON/CV1/IP(12),RP(12)
      COMMON/CV2/BLO(100),ULO(100),NOCIT(100),CO(100)
      COMMON/CV21/FKO
     COMMON/CV3/NV,NCF,PHIT,UZ,US,USP,USM
      COMMON/CV4/X(100),COST
      COMMON/CV5/SIGMA(100,4),TSIG
      COMMON/CV6/XZ(100)
      COMMON/CV7/NF1,NOP,NOPS
      COMMON/CV8/NXBK,XK,NOBOL
      COMMON/CV9/PSIGL( 50),NXBL(50),XNXBL(50),BLIST(50,100)
      COMMON/CV10/FKHL(50)
      COMMON/CV11/SUPC(100,2),UNTC(100,2),XBIJ(50,3)
      COMMON/FV3/RLM(100), PLM(100), SN(100), ISN(100)
      COMMON/TSW/NSWW
C...N(MAX)=100
C...REMOVE ALL REFS TO CONSTRAINT MATRIX A( , )
C...MAX. NO OF ROWS IN SIGMA = N
C
      COMMON/C1/NOPP,NNZP
      COMMON/C2/MXNZ,MXRP
      COMMON/C3/ IK(100), IPDC(100),DC(6,100),JC(100),IR(100)
      COMMON/C4/NINDX,NJNDX,CDATE,IJK(100,3),NLIN,NCVX
      COMMON/C5/IA(100),JA(100),BL(100),UL(100),Q(100)
      COMMON/C6/IXA(100),JXA(100),S(100)
      COMMON/C7/D1(100)
      COMMON/C8/ITN (100),IGN (100)
C

      COMMON/FLK/
     A         A(2,100), C(2,100), TAU(100), ABAR(2,100), DUR(100),
     1   LRLS(100,3),     F(2,100), ISTATE(100), IVEC(100),
     2    JVEC(100),M,N,TM ,IFLAG,TO(100),TD(100), ITER,REPS(100),
     3        RLMDA(100), PLMDA(100),SEN(100),ISEN(100),SUMK
C
      COMMON/B1/
     B                                           ENT(100),RLNT(100),
     4      FS(100), EF(100), RLF(100),RLS(100),TS(100), FS(100)
C
C...THE NON-ZERO ELEMENTS IN P(I,J) MUST GENERATE .LE. 99 IJ-PAIRS
C...THE MAXIMUM NO. OF VARIABLES WITH CONCAVE FUNCTIONS IS .LE. 25
C...THE MAXIMUM NO. OF CONSTRAINT REGIONS IS 2
C...THE MAXIMUM NO. OF ROWS IN SUPC AND UNTC IS N
C......
      CALL SETUP
      CALL BBCAV
      CALL TREXD
      CALL CPATH
      CALL COSTC
      CALL WRPT
C
      STOP7777
      END
      SUBROUTINE FXCAV(S1,S2,D)
C.....
      COMMON/CV1/IP(12),RP(12)
      COMMON/CV2/BLO(100),ULO(100),NOCIT(100),CO(100)
      COMMON/CV3/N,NCF,PHIT,UZ,US,USP,USM
      COMMON/CV11/SUPC(100,2),UNTC(100,2),XBIJ(50,3)
      DIMENSION D(6)
C .....
      RN=N + 1
      IF(IP(6).EQ.1) PRINT 900,RN,S1,S2,D
  900 FORMAT(1H0,20HSUBROUTINE FXCAV    /10F10.2)
C...CONCAVE FX, FX1=S1*D(I,J) + C1, WHEN D(I,J).LT. (D2-D1)
C...            FX2=S2*D(I,J) + C2, WHEN D(I,J).GT. (D2-D1) AND .LT. (D3-D2)
C...
C...      WHERE B2=C2 - S2*(D2-D1)
      N=N+1
      NCF=N
      CO(N)=0.
      BLO(N)=0.
      ULO(N)=D(3)-D(1)
      NOCIT(N)=2
      SUPC(N,1)= D(4)
      SUPC(N,2)= D(5) + S2*(D(1) - D(2) )
      UNTC(N,1)= S1
      UNTC(N,2)= S2
      XBIJ(N,1)= (D(2)-D(1))
      XBIJ(N,2)= (D(3)-D(1))
      IF(IP(6).EQ.1) PRINT 900
      RETURN
      END
      SUBROUTINE FXID(K,S1,S2,P)
C...RETURNS
C...K =-1 - CONVEX
C....    0 - LINEAR
C...    +1 - CONCAV
C...      2,3,4  - LINEAR - ONLY F1(X)
C...S1= SLOPE OF F1(X)
C...S2= SLOPE OF F2(X)
C...GIVEN  P( )= (D( ), C( ))
      COMMON/CV1/IP(12),RP(12)
      DIMENSION P(6)
      IF(IP(6).EQ.1) PRINT 900,RK,S1,S2,P
  900 FORMAT(1H0,20HSUBROUTINE FXID     /10F10.2)
      IF( P(1).EQ. P(3)) GO TO 75
      S1= (P(5)-P(4)) / (P(2)-P(1))
      S2= (P(6)-P(5)) / (P(3)-P(2))
      IF( S1.NE.0)GO TO 20
      IF( S2.LE.0)GO TO 20
      K=2
      GO TO 776
   20 CONTINUE
      IF(S2.NE.0)GO TO 40
      IF(S1.GE.0)GO TO 25
      K=3
      GO TO 776
   25 CONTINUE
      IF(S1.EQ.0) GO TO 40
      K=4
      GO TO 776
   40 CONTINUE
      IF((ABS(S1-S2)).GT.RP(6)   ) GO TO 60
      K=0
      GO TO 776
   60 CONTINUE
      IF(S1.LT.S2)GO TO 70
      K=1
      GO TO 776
   70 CONTINUE
      K= -1
      GO TO 776
   75 CONTINUE
      K=0
      S1=0.0
      S2=0.0
  776 CONTINUE
      RK=K
      IF(IP(6).EQ.1) PRINT 900,RK,S1,S2,P
      RETURN
      END
      SUBROUTINE FXLIN(ID,S1,S2,D)
C......
      COMMON/CV1/IP(12),RP(12)
      COMMON/CV2/BLO(100),ULO(100),NOCIT(100),CO(100)
      COMMON/CV3/N,NCF,PHIT,UZ,US,USP,USM
      COMMON/CV11/SUPC(100,2),UNTC(100,2),XBIJ(50,3)
      COMMON/C3/ IK(100), IPDC(100),DC(6,100),JC(100),IR(100)
      COMMON/C4/NINDX,NJNDX,CDATE,IJK(100,3),NLIN,NCVX
      DIMENSION D(6)
C......
      RN=N + 1
      IF(IP(6).EQ.1) PRINT 900,RN,S1,S2,D
  900 FORMAT(1H0,20HSUBROUTINE FXLIN    /10F10.2)
      UL=D(2)-D(1)
      IF(ID.EQ.0)UL=D(3)-D(1)
C...LINEAR-FX, FX=S1*D(I,J) + C1, WHERE D(I,J).LT. (D2-D1) OR
C...                                    D(I,J).LT. (D3-D1)
      N=N+1
      NLIN=NLIN+1
      CO(N)=S1
      BLO(N)=0
      ULO(N)=UL
      NOCIT(N)=0
      SUPC(N,1)=D(4)
      UNTC(N,1)= S1
      IF(IP(6).EQ.1) PRINT 900
      RETURN
      END
      SUBROUTINE FXVEX(S1,S2,D)
C......
      COMMON/CV1/IP(12),RP(12)
      COMMON/CV2/BLO(100),ULO(100),NOCIT(100),CO(100)
      COMMON/CV3/N,NCF,PHIT,UZ,US,USP,USM
      COMMON/CV11/SUPC(100,2),UNTC(100,2),XBIJ(50,3)
      COMMON/C4/NINDX,NJNDX,CDATE,IJK(100,3),NLIN,NCVX
      DIMENSION D(6)
C......
      RN=N + 1
      IF(IP(6).EQ.1) PRINT 900,RN,S1,S2,D
  900 FORMAT(1H0,20HSUBROUTINE FXVEX    /10F10.2)
C...CONVEX FX, FX1=S1*(D(I,J)-D2) + R1, WHERE D(I,J).LT. (D2-D1)
C...              FX2=S2*(D(I,J) + C2 )    WHERE D(I,J).LT. (D3-D2)
      N=N+1
      N1=N+1
C...N=N+1 AT RETURN
      NCVX=NCVX+1
C...NCVX - NO. OF CONVEX F(X) IN P(I,J)
      CO(N) = S1
      CO(N1)= S2
      BLO(N) = 0.
      BLO(N1)= 0.
      ULO(N) = D(2)-D(1)
      ULO(N1)= D(3)-D(2)
      NOCIT(N) =0
      NOCIT(N1)=0
      SUPC(N,1)=          D(4)-D(5)
      SUPC(N1,1)=D(5)
      UNTC(N,1)= S1
      UNTC(N1,1)=S2
      N=N+1
      IF(IP(6).EQ.1) PRINT 900
      RETURN
      END
      SUBROUTINE MAPDX
C...MAP D(IR,JC)  INTO  X(K)
C...LET IJK(K,3) = I, K-TH INDEX PAIR BECOMES THE I-TH VAR IN BBCAV
C...TRANSLATE (IR,JC) INTO  (I,J)
C
      COMMON/CV1/IP(12),RP(12)
      COMMON/CV3/N,NCF,PHIT,UZ,US,USP,USM
      COMMON/C3/ IK(100), IPDC(100),DC(6,100),JC(100),IR(100)
      COMMON/C4/NINDX,NJNDX,CDATE,IJK(100,3),NLIN,NCVX
      COMMON/C5/IA(100),JA(100),BL(100),UL(100),C(100)
C
      IF(IP(6).EQ.1) PRINT 900,BL,UL
  900 FORMAT(1H0,20HSUBROUTINE MAPDX    /10F10.2)
      N=0
      NCF =0
      NLIN=0
      NCVX=0
      DO 100 K=1,NINDX
      IF(IK(K).NE.(+1))GO TO 100
      I =IPDC(K)
      CALL FXCAV( BL(K),UL(K),DC(1,I ))
      IJK(K,3)=N
  100 CONTINUE
      DO 200 K=1,NINDX
      ID= IK(K)
      IF( (ID.EQ.0).OR.(ID.GT.1))GO TO 150
      GO TO 200
  150 CONTINUE
      I =IPDC(K)
      CALL FXLIN(ID,BL(K),UL(K),DC(1,I) )
      IJK(K,3)=N
  200 CONTINUE
      DO 300 K=1,NINDX
      IF(IK(K).NE.(-1))GO TO 290
      I = IPDC(K)
      CALL FXVEX( BL(K),UL(K),DC(1,I ))
      IJK(K,3)=N-1
      IJK(K+1,3)=N
  290 BL(K)=0.0 $ UL(K)=0.0
  300 CONTINUE
      CALL NEWIJ
      IF(IP(6).EQ.1) PRINT 900
      RETURN
      END
      SUBROUTINE MAPIS(IS,I)
      COMMON/CV1/IP(12),RP(12)
      COMMON/C3/ IK(100), IPDC(100),DC(6,100),JC(100),IR(100)
      COMMON/C4/NINDX,NJNDX,CDATE,IJK(100,3),NLIN,NCVX
      IF(IP(6).EQ.1) PRINT 900, IS,I
  900 FORMAT(1H0,20HSUBROUTINE MAPIS     /20I5)
      K=IJK(I,2)
      IS= IPDC(K)
      IF(IP(6).EQ.1) PRINT 900, IS,I
      RETURN
      END
      SUBROUTINE NEWIJ
C.......
      COMMON/CV1/IP(12),RP(12)
      COMMON/C3/ IK(100), IPDC(100),DC(6,100),JC(100),IR(100)
      COMMON/C4/NINDX,NJNDX,CDATE,IJK(100,3),NLIN,NCVX
      COMMON/C5/IA(100),JA(100),BL(100),UL(100),C(100)
      COMMON/CV5/SIGMA(100,4),TSIG
C.......
      IF(IP(6).EQ.1) PRINT 900
  900 FORMAT(1H0,20HSUBROUTINE NEWIJ    /10F10.2)
      DO 100 I=1,NINDX
      IJK(I,2)=I
      IJK(I,1)=0
  100 CONTINUE
      LT=1
      IA(1)=1
      DO 200 K=1,NINDX
C...SET IA
      IRK=IR(K)
      DO 110 I=1,LT
      IF(IRK.NE.IJK(I,1)) GO TO 110
      GO TO 200
  110 CONTINUE
      IJK(LT,1)=IRK
      LT=LT+1
  200 CONTINUE
      IA(NINDX+1)=LT
C.......IA(NINDX+1) =NO. OF UNIQUE  I-INDICES IN IA( )  PLUS  ONE
      LT=LT-1
      DO 207 K=1,NINDX
      IRK=IR(K)
      DO 205 I=1,LT
      IF(IRK.NE.IJK(I,1)) GO TO 205
      IA(K)=IJK(I,2)
      GO TO 207
  205 CONTINUE
  207 CONTINUE
C...SET JA
      MXNX=IJK(LT+1,2)
      DO 300 K=1,NINDX
      JCK= JC(K)
      DO 210 I=1,LT
      IF(JCK.NE.IJK(I,1))GO TO 210
      JA(K)= IJK(I,2)
      GO TO 300
  210 CONTINUE
      JA(K)=MXNX
  300 CONTINUE
C...DEFINE  X(I) TO  J(TH) PHI(D) IN  IJK(*,2)
      DO 400 K=1,NINDX
      J = IJK(K,3)
      IJK(J,2)=K
  400 CONTINUE
      IF(IP(6).EQ.1) PRINT 910,(IA(I),I=1,NINDX)
      IF(IP(6).EQ.1) PRINT 910,(JA(I),I=1,NINDX)
  910 FORMAT(1H0,20I5)
      DO 410 I=1,NINDX
      SIGMA(I,1)=IA(I)
      SIGMA(I,2)=JA(I)
  410 CONTINUE
      DO 420 I=1,NINDX
      J=IJK(I,2)
      IA(I)=SIGMA(J,1)
      JA(I)=SIGMA(J,2)
      SIGMA(J,1)=0.0
      SIGMA(J,2)=0.0
  420 CONTINUE
      RETURN
      END
      SUBROUTINE PIJ
C...MXNZ - MAX NO OF NON-ZERO ENTRIES IN PRECEDENCE ARRAY
C...MXRP - MAX NO OF ROWS IN PRECEDENCE ARRAY
C...NNZP - NO OF NON-ZERO ENTRIES
C...NORP - NO OF ROWS
C...NOJ(I) - NO OF J-INDICES IN I(TH) ROW
C...INITJ(I) - WORD LOC IN JC-ARRAY FOR INITIAL J-INDEX IN I(TH) ROW
C...DC( , )  -  ARRAY OF (  DIJ(3), CIJ(3)  )
C...JC(  ) - ARRAY OF J-INDICES
C...IR( ) - ARRAY OF I-INDICES
C
      COMMON/CV1/IP(12),RP(12)
      COMMON/C1/NORP,NNZP
      COMMON/C2/MXNZ,MXRP
      COMMON/C3/ IK(100), IPDC(100),DC(6,100),JC(100),IR(100)
      COMMON/C4/NINDX,NJNDX,CDATE,IJK(100,3),NLTN,NCVX
      COMMON/C5/IA(100),JA(100),BL(100),UL(100),C(100)
      COMMON/C6/IXA(100),JXA(100),S(100)
      COMMON/C8/ITN (100),IGN (100)
      DIMENSION D(6)
C
   20 FORMAT(8I5)
   45 FORMAT(2A10)
   50 FORMAT(2I5,7F10.2)
C
      IF(IP(6).EQ.1) PRINT 900
  900 FORMAT(1H0,20HSUBROUTINE PIJ      /10F10.2)
C
C
      READ(5,20)MXNZ,MXRP
      READ(5,20)NORP,NNZP
      READ(5,5) CDATE
    5 FORMAT(F10.2)
C
C
      IF( IP(6).EQ.1) PRINT 920, MXNZ, MXRP
  920 FORMAT(1H0,20HSUBROUTINE PIJ      /20I5)
      IF( IP(6).EQ.1) PRINT  50, NORP, NNZP, CDATE
      IF(  (NORP.GT.MXRP).OR.(NNZP.GT.MXNZ) )STOP0012
        DO 10 I=1,MXRP
      IK(I)=0
      IPDC(I)=0
   10 CONTINUE
      DO 15 I=1,MXNZ
      IP(I)=0
      JC(I)=0
      DO 15 J=1,6
      DC(J,I)=0.0
   15 CONTINUE
      K=0
      DO 200 IFN=1,NNZP
      K=K+1
C
C
      READ(5,45) ITN(IFN), IGN(IFN)
      IF(ENDFILE5)777,30
   30 CONTINUE
```

```
      READ(5,50)IX,    JX,    (DC(J,IFN), J=1,6), S(IFN)
C
C
      IXA(IFN)=IX
      JXA(IFN)=JX
      IF(IP(6).EQ.1) PRINT 920,IX, JX
      IX=IX*100
      CALL FXID(ID,S1,S2,DC(1,IFN))
      BL(K)=S1 $ UL(K)=S2 $ IPDC(K)=IFN
      IF(ID.NE.(-1))GO TO 100
      IR(K)=IX
      JC(K)=IX+JX
      IK(K)=-1
      K=K+1
      IPDC(K)=IFN
      IR(K)=IX+JX
      JC(K)=JX*100
      IK(K)=-2
      GO TO 200
  100 CONTINUE
      IR(K)=IX
      JC(K)=JX*100
      IK(K)=ID
  200 CONTINUE
      NINDX=K
      IF(IP(6).EQ.1) PRINT 900
      RETURN
  777 CONTINUE
      STOP 0016
      END
      SUBROUTINE PRESET
C......
      COMMON/CV1/IP(12),RP(12)
      COMMON/CV2/BLO(100),ULO(100),NOCTT(100),CO(100)
      COMMON/CV3/N,NCF,PHIT,UZ,US,USP,USM
      COMMON/CV4/X(100),COST
      COMMON/CV5/SIGMA(100,4),TSIG
      COMMON/CV6/XZ(100)
      COMMON/CV7/NF1,NOP,NOPS
      COMMON/CV8/NXBK,XK,NOBOL
      COMMON/CV9/PSIGL( 50),NXBL(50),XNXBL(50),BLIST(50,100)
      COMMON/CV10/FKBL(50)
C......
C
      IF(IP(6) .EQ. 1) WRITE(6,999)
  999 FORMAT(1H-,124X,6HPRESET)
      NORA = IP(2)
      MMAX = IP(4)
      NMAX = IP(5)
      IBMAX = IP(9)
      JBMAX = IP(10)
C
      DO 10 J = 1,NMAX
      X(J) =0.0
      XZ(J)=0.0
      DO  7 I =1,4
    7 SIGMA(J,I) = 0.0
   10 CONTINUE
C
      PHIT=0
      UZ=0.0
      USP= 0.0
      USM= 0.0
      COST=0.0
      TSIG=0.0
      NF1=0
      CFX=0.0
      IOPT=0
      NOP=0
      NXBP=0
      XK =0.0
      NOBOL=0
C
      DO 20 I =1,IBMAX
      FKBL(I) = 0.0
      PSIGL(I)= 0.0
      NXBL(I)= 0
      XNXBL(I)=0.0
      DO 20 J=1,JBMAX
      BLIST(I,J) = 0.0
   20 CONTINUE
C
C
      RETURN
      END
      SUBROUTINE RNDOFF(I,IV)
C......
       COMMON/CV1/IP(12),RP(12)
       COMMON/CV2/BLO(100),ULO(100),NOCIT(100),CO(100)
       COMMON/CV4/X(100),COST
       COMMON/CV11/SUPC(100,2),UNTC(100,2),XBIJ(50,3)
C......
C
  400  ID = I
      XR = X(I)
       NRI = NOCIT(I)
       XBND = XBIJ(I,NRI)
       RHOB = RP(3)
      DIF = XR - XBND
      RDIF = DIF / XBND
      IF(RDIF .LT. RHOB) GO TO 777
      WRITE(6,405)
  405 FORMAT(1H-,38HRATIO OF DIFFERENCE .GE. RHOB .....EOP)
       WRITE(6,410)I,XR,XBND,RHOB
  410  FORMAT(1H0,3HI =,I3,6X,6HX(I) =,F12.7,6X,6HUPBND=,E12.7,6X,
     16HRHOB =,E12.7)
      CALL EXIT
  777 RETURN
      END
      SUBROUTINE SET(TMMAX)
      COMMON /TSW/ NSWW
      COMMON/TMX/TMO,EXT
C
C  SECOND GIVES JOB CPU EXECUTION TIME IN 1/1000 OF A SECOND
C
      CALL SECOND(TMO)
      EXT = TMMAX + TMO
      NSWW = 1
      RETURN
      END
      SUBROUTINE SETD(D,JS)
      COMMON/CV1/IP(12),RP(12)
      COMMON/C3/ IK(100), IPDC(100),DC(6,100),JC(100),TR(100)
      DIMENSION D(6)
      IF(IP(6).EQ.1) PRINT 900
  900 FORMAT(1H0,20HSUBROUTINE SETD     /10F10.2)
      DO 100 I=1,6
      D(I)= DC(I,JS)
  100 CONTINUE
      IF(IP(6).EQ.1) PRINT 900
      RETURN
      END
      SUBROUTINE SETUP
C...DEFINE INPUT PARAMETERS TO RBCAV
C...THIS CODE IS NOT INTENDED TO BE A GENERALIZED ALGORITHM
C...MAX-N = 100,  MAX-NCF = 50
C
      COMMON/CV1/IP(12),RP(12)
      COMMON/CV2/BLO(100),ULO(100),NOCIT(100),CO(100)
      COMMON/CV3/N,NCF,PHIT,UZ,US,USP,USM
      COMMON/CV4/X(100),COST
      COMMON/CV6/XZ(100)
      COMMON/CV11/SUPC(100,2),UNTC(100,2),XBIJ(50,3)
      COMMON/C1/NORP,NNZP
      COMMON/C2/MXNZ,MXRP
      COMMON/C3/ IK(100), IPDC(100),DC(6,100),JC(100),IR(100)
      COMMON/C4/NINDX,NJNDX,CDATE,IJK(100,3),NLIN,NCVX
      COMMON/C5/IA(100),JA(100),BL(100),UL(100),C(100)
      COMMON/C7/D1(100)
C
C
C...USER DEFINES IP(5, 6, 7, 9, 10 AND 12)
      READ(5,2) IP(5),IP(6),IP(7),IP(9),IP(10),IP(12)
    2 FORMAT(6I5)
C...USER DEFINES RP(1, 2, 3, 4 AND 6)
      READ(5,3)RP(1),RP(2),RP(3),RP(4),RP(6),RP(7)
    3 FORMAT(6E10.0)
C
      IF(IP(6).EQ.1) PRINT 900
  900 FORMAT(1H0,20HSUBROUTINE SETUP    /10F10.2)
      DO 5 I=1,100
      NOCIT(I)=0
      CO(I)=0.
      BLO(I)=0.
      ULO(I)=0.
      DO 5 J=1,2
      SUPC(I,J)=0.
      UNTC(I,J)=0.
    5 CONTINUE
C
C
      CALL PIJ
   20 FORMAT(1H1,10HDMP NO. = ,F3.0/(20I5/))
   29 FORMAT (1H1,10HDMP NO. = ,F3.0/(10F10.2/))
C
      DMP =1.0
      WRITE(6,20) DMP, IK
C
      DMP=2.
      WRITE(6,20) DMP, IPDC
C
      DMP=3.
      WRITE(6,29) DMP,DC
C
      DMP=4.
      WRITE(6,20)DMP,JC
C
      DMP=5.
      WRITE(6,20) DMP,IR
```

```
C
C
C
C
      CALL MAPDX
C
      DMP=6.
      WRITE(6,20)DMP,IA
C
      DMP=7.
      WRITE(6,20)DMP,JA
      DMP=8.
      PRINT 20,DMP,IJK
      DMP=9.
      PRINT 29,DMP,BLO,ULO,CO
C
      DMP=10.
      PRINT 29,DMP,SUFC,UNTC,XBIJ
C
      DMP=11.
      PRINT 20,DMP,N,NCF,NCVX,NLIN,NOCIT
C
C
C
C
C...DEFINE  IP( ) AND RP( )
C
C   IP1=N, IP2=NORA, IP3=NCF, IP4=MMAX, IP5=NMAX, IP6=LPIN, IP7=LPOUT
C   IP8=ICTU, IP9=IBMAX, IP10=JBMAX, IP11=ICK, IP12=MXNOP
C
C   N - TOTAL NO. OF VARIABLES
C   NORA - NO. OF ROWS IN A-MATRIX
C   NCF - NO. OF VARIABLES W/CONCAVE-F(X)
C   MMAX - MAX. NO. OF CONSTRAINTS FOR JPLP
C   NMAX - MAX. NO. OF VARIABLES FOR JPLP
C   LPIN - IF (1) WRITE LP INPUT FOR EACH PROBLEM
C   LPOUT - IF (1) WRITE LP OUTPUT FOR EACH PROBLEM
C   ICTU - IF (1) CONSTRAIN COST-F(X) .LT. U0
C   IBMAX - MAX NO. OF ROWS IN BLIST
C   JBMAX - MAX NO. OF COLUMNS IN BLIST
C   ICK - IF (1) SET PRINT = .TRUE. IN JPLP
C   MXNOP - MAX. NO. OF LP PROBS. SOLVED BEFORE CALLING EXIT
C
C
C
C   RP1=EPSI, RP2=TMMAX, RP3=THETA, RP4=TRACE, RP5=SUM EK-LIN, RP6=LIN DLOPF DIF
C   RP7=TS ZERO TOL
C   EPSI - ADJUSTMENT FACTOR FOR U0
C   TMMAX - MAX.  BB-EXCT TIME IN SECONDS
C   THETA - X(I) ZERO RNDOFF
C   TRACE - IF(1) TRACE SOLUTION, USING LPIN, LPOUT, AND ICK CODES
C           IF(0) SKIP ALL INTERMEDIATE PRINT OUT
C
      IP(1) = N
      IP(2) = 1000000
      IP(3) = NCF
      IP(4) = 1000000
      IP(8) = 0
      IP(11)= 0
```

```
C      RP(5) IS DEFINED LATER IN THIS SUBR.
C
C
      CALL PRESET
C
C
C...12 MAY 70  DEFINE SUM OF E(K) FOR LINEAR PHI(X) AND STORE IT IN RP(5)
      NDX=NCF+1
      SEKL=0.
      DO 28 I=NDX,N
      X(I) = 0.0
      CALL GETPHI(I,X,XZ,EKL)
      SEKL=SEKL + EKL
   28 CONTINUE
      RP(5)=SEKL
C
C
C...DEFINE D1( ), THE FULKERSON LOWER BOUND ADJUSTMENTS
      DO 281 I=1,N
      D1(I)=0.0
  281 CONTINUE
C...ADJUST LOWER BOUNDS OF VARIABLES WITH CONCAVE AND LINEAR FUNCTIONS.
      NX1=NCF+NLIN
      DO 282 I=1,NX1
      CALL MAPIS(IS,I)
      CALL SETD(X,IS)
      D1(I)=X(1)
  282 CONTINUE
C...ADJUST LOWER BOUNDS OF 1ST VAR OF PSEUDO VAR PAIRS WITH CONVEX FCNS.
      NX1=NX1+1
      DO 283 I=NX1,N,2
      CALL MAPIS(IS,I)
      CALL SETD(X,IS)
      D1(I)=X(1)
  283 CONTINUE
C
C...
      DMP=12.
      PRINT 29, DMP, (D1(I),I=1,N)
C
C
      CALL SET(RP(2))
C
C
      WRITE(6,30)(IP(I),I=1,12)
   30 FORMAT(1H1,20HINTEGER PARAMETERS =,12I6)
      WRITE(6,40)(RP(I),I=1,6)
   40 FORMAT(1H-,17HREAL PARAMETERS =,6F18.8)
C
      WRITE(6,90)
   90 FORMAT(1H-,50H X(J) LOWER-UPPER BOUNDS AND NO. CONCAVE INTERVALS)
      DO 100 J = 1,N
      WRITE(6,95)J,BLO(J),ULO(J),NOCIT(J)
   95 FORMAT(1H0,2X,I3,3X,2F12.4,6X,I3)
  100 CONTINUE
      NOBOL = 0
      IF(IP(6).EQ.1) PRINT 900
      RETURN
      END
      SUBROUTINE GETPHI(KFX,XPHI,PHI,SUMPHI)
C......
      COMMON/CV1/IP(12),RP(12)
      COMMON/CV2/BLO(100),ULO(100),NOCIT(100),CO(100)
      COMMON/CV3/N,NCF,PHIT,U7,US,USP,USM
      COMMON/CV11/SUPC(100,2),UNTC(100,2),XBIJ(50,3)
C......
C
C  IF (KFX)  1. .GT.  0. EVALUATE  KFX(TH) F(X).
C            2. .LT.  0. EVALUATE  FX(1) TO FX(IFX). (IFX = -KFX).
C            3. .EQ.  0. INVALID KFX **** UEP.
C
      DIMENSION XPHI(100), PHI(100)
      IF(IP(6) .EQ. 1) WRITE(6,999)
  999 FORMAT(1H-,124X,6HGETPHI)
      THETA = RP(3)
      SUMPHI = 0.0
      IF(KFX)200,770,100
  100 NDX1 = KFX
      NDX2 = KFX
      PHI(KFX) = 0.0
      GO TO 211
C
C -- EVALUATE PHI(XI) IN THE J(TH) XI-INTERVAL
C
  200 IFX =(-1) * KFX
      DO 210 I = 1,N
      PHI(I) = 0.0
  210 CONTINUE
      NDX1 =1
      NDX2 =IFX
  211 CONTINUE
      IF(IP(6).NE.1)GO TO  212
      WRITE(6,778)
      IF( KFX .GT. 0) IFX = KFX
      WRITE(6,780) (PHI(I),I=1,IFX), SUMPHI
      WRITE(6,780) (XPHI(I),I=1,IFX)
  212 CONTINUE
      DO 2200 I =NDX1,NDX2
      NOCI = NOCIT(I)
      IF(NOCI .LE. 0) GO TO 2150
      IF(XPHI(I) .GT. XBIJ(I,NOCI)) CALL RNDOFF(I,3)
      IF(XPHI(I) .LT. - RP(3)) GO TO 3010
      DO 2120 K=1,NOCI
      IF( XPHI(I) .GT. XBIJ(I,K) ) GO TO 2120
      J = K
      GO TO 2140
```

```
 2120   CONTINUE
 2140   CONTINUE
        PHI(I) = SUPC(I,J) + UNTC(I,J) * XPHI(I)
        GO TO 2170
 2150   CONTINUE
C
C -- USER EVALUATES LINEAR PHI - EQS USING INTERNALLY SET CONSTANTS
C      OR DATA INPUT FROM USER READIN.
         PHI(I)=SUPC(I,1) + UNTC(I,1)*XPHI(I)
C

        GO TO 2170
 2170   SUMPHI = SUMPHI +  PHI(I)
 2200   CONTINUE
        GO TO 777
  770   WRITE(6,771)
  771   FORMAT(1H1,26HKFX = 0 IN GETPHI **** UEP)
        CALL EXIT
C
  777   CONTINUE
        IF(IP(6) .EQ. 0) GO TO 888
        WRITE(6,778)
  778   FORMAT(1H0,21HPHI AND SUMPHI / XPHI)
        IF( KFX .GT. 0) IFX = KFX
        WRITE(6,780) (PHI(I),I=1,IFX), SUMPHI
        WRITE(6,780) (XPHI(I),I=1,IFX)
  780   FORMAT(1H0, 1P6F18.8)
  888   RETURN
C
 3010   WRITE(6,3011)
 3011   FORMAT(1H-,37H X(I) .LT. LW-BND IN GETPHI ***** UEP)
 7777   CALL EXIT
        CALL EXIT
C
        END
```

```
      SUBROUTINE BBCAV
C
C  LABELLED  COMMON
C.......
      COMMON/CV1/IP(12),RP(12)
      COMMON/CV2/BLO(100),ULO(100),NOCIT(100),CO(100)
      COMMON/CV21/FKO
      COMMON/CV3/N,NCF,PHIT,UZ,US,USP,USM
      COMMON/CV4/X(100),COST
      COMMON/CV5/SIGMA(100,4),TSIG
      COMMON/CV6/XZ(100)
      COMMON/CV7/NF1,NOP,NOPS
      COMMON/CV8/NXBK,XK,NOBOL
      COMMON/CV9/PSIGL( 50),NXBL(50),XNXBL(50),BLIST(50,100)
      COMMON/CV10/FKBL(50)
      COMMON/TSW/NSWW
C...N(MAX)=100
C...REMOVE ALL REFS TO CONSTRAINT MATRIX A( , )
C...MAX, NO OF ROWS IN SIGMA = N
      COMMON/FLK/
     A        A(2,100), C(2,100), TAU(100), ABAR(2,100), DUR(100),
     1   LBLS(100,3),    F(2,100), ISTATE(100), IVEC(100),
     2    JVEC(100),M,NB,TM,IFLAG,TO(100),TD(100), ITER,REPS(100),
     3        RLMDA(100), PLMDA(100),SEN(100),ISEN(100),SUMK
      COMMON/FV3/RLM(100), PLM(100), SN(100), ISN(100)
C
C.......
      INTEGER UB,CT,BV
      DIMENSION TMP(100),TSTO(150)
      DIMENSION BLT(100),ULT(100),CT(100)
      DIMENSION STR(2,150),CST(2),IBR(2),XTMP(2),ET(2)
C
C
      IF(IP(6).EQ.1) PRINT 900
  900 FORMAT(1H0,20HSUBROUTINE BBCAV    /10F10.2)
    7 CONTINUE
      CI = 4
      UB = 3
      LB = 2
      BV = 1
      MNC =(-1)* NCF
      MNX = (-1)* N
      EPSI = RP(1)
      NOPS = 1
      NCF4 = NCF * 4
C
      CALL BOX1
C
C  SOLVE 1ST LP-PROBLEM
C
   55 CONTINUE
      BBK = BLO(NXBK)
      UBK = ULO(NXBK)
      US = USP
      IF( UZ .LT. 0.0) US = USM
C
C  CLEAR TMP, TSTO, BLT, ULT, CT, STR, CST, IBR, AND XTMP
```

```
C
      DO 10 I = 1,N
      TMP(I) = 0.0
      BLT(I) = 0.0
      ULT(I) = 0.0
      CT(I) = 0.0
   10 CONTINUE
      DO 20 I = 1,2
      CST(I) = 0.0
      IBR(I) = 0
      XTMP(I) = 0.0
      DO 19 J = 1,NCF4
      STR(I,J) = 0.0
   19 CONTINUE
   20 CONTINUE
      DO 30 I = 1,NCF4
   30 TSTO(I) = 0.0
C
C  REDEFINE SIGMA FOR 1ST-LP-PR FROM KO-DATA
C
      DO 553 I = 1,N
      SIGMA(I,LB) = BLO(I)
      SIGMA(I,UB) = ULO(I)
      SIGMA(I,CI)= CO(I)
  553 CONTINUE
      XKSTO = XK
      XK = XK - BBK
      SIGMA(NXBK,UB)= XK
      TSIG =EKO
C
      IPB1 = 0
      IPB2 = 0
      BLT(NXBK)= SIGMA(NXBK,LB)
      ULT(NXBK)=XK + BLT(NXBK)
      CALL GETC (NXBK,BLT,ULT,CT)
      SIGMA(NXBK,CI) = CT(NXBK)
      NOP = NOP + 1
      CALL GETLPT
      COST1 = COST
      IF(NF1 .EQ. 1)GO TO 57
      IF(NPHASE .NE. 5)GO TO 56
      WRITE(6,551)
  551  FORMAT(1H-15HNF-STP1 *** NSP)
      GO TO 90
   56 IF(IOPT .NE. 1)GO TO 26
      WRITE(6,561)
  561 FORMAT(1H-18HUNBND-SOL1 *** NSP)
      GO TO 90
   57 CONTINUE
      NOPS = NOPS + 1
      DO 571 I = 1,N
      X(I) = X(I) + BLO(I)
  571 CONTINUE
      MNX = (-N)
      CALL GETPHI(MNX,X,TMP,PHIT)
C
      WRITE(6,573) PHIT
  573 FORMAT(1H0,11HPHI(XADJ) =,1PE18.7)
```

```
      WRITE(6, 575) ( X(I), I=1,N )
  575 FORMAT( 1H0,6HXADJ =,3X,1P6E18.7 / ( 10X, 1P6E18.7 ) )
C
      IF(PHIT .GE. US)GO TO 70
C
C  PHIT .LT. UZ FOR 1ST-PROBLEM
C
      UZ = PHIT
      DO 58 I = 1,N
      RLM(I)=RLMDA(I)
      PLM(I)=PLMDA(I)
      SN(I)=SFN(I)
      TSN(I)=TSFN(I)
   58 XZ(I)    = X(I)
      USP = (UZ/(1.0 + EPSI))
      USM = (UZ/(1.0 - EPSI))
      US = USP
      IF(UZ .LT. 0.0)US = USM
   70 CONTINUE
      IF(COST1 .GE. US)GO TO 90
      IPB1 = 1
      DO 701 I = 1,N
C    10  SEP  68
C...AT THIS POINT X( ) IS THE ACTUAL VALUE OF X( )
C
      TMP(I)=X(I) - BLO(I)
  701 CONTINUE
      CALL NXBRN( TMP, SIGMA, NXB)
C
C  PUT NODE INTO TEMPORARY STORAGE VECTOR-TSTO, NXB IN NXBT,
C  X(NXB) IN XNXBT, AND TSIG IN ENXBT
C
      DO 80 IT = 2,4
      K1 = (IT-2)*NCF + NCF
      DO 80 I = 1,NCF
      K1 = K1 + 1
      TSTO( K1 ) = SIGMA(I,IT)
   80 CONTINUE
      NXBT  = NXB
      XNXBT = X(NXB)
      ENXBT = TSIG
C
C  PREPARE TO SOLVE 2ND PROBLEM
C  DEFINE UBK, BBK AND NORA
C  STORE ACTUAL LOWER BOUND
C  REDEFINE SIGMA FOR LP - TBL
C
   90 CONTINUE
      BBK = BLO(NXBK)
      UBK = ULO(NXBK)
      DO 91 I = 1,N
      BLT(I) = 0.0
      ULT(I) = 0.0
      CT(I) = 0.0
      TMP(I) = 0.0
   91 CONTINUE
C
C  REDEFINE SIGMA FOR 2ND-LP-PB FROM KO-DATA
```

```
C
      DO 96 I = 1,N
      SIGMA(I,LB) = BLO(I)
      SIGMA(I,UB) = ULO(I)
      SIGMA(I,CT) = CO(I)
   96 CONTINUE
      BBK2 = BBK + XK
      UBK2 = UBK-XK
      SIGMA(NXBK,UB) = UBK2
      SIGMA(NXBK,LB) = BBK2
      BLT(NXBK) = BBK
      CALL GETPHI(NXBK,BLT,TMP,DMY)
      PH1 = TMP(NXBK)
      BLT(NXBK) = BBK2
      CALL GETPHI(NXBK,BLT,TMP,DMY)
      PH2 = TMP(NXBK)
      IP2 = 0
      TSIG = FKO  - PH1 + PH2
      BLT(NXBK) = BBK2
      ULT(NXBK) = BBK2 + UBK2
      CALL GETC (NXBK,BLT,ULT,CT)
      SIGMA(NXBK,CT) = CT(NXBK)
      NOP = NOP + 1
      CALL GETLPT
      COST2 = COST
      IF(NFI .EQ. 1)GO TO 104
      IF(NPHASE .NE. 5)GO TO 98
      WRITE(6,97)
   97 FORMAT(1H-,15HNF-STP2 *** NSP)
      GO TO 130
   98 IF(IOPT .NE. 1)GO TO 26
      WRITE(6,99)
   99 FORMAT(1H-,18HUNBND-SOL2 *** NSP)
      GO TO 130
  104 CONTINUE
      NOPS = NOPS + 1
      DO 105 I = 1,N
      TMP(I) = X(I) + SIGMA(I,2)
  105 CONTINUE
C...AT THIS POINT TMP( ) IS THE ACTUAL VALUE OF X( ).
      CALL GETPHI(MNX,TMP,ULT,PHIT)
C
      WRITE(6,573) PHIT
      WRITE(6, 575)(TMP(I), I=1,N )
C
      IF(PHIT .GE. US)GO TO 109
      UZ = PHIT
      DO 107 I = 1,N
      BLM(I)=BLMDA(I)
      BLM(I)=BLMDA(I)
      SN(I)=SFN(I)
      TSN(I)=TSFN(I)
  107 XZ(I)     = TMP(I)
      USP = ( UZ /(1.0 + EPSI))
      USM = ( UZ /(1.0 - EPSI))
      US = USP
      IF(UZ .LT. 0.0)US = USM
  109 CONTINUE
```

```
      IF(COST2 .GE. US)GO TO 130
C
C   10  SEP  68
      CALL NXBRN(    X, SIGMA, NXB)
      IPB2 = 1
  130 IPB = IPB1  + IPB2
C
C
      IF( IPB - 1 )180,150,160
  150 IF(IPB1 .EQ.  0)GO TO 155
      K = 1
C
C  1ST PROB. YIELDS A NODE FOR BLIST
C
  152 CST(K) = COST1
      IBR(K) = NXBT
      XTMP(K)=  XNXBT
      ET(K) = ENXBT
      DO 153 J = 1,NCF4
      STR(K,J) = TSTO(J)
  153 CONTINUE
      IF(IPB .EQ. 2)GO TO 166
      IT1 = 1
      IT2 = 1
      GO TO 170
  160 IF(COST1 .NE. COST2)GO TO 161
      K=1
      L=2
      GO TO 165
  161 IF(COST1 .GT. COST2)GO TO 162
      K=2
      L=1
      GO TO 165
  162 K = 1
      L = 2
  165 GO TO 152
  166 GO TO 156
  167 IT1 = 1
      IT2 = 2
      GO TO 170
  155 L = 2
  156 CST(L) = COST2
      IBR(L) = NXB
      XTMP(L) = X(NXB) + SIGMA(NXB,LB)
      ET(L) = TSIG
      DO 157 I=1,NCF
  157 STR(L,I) = SIGMA(I,1)
     DO 158 IT = 2,4
      K1= (IT-2) * NCF + NCF
      DO 158 I = 1,NCF
      K1 = K1 + 1
      STR(L,K1)=SIGMA(I,IT)
  158 CONTINUE
      IF(IPB .EQ. 2)GO TO  167
      IT1 = 2
      IT2 = 2
      GO TO 170
C
```

```
C  MOVE ONE AND/OR TWO BRANCHES ONTO TREE FROM TEMP_STORAGE
C
 170   CONTINUE
       DO 178 IK = IT1,IT2
       NOBOL  = NOBOL  + 1
       PSIGL (NOBOL) = CST(IK)
       NXBL(NOBOL) = IBR(IK)
       XNXBL (NOBOL) = XTMP(IK)
       EKBL(NOBOL) = ET(IK)
       DO 168 J =  1,NCF4
 168   BLIST(NOBOL,J) = STR(IK,J)
 178   CONTINUE
 180   LBB = NOBOL
       CALL PSHDWN
       IF(NOBOL.EQ.0)GO TO 26
       IF(PSIGL (LBB ).GE.US)GO TO 180
       GO TO 55
  26   CONTINUE
       IF(IP(6).EQ.1) PRINT 900
       RETURN
       END
```

```
      SUBROUTINE BOX1
C
C  LABELLED  COMMON
C......
      COMMON/CV1/IP(12),RP(12)
      COMMON/CV2/BLO(100),ULO(100),NOCTT(100),CO(100)
      COMMON/CV21/FKO
      COMMON/CV3/N,NCF,PHIT,U7,US,USP,USM
      COMMON/CV4/X(100),COST
      COMMON/CV5/SIGMA(100,4),TSIG
      COMMON/CV6/X7(100)
      COMMON/CV7/NF1,NOP,NOPS
      COMMON/CV8/NXBK,XK,NOBOL
      COMMON/CV9/PSIGL( 50),NXBL(50),XNXBL(50),BLIST(50,100)
      COMMON/TSW/NSWW
      COMMON/FLK/
     A        A(2,100), C(2,100), TAU(100), ABAR(2,100), DUR(100),
     1   LBLS(100,3),     F(2,100), ISTATE(100), IVEC(100),
     2    JVEC(100),M,NB,TM,IFLAG,TO(100),TD(100), ITER,REPS(100),
     3        RLMDA(100), PLMDA(100),SEN(100),ISEN(100),SUMK
      COMMON/FV3/RLM(100), PLM(100), SN(100), ISN(100)
C...N(MAX)=100
C...REMOVE ALL REFS TO CONSTRAINT MATRIX A( , )
C...MAX. NO OF ROWS IN SIGMA = N
C
C......
C
C
      INTEGER UB,CI,BV
      DIMENSION TMP(100)
C
C  BOX NO. 1  (NOP = 1)
C
      IF(IP(6).EQ.1) PRINT 900
  900 FORMAT(1H0,20HSUBROUTINE BOX1    /10F10.2)
       CI = 4
      UB = 3
      LB = 2
      BV = 1
      MNC =(-1)* NCF
      MNX = (-1)* N
      CALL GETC (MNX,BLO,ULO,CO)
      CALL GETPHI(MNC,BLO,TMP,ESIG)
      FKO = ESIG
C  SET ISIGMA FOR 1ST LP PROB.
      DO 10 I = 1,N
      TMP(I) = 0.0
       SIGMA(I,LB)=BLO(I)
      SIGMA(I,UB)=ULO(I)
      SIGMA(I,CI)=CO(I)
   10 CONTINUE
      TSIG = FKO
C
      NOP = 1
      CALL GETLPT
      IF(NF1 .EQ. 1)GO TO 28
      IF(NPHASE .NE. 5)GO TO 24
      WRITE(6,22)
   22 FORMAT(1H1,29HNF-STP WHEN NOP = 1 ***** UFP)
      GO TO 7
   24 IF(IOPT .NE. 1)GO TO 26
      WRITE(6,25)
   25 FORMAT(1H1,29HUNBND-SOL WHEN NOP = 1 ** UFP)
      GO TO 7
   26 CALL EXIT
C
   29 CONTINUE
      DO 30 I=1,N
      BLM(I)=BLMDA(I)
      PLM(I)=PLMDA(I)
      SN(I)=SEN(I)
      ISN(I)=ISEN(I)
      XZ(I) = X(I)
   30 CONTINUE
      CALL GETPHI(MNX,X,TMP,UZ)
      EPSI = BP(1)
      USP= (UZ /(1.0 + EPSI))
      USM= (UZ /(1.0 - EPSI))
      EKO = TSIG
C
C   10  SEP  68
      CALL NXBRN(X,SIGMA,NXB)
      XK = X(NXB)
      XNXBL(1) = X(NXB)
      NXBK  = NXB
C
   50 CONTINUE
      IF(IP(6).EQ.1) PRINT 900
  777 RETURN
    7 STOP666
      END
```

```
FUNCTION FLKLB(S, D, I, B4)
COMMON/CV2/BLO(100),,ULO(100),NOCIT(100),CO(100)
FLKLB=D
IF( NOCIT(I).GT.0)FLKLB=FLKLB+B4*S
RETURN
END
```

```
      SUBROUTINE GETASQ(NOES,ELM,JSQ)
C...SHELL METHOD OF HALVING
C
C  GETASQ(NOES,ELM,JSQ)  SORTS ELM(J),J=1,NOES IN AN ASCENDING SEQUENCE
C  PRESET INITIAL POSITION CODE OF ELM(J)
C  JSQ(J) PRESET TO (-1) WHEN ELM(J) IS UNDEFINED (I.E. INFINITE)
C
      DIMENSION ELM(1), JSQ(1)
      L = 1
    7 L = 2 * L
      IF( L .LE. NOES) GO TO 7
      L = L - 1
   10 L = L / 2
      DO 20 K2 = 1, NOES
      K1 = K2
   15 K3 = K1 + L
      IF( K3 .GT. NOES) GO TO 30
      IF( ELM(K1).LE. ELM(K3) ) GO TO 20
      RT = ELM(K1)
      ELM(K1 ) = ELM(K3)
      ELM(K3) = RT
      RT = JSQ(K1)
      JSQ(K1) = JSQ(K3)
      JSQ(K3) = RT
      K1 = K1 - L
      IF( K1 .GE. 1) GO TO 15
   20 CONTINUE
   30 IF( L .GT. 1) GO TO 10
      RETURN
      END
```

END

```
      SUBROUTINE GETC (KCX,BLT,ULT,CT)
      COMMON/CV1/IP(12),RP(12)
      COMMON/CV3/N,NCF,PHIT,UZ,US,USP,USM
      DIMENSION BLT(01),ULT(01),CT(01),FX1(50),FX2(50)
C
C  IF (KCX) 1. .GT. 0, EVALUATE KCX(TH) C(X)-SLOPE.
C           2. .LT. 0, EVALUATE CX(1) TO CX(IFX), (IFX = -KFX).
C           3. .EQ. 0, INVALID KCX **** UEP.
C
      IF(IP(6) .EQ. 1) WRITE(6,999)
  999 FORMAT(1H-,124X,6HGETC  )
      IF(KCX .GT. N)  GO TO 770
      IF(KCX)200,770,100
  100 FX1(KCX) = 0.0
     FX2(KCX) = 0.0
      CALL GETPHI(KCX,BLT,FX1,DMY)
      CALL GETPHI(KCX,ULT,FX2,DMY)
      NDX1 = KCX
      NDX2 = KCX
      GO TO 220
  200 ICX =(-1) * KCX
      IF( ICX .GT. N) GO TO 770
      DO 210 I = 1,NCF
      FX1(I) = 0.0
      FX2(I) = 0.0
      CT(I)  = 0.0
  210 CONTINUE
      CALL GETPHI(KCX,BLT,FX1,DMY)
      CALL GETPHI(KCX,ULT,FX2,DMY)
      NDX1 = 1
      NDX2 = ICX
  220 DO 225 J = NDX1,NDX2
      DIF = ULT(J) - BLT(J)
      IF(DIF    .EQ. 0.0) GO TO 225
      CT(J) = (FX2(J) - FX1(J)) / DIF
  225 CONTINUE
      GO TO 777
  770 WRITE(6,771)KCX
  771 FORMAT(1H1,13HINVALID KCX =,I3,10H  **** UEP)
      CALL EXIT
C
  777 CONTINUE
      IF(IP(6) .EQ. 0) GO TO 888
      WRITE(6,778) (CT(I),I=1,N)
  778 FORMAT(1H0,15HCT(I) IN GETC =,6F12.4/(16X,6F12.4))
  888 RETURN
C
      END
      SUBROUTINE GETLPT
C...GEN INPUT TO FULKERSON ALGORITHM
      COMMON/CV1/IP(12),RP(12)
      COMMON/CV3/NV,NCF,PHIT,UZ,US,USP,USM
      COMMON/CV4/X(100),COST
      COMMON/CV5/SIGMA(100,4),TSIG
      COMMON/CV7/NF1,NOP,NOPS
      COMMON/C4/NINDX,NJNDX,CDATE,IJK(100,3),NLIN,NCVX
      COMMON/C5/IA(100),JA(100),BL(100),UL(100),Q(100)
      COMMON/C7/D1(100)
      COMMON/FLK/
     A       A(2,100), C(2,100), TAU(100), ABAR(2,100), DUR(100),
     1   LBLS(100,3),    F(2,100), ISTATE(100), IVEC(100),
     2    JVEC(100),M,N,TM ,IFLAG,TO(100),TD(100), ITER,REPS(100),
     3       RLMDA(100), PLMDA(100),SEN(100),ISEN(100),SUMK
C...N AND M ARE ASSUMED TO BE .LT. 100 EACH
C...N - NO. OF UNIQUE I-INDICES IN IA
C...M - NUMBER OF (I,J)
C...DUR( ) - X( ) --- DEPENDENT VARIABLE
C...A(1,J) - SIGMA(I,3) --- UPPER BNDS
C...A(2,J) - SIGMA(I,2) --- LOWER BNDS
C...IVEC( ) - IA( )  ALL-I
C...JVEC( ) - JA( ) ALL-J
C...ENT( ) - T( )
C...C(1,J) - SIGMA(I,4) --- SLOPES
      IF(IP(6).EQ.1) PRINT 900
  900 FORMAT(1H0,20HSUBROUTINE GETLPT   /10F10.2)
      M=IP(1)
       N = IA(NINDX+1)
       TM=CDATE
       B4=+1.
      DO 100 I=1,M
       A(1,I)=SIGMA(I,3) + D1(I)
       A(2,I)=FLKLB( SIGMA(I,2), D1(I), I, B4)
      C(1,I)=SIGMA(I,4)
       IF( SIGMA(I,4).LT.0 ) C(1,I) = -C(1,I)
      IVEC(I)=IA(I)
      JVEC(I)=JA(I)
  100 CONTINUE
       DO 110 I=1,NCF
       A(1,I) = SIGMA(I,2)  + A(1,I)
  110  CONTINUE
       SUMK=TSIG+RP(5)
C
       CALL FLKSON

       NF1=1
C...COMPUTE LOWER BOUND COST
       COST=0.0
       B4=-1.
       DO 200 I=1,M
       X(I)=FLKLB( SIGMA(I,2), (DUR(I)-D1(I)), I, B4)
       COST=COST+  X(I)*SIGMA(I,4)
  200  CONTINUE
C...ADJUST LOWER BOUND COST
C...12 MAY 76 ADD RP(5), THE SUM OF E(K) FOR LINEAR PHI(X)
C...RP(5) IS COMPUTED IN SUBR SETUP
```

```
      COST=COST+TSIG*RP(5)
      IF(IP(6).EQ.1) PRINT 900, COST, TSIG, RP(5)
      IF( IP(7).EQ.1) CALL TABOUT(2)
      RETURN
      END
```

```
      SUBROUTINE NXBRN(XT,SIGMAT,NXB)
      COMMON/CV1/IP(12),RP(12)
      COMMON/CV3/N,NCF,PHIT,UZ,US,USP,USM
     DIMENSION XT(100),BLT(50),CT(50),TEMP(50),SIGMAT(100,4)
     DIMENSION  FX1(50),FX2(50),DIF(50),NDX(50)
      IF(IP(6) .EQ. 1) WRITE(6,999)
  999 FORMAT(1H-,124X,6HNXBRN )
      NXB =  0
C  NXBRN GIVES BEST BRNCH-CANDIDATE FOR  XT(I)
      DO 5  J = 1,NCF
      TEMP(J) = 0.0
      FX2(J)= 0.0
      FX1(J)= 0.0
      DIF(J)= 0.0
      NDX(J)= 0
      BLT(J)=  SIGMAT(J,2)
      CT(J) =  SIGMAT(J,4)
    5 CONTINUE
      DO 20 J = 1,NCF
      TEMP(J) = XT(J) + BLT(J)
   20 CONTINUE
      NEX =(-1) * NCF
      CALL GETPHI(NEX,TEMP,FX2,DMY)
      CALL GETPHI(NEX,BLT,FX1,DMY)
      DO 30 J = 1,NCF
      DIF(J) = FX2(J) - FX1(J) - CT(J)*XT(J)
      NDX(J) = J
   30 CONTINUE
      CALL GETASQ(NCF,DIF,NDX)
      NXB = NDX(NCF)
      RETURN
      END
      SUBROUTINE PSHDWN
C
C LABELLED COMMON
C......
      COMMON/CV1/IP(12),RP(12)
      COMMON/CV2/BLO(100),ULO(100),NOCTT(100),CO(100)
      COMMON/CV3/N,NCF,PHIT,UZ,US,USP,USM
      COMMON/CV4/X(100),COST
      COMMON/CV5/SIGMA(100,4),TSIG
      COMMON/CV6/XZ(100)
      COMMON/CV7/NFI,NOP,NOPS
      COMMON/CV8/NXBK,XK,NOBOL
      COMMON/CV9/PSIGL( 50),NXBL(50),XNXBL(50),BLIST(50,100)
      COMMON/CV10/EKBL(50)
C......
C
      IF(IP(6) .EQ. 1) WRITE(6,999)
  999 FORMAT(1H-,124X,6HPSHDWN)
   10 IF(NOBOL .GT. 0)GO TO 20
      WRITE(6,11)
   11 FORMAT(1H-43HPUSH-DOWN LIST IS EMPTY **** END OF PROBLEM)
      RP(4) = + 1.0
      CALL TABOUT(3)
      RETURN
   20 NXBK = NXBL(NOBOL)
      EKO = EKBL(NOBOL)
      XK   = XNXBL(NOBOL)
      DO 40 I = 1,NCF
      K1 =NCF  + I
      K2 = K1 + NCF
      K3 =  K2 + NCF
      BLO(I) = BLIST(NOBOL,K1)
      ULO(I) = BLIST(NOBOL,K2)
      CO(I) = BLIST(NOBOL,K3)
   40 CONTINUE
      NOBOL = NOBOL - 1
      RETURN
      END
      SUBROUTINE TIMEC
       COMMON /TSW/ NSWW
       COMMON/TMX/TMO,EXT
C
C  SECOND GIVES JOB CPU EXECUTION TIME IN 1/1000 OF A SECOND
C
       CALL SECOND(SECS)
       X = SECS - TMO
      WRITE(6,666)  X
  666 FORMAT(12H0EXCT-TIME =,F9.3,8H SECONDS)
       IF(SECS .LT. EXT) GO TO 100
       NSWW = 2
       STOP0666
  100  RETURN
       END
      SUBROUTINE TABOUT(IRT)
C
C  LABELLED COMMON
C......
      COMMON/CV1/IP(12),RP(12)
      COMMON/CV2/BLO(100),ULO(100),NOCTT(100),CO(100)
      COMMON/CV3/N,NCF,PHIT,UZ,US,USP,USM
      COMMON/CV4/X(100),COST
      COMMON/CV5/SIGMA(100,4),TSIG
      COMMON/CV6/XZ(100)
      COMMON/CV7/NF1,NOP,NOPS
      COMMON/CV8/NXBK,XK,NOBOL
      COMMON/CV9/PSIGL( 50),NXBL(50),XNXBL(50),BLIST(50,100)
      COMMON/CV10/EKBL(50)
      COMMON/CV11/SUPC(100,2),UNTC(100,2),XBIJ(50,3)
C
C  IF IRT  =  1, WRITE LP-INPUT.
C             2, WRITE LP-OUTPUT = F(ZETAJ)
C             3, WRITE GEN. TAB-OUTPUT
C
      DIMENSION PRNNT(4)
      DATA BLANK /1H /
C
      IF(NOP .GT. 1) GO TO 90
      GO TO 100
 90   CONTINUE
      IF(RP(4) .EQ. 0.0) GO TO 777
 100  CONTINUE
      WRITE(6,101)
 101  FORMAT(1H1,30HTABOUT - GENERAL - INFORMATION)
      CALL TIMEC
      WRITE(6,103)NOP,NF1,NPHASE,NOPS
 103  FORMAT(1H0,5HNOP =,I3,6X,5HNF1 =,I3,6X,8HNPHASE =,I3,
     1 6X,6HNOPS =,I3)
      WRITE(6,105)UZ,USP,USM
 105  FORMAT(1H0,6HU 0  =,1PE18.7,6X,5HUSP =,1PE18.7,6X,5HUSM =,
     1 1PE18.7)
      WRITE(6,107)(XZ(I),I=1,N)
 107  FORMAT(1H0,6HX 0  =,6F20.7/(7X,6F20.7))
      WRITE(6,109)
 109  FORMAT(1H0,10HSIGMA(I,J),13X,5HRHS-B,12X,6HLW-BND,12X,6HUP-BND,
     111X,7HC-SLOPE)
C
      DO 115 I=1,N
      WRITE(6,113) I, (SIGMA(I,J),J=1,4)
 113  FORMAT(1H ,5X,I2,3X,4F18.5)
 115  CONTINUE
C
      WRITE(6,117)TSIG
 117  FORMAT(1H0,6HE(K) =,F18.6)
      NDX = NOBOL + 1
      WRITE(6,119)NOBOL,NXBK,XNXBL(NDX)
 119  FORMAT(1H0,7HNOBOL =,I2,6X,6HNXBK =,I2,6X,7HXBRN  =,F18.6)
C
C
      IT = NOBOL + 1
      DO 131 I=1,IT
      WRITE(6,121)I,PSIGL(I),NXBL(I),XNXBL(I),EKBL(I)
  121 FORMAT(1H-,6HNODE =,I4,6X,6HCOST =,F20.6,6X,9HNX-BRN =,I4,6X,
     1 7HX-BRN =,F20.6,6X,6HE(K) =,F20.6)
      WRITE(6,123)(BLIST(I,J),J = 1,NCF)
  123 FORMAT(1H0,17X,9HRHS-BV  =,5F20.6/(27X,5F20.6))
      J1 = NCF  + 1
      L1 = NCF  + NCF
      WRITE(6,125)(BLIST(I,J),J = J1,L1)
  125 FORMAT(1H0,17X,9HLW-BND  =,5F20.6/(27X,5F20.6))
      J1 = J1 + NCF
      L1 = L1 + NCF
      WRITE(6,127)(BLIST(I,J),J= J1,L1)
  127 FORMAT(1H0,17X,9HUP-BND  =,5F20.6/(27X,5F20.6))
      J1 = J1 + NCF
      L1 = L1 + NCF
      WRITE(6,129)(BLIST(I,J),J= J1,L1)
  129 FORMAT(1H0,17X,9HCSLOPE  =,5F20.6/(27X,5F20.6))
  131  CONTINUE
C
C
      IF(IRT.NE. 1)GO TO 145
C
C
      GO TO 777
C
C
  145 IF(IRT.NE. 2)GO TO 150
C
C
      WRITE(6,147)
  147 FORMAT(1H0,9HLP-OUTPUT)
      WRITE(6,149)COST,(X(I),I =1,N)
  149 FORMAT(1H0,6HCOST =,1PE18.7//9H X  (I) =,0P6F20.7/(9X,6F20.7))
      GO TO 777
C
C
  150 IF(IRT .NE. 3)GO TO 155
      GO TO 777
  155 CONTINUE
      WRITE(6,757)
  757 FORMAT(1H-,35HTABOUT-IRT .NE. 1,2,OR 3 *****  UEP)
      CALL EXIT
  777 RETURN
      END
      SUBROUTINE FLKSON
C       THIS IS THE MAIN SUBROUTINE FOR THE FULKERSON CODE.
C
        COMMON/FLK/
     A        A(2,100), C(2,100), TAU(100), ABAR(2,100), DUR(100),
     1    LABLS(100,3),     F(2,100), ISTATE(100), IVEC(100),
     2     JVEC(100),M,N,TM ,IFLAG,TO(100),TD(100), ITER,REPS(100),
     3        BLMDA(100), PLMDA(100),SEN(100),ISEN(100),SUMK
C
C
C...DEBUG.............................................................
        COMMON/CV1/IP(12),RP(12)
  900   FORMAT(1H0,20HSUBROUTINE FLKSON   /(10F10.2))
  910   FORMAT(1H0,20HSUBROUTINE FLKSON   /(10I10))
        IF(IP(6).EQ.1) PRINT 910, M, N
        IF(IP(6).EQ.1) PRINT 900, TM, SUMK
        IF(IP(6).EQ.1) PRINT 900, (A(2,I),I=1,M)
        IF(IP(6).EQ.1) PRINT 900, (A(1,I),I=1,M)
        IF(IP(6).EQ.1) PRINT 900, (C(1,I),I=1,M)
        IF(IP(6).EQ.1) PRINT 910, (IVEC(I),I=1,M)
        IF(IP(6).EQ.1) PRINT 910, (JVEC(I),I=1,M)
C...DEBUG.............................................................
C...INITIALIZE C(2, ) AND F( , )
        DO 914 I=1,M
        C(2,I)=1000000.
  914   CONTINUE
        DO 915 I=1,2
        DO 915 J=1,M
        F(I,J)=0.
  915   CONTINUE
C         SET UP THE TAU VALUES WITH A SUBROUTINE.
          CALL TAUER
C        PRINT THE TAU VALUES.
       WRITE(6,110)(TAU(I),I=1,N)
 110      FORMAT(1H ,*THE TAU VALUES ARE...*,10F10.2/(1H ,22X,10F10.2)//)
       ITER = 0
       DO 68 IV=1,M
       I=IVEC(IV)
       J=JVEC(IV)
       D=TAU(J) -TAU(I)
 68      DUR(IV)=AMIN1(D,A(1,IV))
       CALL BKPT
       IF(TAU(N).LE.TM) GO TO 35
       CALL INTERP
C         SET UP THE ABAR MATRIX.
       CALL ABARER
 111      FORMAT(1H ,*ROW 1 OF ABAR IS...*,10F10.2/
C
      1(1H ,20X,10F10.2)//)
 112    FORMAT(1H ,*ROW 2 OF ABAR IS...*,10F10.2/(1H ,
      1 20X,10F10.2)//)
C        SET UP THE LABELS MATRIX, AND THE STATE VECTOR.
 9       CALL LBLER
 202    FORMAT(1H ,*LABELS ARE...*,3I15)
 401     FORMAT(1H ,*REPS VECTOR IS...*,6F15.2)
  203       FORMAT(1H , *STATES ARE...*,12I5)
C        GO INTO THE FIRST LABELING.
      INODE = 1
      CALL FLBLER(INODE)
      ISTATE(INODE) = 3
C       DETERMINE WHETHER TO TERMINATE, GO BACK TO FLBLER, OR
C       GO TO SLBLER.
 204        FORMAT(1H ,*STATES ARE NOW...*,12I5)
 4      IF(ISTATE(N) .EQ. 2)GO TO 10
      IS=1
 1        IF(ISTATE(IS) .EQ.2) GO TO 2
      IS=IS +1
      IF(IS .EQ.(N+1)) GO TO  3
      GO TO 1
  2       INODE = IS
      CALL FLBLER(INODE)
      GO TO 4
  3       DO 51 I3 = 1,N
      IF(ISTATE(I3) .EQ. 3) ISTATE(I3) = 2
  51        CONTINUE
C       PICK AN INODE FOR THE SECOND LABELING, IF SUCH A NODE EXISTS.
 52       IS = 1
  6        IF(ISTATE(N) .EQ.2) GO TO 8
  16       IF(ISTATE(IS) .EQ. 2) GO TO 5
      IF(IS .EQ. N) GO TO 7
      IS = IS + 1
      GO TO 16
C       DO A SECOND LABELING.
   5      INODE = IS
      CALL SLBLER(INODE)
 301        FORMAT(1H ,*SECOND LABELS ARE...*,3I15)
 451      FORMAT(1H ,*SECOND REPS ARE...*,6F15.2)
 302         FORMAT(1H ,*SECOND STATES ARE...*,12I5)
      GO TO 52
C       WHEN CONTROL GOES TO 7 BELOW, YOU DO A NODE NUMBER CHANGE.
  7     CALL NONOCH
      CALL RMPT
      CALL INTERP
      IF(IFLAG.EQ.1)GO TO 39
      GO TO 9
C       WHEN CONTROL GOES TO 8 BELOW, YOU DO A FLOWCHANGE.
  8      CALL FLCH
      GO TO 9
C       WHEN CONTROL COMES TO 34 BELOW, YOU WANT TO GET OUT OF
C          ELKSON AND GO ON TO SOMETHING ELSE.
 10        WRITE(6,33)
 33        FORMAT(1H ,*YOU HAVE TERMINATED WITHOUT INTERPOLATING.*)
      GO TO 39
 35        WRITE(6,38)
 38        FORMAT(1H ,*TM IS GREATER THAN MAX TAU(N).*)
  39      CONTINUE
      CALL MAXCAL
  34      RETURN
      END
      SUBROUTINE SLBLER(INODE)
C      THIS PERFORMS THE SECOND LABELING OPERATION,
C      STARTING FROM A GIVEN INITIAL NODE.
C
       COMMON/BLK/
     A        A(2,100), C(2,100), TAU(100), ABAR(2,100), DUR(100),
     1   LBLS(100,3),     F(2,100), ISTATE(100), IVEC(100),
     2    JVEC(100),M,N,TM ,IFLAG,TO(100),TD(100), ITER,REPS(100),
     3        BLMDA(100), PLMDA(100),SEN(100),ISEN(100),SUMK
C
C      BEGIN SEARCHING FOR CONDITION 3.7.
      DO 1 IV =1, M
      IF(IVEC(IV) .NE. INODE) GO TO 1
      I = IVEC(IV)
      J = JVEC(IV)
      IF(ISTATE(J) .NE. 1) GO TO 1
      DO 1 K=1,2
      AB = ABAR(K,IV)
      IF(ABS(AB) .GT. 1.E-5) GO TO 1
      FF = F(K,IV)
      DIF = C(K,IV) - FF
      IF(DIF .LT. 1.E-6) GO TO 1
      REPSI = REPS(I)
      REPSJ=AMIN1(REPSI,DIF)
      LBLS(J,1) = I
      LBLS(J,2) = K
      LBLS(J,3) = 1
      REPS(J)=REPSJ
      ISTATE(J) = 2
    1  CONTINUE
C      BEGIN SEARCHING FOR CONDITION 3.8.
C
      DO 2 JV = 1,M
      IF(JVEC(JV) .NE. INODE) GO TO 2
      J = IVEC(JV)
      I = JVEC(JV)
      IF(ISTATE(J) .NE.1) GO TO 2
      DO 2 K = 1,2
      AB = ABAR(K,JV)
      IF(ABS(AB) .GT. 1.E-5) GO TO 2
      IF(F(K,JV) .LE. 1.E-5) GO TO 2
      RLBLS=REPS(I)
      REPSJ=AMIN1(RLBLS,F(K,JV))
      LBLS(J,1) = I
      LBLS(J,2) = K
      LBLS(J,3) = -1
      REPS(J) = REPSJ
      ISTATE(J) = 2
    2  CONTINUE
      ISTATE(INODE) = 3
      RETURN
      END
      SUBROUTINE NONOCH
C        THIS PERFORMS THE NODE NUMBER CHANGE AND PRINTS
C           START TIMES AND DURATION TIMES.
C
       COMMON/FLK/
     A         A(2,100), C(2,100), TAU(100), ABAR(2,100), DUR(100),
     1    LBLS(100,3),     F(2,100), ISTATE(100), IVEC(100),
     2     JVEC(100),M,N,TM ,IFLAG,TO(100),TD(100), ITER,REPS(100),
     3         RLMDA(100), PLMDA(100),SEN(100),ISEN(100),SUMK
C
      DEL1 = 1.E5
      DEL2 = 1.E5
C       BEGIN COMPUTING DEL1 AND DEL2.
      DO 1 ISCN = 1,M
      I = IVEC(ISCN)
      J = JVEC(ISCN)
      IDUM = ISTATE(I)
      GO TO (2,3,3),IDUM
    2   IF(ISTATE(J) .EQ.1) GO TO 1
      DO 4 K=1,2
      AB = ABAR(K,ISCN)
      IF(AB .LT. 1.E-7) GO TO 4
      DEL2 = AMIN1(DEL2,  AB)
    4   CONTINUE
    3   IF(ISTATE(J) .NE.1) GO TO 1
      DO 46 K = 1,2
      AB =ABAR(K,ISCN)
      IF(AB .GT. -1.E-7) GO TO 46
      DEL1 = AMIN1(DEL1,-AB)
   46     CONTINUE
    1   CONTINUE
   47     FORMAT(1H ,*DEL1 IS...*,F20.2)
   48     FORMAT(1H ,*DEL2 IS...*,F20.2)
C       COMPUTE DEL.
C
      DEL = AMIN1(DEL1, DEL2)
   49      FORMAT(1H ,*DEL IS...*,F20.2)
C       RECOMPUTE THE TAU VALUES.
      DO 6 ISCN = 1,N
      IF(ISTATE(ISCN) .NE.1) GO TO 6
      TAU(ISCN) = TAU(ISCN) - DEL
    6   CONTINUE
C       RECALCULATE THE ABAR MATRIX.
C
      CALL ABARER
      WRITE(6,7) (TAU(I), I = 1,N)
    7   FORMAT(1H , *TAU VALUES ARE...*,9F10.2/(1H ,17X,9F10.2)//)
C       CALCULATE JOB DURATION TIMES.
      DO 8 ISCN = 1,M
      I = IVEC (ISCN)
      J = JVEC(ISCN)
      D = TAU(J) - TAU(I)
    8   DUR(ISCN) = AMIN1(D,A(1,ISCN))
C       PRINT DURATION TIMES.
      WRITE(6,9)(DUR(I), I = 1,M)
    9   FORMAT(1H ,*DURATION TIMES WERE...*,9F10.2/(1H ,22X,9F10.2)//)
      RETURN
      END
      SUBROUTINE ABARER
C      THIS SUBROUTINE CALCULATES THE ABAR MATRIX.
C
       COMMON/FLK/
     A          A(2,100), C(2,100), TAU(100), ABAR(2,100), DUR(100),
     1    LBLS(100,3),    F(2,100), ISTATE(100), IVEC(100),
     2      JVEC(100),M,N,TM ,IFLAG,TO(100),TD(100), ITER,REPS(100),
     3          RLMDA(100), PLMDA(100),SEN(100),ISEN(100),SUMK
C
      DO 1 IV = 1,M
      I = IVEC(IV)
      J= JVEC(IV)
      ABAR(1,IV) = A(1,IV) + TAU(I) - TAU(J)
    1  ABAR(2,IV) = A(2,IV) + TAU(I) - TAU(J)
      RETURN
      END
```

```
      SUBROUTINE TAUER
C      THIS SUBROUTINE CALCULATES THE INITIAL TAU VALUES.
C
       COMMON/FLK/
     A          A(2,100), C(2,100), TAU(100), ABAR(2,100), DUR(100),
     1    LBLS(100,3),    F(2,100), ISTATE(100), IVEC(100),
     2      JVEC(100),M,N,TM ,IFLAG,TO(100),TD(100), ITER,REPS(100),
     3          RLMDA(100), PLMDA(100),SEN(100),ISEN(100),SUMK
C
      TAU(1) = 0.
      DO 1 J=2, N
      TARG = 0.
      DO 3 IV =1, M
      IF(JVEC(IV) .EQ. J) GO TO 2
      GO TO 3
    2  I = IVEC(IV)
      TARG = AMAX1(TARG,(TAU(I) + A(1,IV)))
    3  CONTINUE
      TAU(J) = TARG
    1  CONTINUE
      RETURN
      END
      SUBROUTINE LBLER
C       THIS SUBROUTINE SETS UP THE LABELS MATRIX AND THE STATE
C          VECTOR.
C
       COMMON/FLK/
     A          A(2,100), C(2,100), TAU(100), ABAR(2,100), DUR(100),
     1    LBLS(100,3),     F(2,100), ISTATE(100), IVEC(100),
     2     JVEC(100),M,N,TM ,IFLAG,TO(100),TD(100), ITER,REPS(100),
     3         BLMDA(100), PLMDA(100),SEN(100),ISEN(100),SUMK
C
      DO 2 J=1,3
      DO 1 I= 1,N
    1   LBLS(I,J) = -1
    2   CONTINUE
      DO 40 I =1,N
   40   REPS(I)=-1.
      REPS(1) = 1000000.
      DO 3 I = 1,N
    3  ISTATE(I) = 1
      ISTATE(1) = 2
      RETURN
      END
```

```
      SUBROUTINE BKPT
C          THIS TABULATES LAMBDA AND PLAMBDA.
C
       COMMON/FLK/
     A          A(2,100), C(2,100), TAU(100), ABAR(2,100), DUR(100),
     1    LBLS(100,3),     F(2,100), ISTATE(100), IVEC(100),
     2     JVEC(100),M,N,TM ,IFLAG,TO(100),TD(100), ITER,REPS(100),
     3         BLMDA(100), PLMDA(100),SEN(100),ISEN(100),SUMK
C
      ITER =ITER +1
      BLMDA(ITER) = TAU(N)
      SUM = 0.
      DO 25 IV = 1,M
   25     SUM = SUM +C(1,IV)*DUR(IV)
      PLMDA(ITER) = SUMK -SUM
      WRITE(6,26) (BLMDA(I),I = 1,ITER)
   26    FORMAT(1H ,*BLMDAS ARE...*,15F6.1)
      WRITE(6,27) (PLMDA(I),I = 1,ITER)
   27    FORMAT(1H , *PLMDAS ARE...*,15F6.2)
      RETURN
      END
      SUBROUTINE FLBLER(INODE)
C     THIS PERFORMS THE FIRST LABELING.
C
       COMMON/FLK/
     A         A(2,100), C(2,100), TAU(100), ABAR(2,100), DUR(100),
     1    LBLS(100,3),     F(2,100), ISTATE(100), IVEC(100),
     2      JVEC(100),M,N,TM ,IFLAG,TO(100),TD(100), ITER,REPS(100),
     3         RLMDA(100), PLMDA(100),SEN(100),ISEN(100),SUMK
C
      DO 1 IV = 1,M
      I = IVEC (IV)
      IF(I .NE. INODE) GO TO 1
      J = JVEC(IV)
      IF((ISTATE(J)) .NE. 1) GO TO 1
      AB = ABAR(2,IV)
      IF((ABS(AB)) .GE. 1.E-5) GO TO 1
      LBLS(J,1) = I
      LBLS(J,2) = 2
      LBLS(J,3) = 1
      REPS(J)= 1000000.
      ISTATE(J) = 2
    1  CONTINUE
      ISTATE(INODE) = 3
      RETURN
      END
      SUBROUTINE INTERP
C        THIS PERFORMS THE INTERPOLATION, GIVEN A TIME TM.
C
       COMMON/FLK/
     A          A(2,100), C(2,100), TAU(100), ABAR(2,100), DUR(100),
     1    LBLS(100,3),    F(2,100), ISTATE(100), IVEC(100),
     2     JVEC(100),M,N,TM ,IFLAG,TO(100),TD(100), ITER,REPS(100),
     3          RLMDA(100), PLMDA(100),SEN(100),ISFN(100),SUMK
C
      IF(TAU(N) .LE.TM) GO TO 1
      DO 2 I =1,N
    2  TO(I) = TAU(I)
      IFLAG = 0
      GO TO 3
    1  RATIO = (TO(N) -TM)/(TO(N) -TAU(N))
      DO 4 I = 1,N
    4   TD(I) = TO(I) - RATIO*(TO(I) -TAU(I))
      DO 5 IV = 1,M
      I =IVEC(IV)
      J = JVEC(IV)
      AV = A(1,IV)
      DIF = TD(J) -TD(I)
    5  DUR(IV) = AMIN1(AV,DIF)
      WRITE(6,6)(TD(I),I = 1,N)
    6   FORMAT(1H ,*INTERPOLATED NODE TIMES...*,9F10.2)
      WRITE(6,7)(DUR(I),I=1,M)
    7   FORMAT(1H ,*INTERPOLATED DURATIONS...*,9F10.2)
      IFLAG = 1
    3    RETURN
      END
      SUBROUTINE FLCH
C        THIS PERFORMS THE FLOWCHANGE
C
       COMMON/FLK/
     A          A(2,100), C(2,100), TAU(100), ABAR(2,100), DUR(100),
     1    LBLS(100,3),    F(2,100), ISTATE(100), IVEC(100),
     2      JVEC(100),M,N,TM ,IFLAG,TO(100),TD(100), ITER,REPS(100),
     3         RLMDA(100), PLMDA(100),SEN(100),ISEN(100),SUMK
C
      REPSN=REPS(N)
      IPN = N
    6   I = LBLS(IPN,1)
      IF(LBLS(IPN,3)) 1,1,2
C       WHEN CONTROL COMES TO 2, YOU ADD REPSN TO
C          THE FLOW F(I,P,K).
    2   DO 3 ISCN = 1,M
      IF(IVEC(ISCN) .NE. I) GO TO 3
      IF(JVEC(ISCN) .NE. IPN) GO TO 3
      IPT = ISCN
      GO TO 4
    3   CONTINUE
    4   K=LBLS(IPN,2)
      F(K,IPT) = F(K,IPT) + REPSN
      IF(I .EQ. 1) GO TO 5
      IPN = I
      GO TO 6
C       WHEN CONTROL COMES TO 1 BELOW, YOU SUBTRACT REPSN
C          FROM THE FLOW F(P,I,K).
C
    1    DO 7 ISCN = 1,M
      IF(IVEC(ISCN) .NE. IPN) GO TO 7
      IF(JVEC(ISCN) .NE. I) GO TO 7
      IPT = ISCN
      GO TO 8
    7   CONTINUE
    8   K=LBLS(IPN,2)
      F(K,IPT) = F(K,IPT) - REPSN
      IPN = I
      GO TO 6
     5 CONTINUE
   20        FORMAT(1H ,*FLOW ROW 1 IS...*,6F15.3)
   21        FORMAT(1H ,*FLOW ROW 2 IS...*,6F15.3)
        RETURN
      END
      SUBROUTINE MAXCAL
C        THIS PERFORMS A SENSITIVITY ANALYSIS FOR THE LAST ITERATION.
C
       COMMON/FLK/
     A         A(2,100), C(2,100), TAU(100), ABAR(2,100), DUR(100),
     1    LBLS(100,3),     F(2,100), ISTATE(100), IVEC(100),
     2     JVEC(100),M,N,TM ,IFLAG,TO(100),TD(100), ITER,REPS(100),
     3         RLMDA(100), PLMDA(100),SEN(100),ISEN(100),SUMK
C
      IC = 0
      DO 10 IV = 1,100
 10   ISEN(IV) = 0
      DO 1 IV = 1,M
      I =IVEC(IV)
      J = JVEC(IV)
      IF(J.NE.N) GO TO 1
      IC = IC+1
      ARG = F(1,IV) + F(2,IV) -C(1,IV)
      SEN(IC) = AMAX1(0.,ARG)
      ISEN(IC) = I
C...IF M = 100, CHANGE DIMENSION OF ISEN TO 101.
       ISEN(M+1)=IC
C...RLMH
    1     CONTINUE
      WRITE(6,2)(SEN(I),I=1,IC)
    2     FORMAT(1H ,*SENSITIVITY VALUES ARE...*,10F10.2)
      RETURN
      END
```

PREPARATION OF OUTPUTS OF MATHEMATICAL PROGRAMMING ALGORITHM FOR SUBSEQUENT ROUTINES OF THE MATHEMATICAL PROGRAMMING PACKAGE

```
      SUBROUTINE TRFXD
      COMMON/CV1/IP(12),RP(12)
     COMMON/CV3/NV,NCF,PHIT,UZ,US,USP,USM
      COMMON/CV4/X(100),COST
      COMMON/CV5/SIGMA(100,4),TSIG
      COMMON/CV6/XZ(100)
      COMMON/CV9/PSIGL( 50),NXBL(50),XNBL(50),BLIST(50,100)
      COMMON/FV3/RLM(100), PLM(100), SN(100), ISN(100)
      COMMON/C1/NORP,NNZP
      COMMON/C4/NINDX,NJNDX,CDATE,IJK(100,3),NLIN,NCVX
      COMMON/C5/IA(100),JA(100),BL(100),UL(100),CV(100)
      COMMON/C6/IXA(100),JXA(100),S(100)
      COMMON/C7/D1(100)
C
      COMMON/FLK/
     A        A(2,100), C(2,100), TAU(100), ABAR(2,100), DUR(100),
     1   LBLS(100,3),     F(2,100), ISTATE(100), IVEC(100),
     2    JVEC(100),M,N,TM ,IFLAG,TO(100),TD(100), ITER,REPS(100),
     3        RLMDA(100), PLMDA(100),SEN(100),ISEN(100),SUMK
C
      DIMENSION CIJ(100)
      EQUIVALENCE          (SIGMA(1,1),CIJ(1))
C...DUR(IJ) AND C(IJ) ARE TO BE PASSED ON TO PHASE 5
      IF(IP(6).EQ.1) PRINT 900
  900 FORMAT(1H0,20HSUBROUTINE TRFXD    /(1X,10F10.2))
C...CONCAVE AND LINEAR
      NX2=NCF + NLIN
      DO 200 I= 1, NX2
      CALL MAPIS(IS,I)
      DUR(IS) = XZ(I) + D1(I)
      CALL GETPHI(I,XZ,   BLIST,CIJ(IS))
  200 CONTINUE
C...CONVEX
      NX1=NX2+1
      DO 300 I=NX1,NV,2
      CALL MAPIS(IS,I)
      X1 = XZ(I) + D1(I)
      X2 = XZ(I+1)
      DUR(IS)=X1+X2
      CALL GETPHI(I,XZ,   BLIST,C1)
      CALL GETPHI(I+1,XZ,     BLIST,C2)
      CIJ(IS)=C1+C2
  300 CONTINUE
C
```

```
      DO 410 I=1,M
      IVEC(I)=IXA(I)
      JVEC(I)=JXA(I)
 410  CONTINUE
      IC = ISEN(NINDX+1)
      DO 420 I=1,IC
      NX2=ISN(I)
      DO 412 J=1,NINDX
      IF(N.NE.JA(J)) GO TO 412
      IF( NX2 .NE. IA(J)) GO TO 412
      CALL MAPIS( NX1, J)
      ISN(I) = IXA(NX1)
 412  CONTINUE





 420  CONTINUE
      M = MNZP
      N= NORP
C
      IF(IP(6).EQ.1)PRINT 910,M, N, (IVEC(I),JVEC(I),I=1,M)
 910  FORMAT(1H0,20HSUBROUTINE TREXD    /(1X,20I5))
      IF(IP(6).EQ.1) PRINT 900, (SIGMA(I,1),DUR(I),I=1,M)
      RETURN
      END
      SUBROUTINE CPATH
C       THIS DOES THE CPM CALCULATIONS.
C
       COMMON/FLK/
     A         A(2,100), C(2,100), TAU(100), ABAR(2,100), DUR(100),
     1    IBLS(100,3),     F(2,100), ISTATE(100), IVEC(100),
     2     JVEC(100),M,N,TM ,IFLAG,TO(100),TD(100), ITER,REPS(100),
     3        RLMDA(100), PLMDA(100),SEN(100),ISEN(100),SUMK
C
       COMMON/B1/
     B                                             ENT(100),RLNT(100),
     4      ES(100), EF(100), RLF(100),RLS(100),TS(100), FS(100)
C
C
C       INITIALIZE NODE OCCURRENCE TIMES.
      DO 1 I = 1,N
      ENT(I) = 0.
    1   RLNT(I) = 0.
      TMIN = 0.
C       FORWARD PASS--CALCULATE EARLY START, EARLY FINISH TIMES
C          FOR EACH JOB, AND EARLY START TIMES FOR EACH NODE.
      DO 5 IV = 1,M
      I = IVEC(IV)
      J = JVEC(IV)
      ES(IV) = ENT(I)
      EF(IV) = ES(IV) + DUR(IV)
      IF(EF(IV) - ENT(J)) 6,6,7
    7   ENT(J)= EF(IV)
    6   IF(EF(IV) -TMIN)5,5,8
    8   TMIN = EF(IV)
    5    CONTINUE
C       BACKWARD PASS--CALCULATE RLF, RLS, AND TS FOR EACH JOB, AND
C       RLNT FOR EACH NODE.
      DO 20 I = 1, N
   20 RLNT(I) = TMIN
      DO 9 IV = 1,M
      L = M +1-IV
      INODE =IVEC(L)
      JNODE = JVEC(L)
      RLF(L) = RLNT(JNODE)
      RLS(L) = RLF(L) - DUR(L)
      IF(RLS(L) - RLNT(INODE)) 10 ,11,11
```

```
10    RLNT(INODE) = RLS(L)
11    TS(L) = RLS(L) -ES(L)
9     CONTINUE
C       CALCULATE FREE SLACK FOR EACH JOB.
     DO 12 IV = 1,M
     J = JVFC(IV)
12    FS(IV) = TS(IV) - RLNT(J) + ENT(J)
     WRITE(6,100)
100   FORMAT(1H ,*EARLY START TIMES WERE...*)
     WRITE(6,101)(ES(I),I = 1,M)
101   FORMAT(1H ,15F6.2)
     WRITE(6,102)
102   FORMAT(1H ,*LATE START TIMES WERE...*)
     WRITE(6,101)(RLS(I),I = 1,M)
     WRITE(6,103)
103   FORMAT(1H ,*FREE SLACKS WERE...*)
     WRITE(6,101)(FS(I),I= 1,M)
      WRITE(6,104)
104  FORMAT(1H ,*FF*)
     WRITE(6,101)(FF(I),I = 1,M)
105  FORMAT(1H ,*TS*)
      WRITE(6,105)
     WRITE(6,106)(TS(I),I = 1,M)
106  FORMAT(1H , 6F18.14)
     RETURN
     END
```

```
      SUBROUTINE COSTC
C
      COMMON/C1/NORP,NNZP
      COMMON/C4/NINDX,NJNDX,CDATE,IJK(100,3),NLIN,NCVX
      COMMON/C6/IXA(100),JXA(100),S(100)
C
      COMMON/CV1/IP(12),RP(12)
     COMMON/CV3/NV,NCF,PHIT,UZ,US,USP,USM
      COMMON/CV4/X(100),COST
      COMMON/CV5/SIGMA(100,4),TSIG
      COMMON/CV9/PSIGL( 50),NXBL(50),XNBL(50),BLIST(50,100)
C
      COMMON/FLK/
     A         A(2,100), C(2,100), TAU(100), ABAR(2,100), DUR(100),
     1   LBLS(100,3),    F(2,100), ISTATE(100), IVEC(100),
     2    JVEC(100),M,N,TM ,IFLAG,TO(100),TD(100), ITER,REPS(100),
     3        RLMDA(100), PLMDA(100),SEN(100),ISEN(100),SUMK
C
      COMMON/B1/
     B                                                ENT(100),RLNT(100),
     4      FS(100), EF(100), RLF(100),RLS(100),TS(100), FS(100)
C
      DIMENSION DIJ(100), CIJ(100), T(100)
      EQUIVALENCE(CIJ,SIGMA(1,1)), (DIJ,DUR),          (T,ENT)
      DIMENSION AC(100,10), BC(100,10), CC(100,10), CS(100,10),CK(10)
      DIMENSION RN(10), ALP(10), UB(10)
C...THIS EQUIVALENCE REQUIRES THAT BLIST(50, ) CONTAIN AT LEAST 4020 WORDS
      EQUIVALENCE (BLIST(1,1),AC(1,1)), (BLIST(1,21),BC(1,1)),
     1              (BLIST(1,41),CC(1,1)), (BLIST(1,61),CS(1,1)),
     2 (BLIST(1,81),CK), (BLIST(11,81),UB)
C
      IF(IP(6).EQ.1) PRINT 900
  900 FORMAT(1H0,20HSUBROUTINE COSTC    /10F10.2)
C...DEFINE KCAP TO BE MIN-INTEGER .GE. CDATE   (.LE. 10)
      KCAP = (CDATE + .99)
      DO 100 I=1,NNZP
      DO 100 J=1,10
C...KCAP MUST BE .LE. 10
      AC(I,J)=0.0
      BC(I,J)=0.0
      CC(I,J)=0.0
      CS(I,J)=0.0
  100 CONTINUE
      TC=0.0
      DO 200 I=1,NNZP
      IX=IXA(I)
C...ASSUME  T(I) CORRESPONDS TO I(TH)ROW  OF PIJ( ).
      TX=T(IX)
```

```
 TXD=TX+DIJ(I)
 TC=TC+CIJ(I)
 DO 200 K=1,KCAP
 IF((TX.GE.K-1).AND.(TX.LT.K))AC(I,K)=1.0
 IF((TXD.GE.K-1).AND.(TXD.LT.K))BC(I,K)=1.0
 IF((TX.LT.K-1).AND.(TXD.GE.K)) CC(I,K)=1.0
 IF(DIJ(I).EQ. 0.0)GO TO 200
 CS(I,K) =(CIJ(I)/DIJ(I))*(AC(I,K) * (K-TX) +
1          BC(I,K)*(TXD + 1-K) +
2                   CC(I,K) - AC(I,K)*BC(I,K))
  200   CONTINUE
C
        IF(IP(6).NE.1)GO TO 230
        PRINT 220, (T(I),I=1,M)
  220   FORMAT(1X,10F10.2)
        PRINT 220,((AC(I,J),J=1,10),I=1,M)
        PRINT 220,((BC(I,J),J=1,10),I=1,M)
        PRINT 220,((CC(I,J),J=1,10),I=1,M)
        PRINT 220,((CS(I,J),J=1,10),I=1,M)
        PRINT 220, (S(I),I=1,M)
  230   CONTINUE
C
        TSK=0.0
        DO 300 K=1,KCAP
        CK(K)=0.0
        DO 290 I=1,NNZP
        RN(K)= AC(I,K)+ BC(I,K) + CC(I,K) - AC(I,K)* BC(I,K)
        CK(K)=CK(K) + CS(I,K)
  290   CONTINUE
        TSK=TSK + CK(K)
  300   CONTINUE
        IF(IP(6).EQ.1) PRINT 220,  TC, TSK
        IF( ABS(TC-TSK).GT.0.05)STOP0017
        DO 320 I=1,NNZP
        TMP=0.0
        DO 310 K=1,KCAP
        TMP=TMP + CS(I,K)
        IF(IP(6).EQ.1) PRINT 220,  CIJ(I), TMP
  310   CONTINUE
        IF( ABS(CIJ(I)-TMP).GT.0.05)STOP0020
  320   CONTINUE
  400   CONTINUE
C
C
C...STATISTICAL ANALYSIS
C
C
        ALP(1) = 1.65   $  X(1) = .95
        ALP(2) = 1.75   $  X(2) = .96
        ALP(3) = 1.88   $  X(3) = .97
        ALP(4) = 2.05   $  X(4) = .98
        ALP(5) = 2.33   $  X(5) = .99
```

```
      ALP(6) = 3.87   $  X(6) = 1.0
      URCOST=0.0
      TMP = 0.0
      DO 410 I=1,NNZP
      TMP=TMP+(  S(I)* CIJ(I)/1.3 )**2
  410 CONTINUE
      URCOST= TC + 1.65 * (SQRT(TMP))
      USP=URCOST
      TMP =KCAP
      TMP= 1.0/TMP
      TMP = (.95)** (TMP)
      DO 430 I=1,6
      IF(TMP.GT. X(I))GO TO 430
      IF(TMP.GE. X(I-1))GO TO 450
      STOP 0021
  430  CONTINUE
       STOP 0022
  450  CONTINUE
C..    AK
       NX1= I-1
       NX2= I
       AK =  ALP(NX1)+((ALP(NX2) - ALP(NX1))/(X(NX2)-X(NX1))) *
     1       (TMP - X(NX1))
       DO 460  K = 1,KCAP
       TMP =0.0
       DO 455  I = 1,NNZP
       IF(CIJ(I).EQ.0)GO TO 455
       TMP = TMP + ((S(I)*CIJ(I)/1.3)**2) * CS(I,K)/CIJ(I)
  455  CONTINUE
       UR(K)= CK(K)+AK*SQRT(TMP)
  460  CONTINUE
       IF(IP(6).NE.1)GO TO 470
       PRINT 220, TC, URCOST, AK
       PRINT 220, (CK(I),I=1,KCAP)
       PRINT 220, (UR(I),I=1,KCAP)
  470  CONTINUE
       IF(IP(6).EQ.1) PRINT 900
       RETURN
       END
      SUBROUTINE WRPT
C...ALL LABELLED COMMONS
      COMMON/CV1/IP(12),RP(12)
      COMMON/CV2/BLO(100),ULO(100),NOCTT(100),CO(100)
     COMMON/CV3/NV,NCF,PHIT,UZ,US,USP,USM
      COMMON/CV4/X(100),COST
      COMMON/CV5/SIGMA(100,4),TSIG
      COMMON/CV6/X7(100)
      COMMON/CV7/NF1,NOP,NOPS
      COMMON/CV8/NXBK,XK,NOBOL
      COMMON/CV9/PSIGL( 50),NXBL(50),XNXBL(50),BLIST(50,100)
      COMMON/CV10/FKBL(50)
      COMMON/CV11/SUPC(100,2),UNTC(100,2),XBIJ(50,3)
      COMMON/FV3/RLM(100), PLM(100), SN(100), ISN(100)
      COMMON/TSW/NSWW
C...
      COMMON/C1/NORP,NNZP
      COMMON/C2/MXNZ,MXRP
      COMMON/C3/ IK(100), IPDC(100),DC(6,100),JC(100),IR(100)
      COMMON/C4/NINDX,NJNDX,CDATE,IJK(100,3),NLIN,NCVX
      COMMON/C5/IA(100),JA(100),BL(100),UL(100),Q(100)
      COMMON/C6/IXA(100),JXA(100),S(100)
      COMMON/C7/D1(100)
      COMMON/C8/ITN (100),IGN (100)
C
      COMMON/FLK/
     A        A(2,100), C(2,100), TAU(100), ABAR(2,100), DUR(100),
     1   LBLS(100,3),    F(2,100), ISTATE(100), IVEC(100),
     2    JVEC(100),M,N,TM ,IFLAG,TO(100),TD(100), ITER,REPS(100),
     3       RLMDA(100), PLMDA(100),SEN(100),ISEN(100),SUMK
C
      COMMON/B1/
     B                                               ENT(100),RLNT(100),
     4      FS(100), EF(100), RLF(100),RLS(100),TS(100), FS(100)
C
C......
C...THIS EQUIVALENCE REQUIRES THAT BLIST(50, ) CONTAIN AT LEAST 4020 WORDS
      DIMENSION AC(100,10), BC(100,10), CC(100,10), CS(100,10)
      DIMENSION CK(10), UB(10)
      EQUIVALENCE (BLIST(1,1),AC(1,1)), (BLIST(1,21),BC(1,1)),
     1             (BLIST(1,41),CC(1,1)), (BLIST(1,61),CS(1,1)),
     2             (BLIST(1,81),CK(1)), (BLIST(11,81),UB(1))
C
      DIMENSION CIJ(100), T(100)
      EQUIVALENCE (SIGMA(1,1),CIJ), (ENT(1),T(1))
C
      DATA KBL/10H          /
      TABLE = 3.
```

```
C......PRINT 20, TABLE
   20  FORMAT(1H1, 5HTABLE, F3.0)
       PRINT 19
   19  FORMAT(1H1/////)
       PRINT 31
   31  FORMAT(1H0,10H TASK NAME,10H    I    J,
     1 60H        D(1)        D(2)        D(3)       C(1)        C(2)        C(3) ,
     2 10H        S(I)/)
       DO 39 I=1,M
C...TABLE 3 - INPUT DATA
       PRINT 32, ITN(I), IXA(I), JXA(I),(DC(K,I),K=1,6), S(I)
   32  FORMAT(1H0,A10,2I5,7F10.2)
   39  CONTINUE
C...TABLE 4 - TASK COSTS AND SCHEDULE
       TABLE = 4.
C......PRINT 20, TABLE
       PRINT 19
       PRINT 41
   41  FORMAT(1H0,10H TASK NAME,10H    I    J,
     1 50H        COST  DURATION INIT-DATE  END-DATE     SLACK/)
       DO 45 I=1,NN7P
       PRINT 42, ITN(I), IXA(I), JXA(I), CIJ(I), DUR(I), ES(I), EF(I),
     1 FS(I)
   42  FORMAT(1H0,A10,2I5, 5F10.2)
   45  CONTINUE
       PRINT 46, UZ, USP
   46  FORMAT(1H0,18HTOTAL PROJECT COST,F10.2//,
     1 37H UPPER LIMIT ON TOTAL PROJECT COST IS, F10.2,
     2 30H  WITH PROBABILITY 95 PERCENT.)
C...TABLE 5 - FINAL GOAL ACHIEVEMENT DATES
       TABLE = 5.
C......PRINT 20, TABLE
       PRINT 19
       PRINT 51
   51  FORMAT(1H0,38H GOAL NAME  NODE NO.  ACHIEVEMENT DATE/)
       DO 53 I=1,M
       IF(JXA(I).NE. N )GO TO 53
       PRINT 52, IGN(I), IXA(I), ES(I)
   52  FORMAT(1H0, A10, I5, 5X, F10.2 )
   53  CONTINUE
C...TABLE 6 - TOTAL YEARLY COSTS OF PROJECT
       KCAP =(CDATE + .99)
       TABLE = 6.
C......PRINT 20, TABLE
       PRINT 19
       PRINT 61
   61  FORMAT(1H0,47H YEAR   PROJECT COST    UP. BND.-COST-.95 PROB./)
       DO 63 I=1,KCAP
       PRINT 62, I, CK(I), UB(I)
   62  FORMAT(1H0, I5,F15.2,10X,F15.2)
   63  CONTINUE
C...TABLE 7 - YEARLY TASK COST
       TABLE = 7.
```

```
C......PRINT 20, TABLE
       PRINT 19
       PRINT 71
   71  FORMAT(1H0,20H TASK NAME     I     J,
     1 50H         YR1         YR2         YR3         YR4         YR5,
     2 50H         YR6         YR7         YR8         YR9        YR10/)
       DO 73 I=1,M
       PRINT 72, ITN(I), IXA(I), JXA(I), (CS(I,J),J=1,KCAP)
   72  FORMAT(1H0,A10,2I5,10F10.2/(1X,20X,10F10.2))
   73  CONTINUE
C...TABLE 8 - OBLIGATIONS AND TASKS STARTING, FINISHING, ACTIVE AND CONTINUING
       TABLE = 8.
       DO  87 J=1,KCAP
C......PRINT 20, TABLE
       PRINT 19
       PRINT 80, J
   80  FORMAT(1H0,4HYEAR,I3)
       PRINT 81
   81  FORMAT(1H0,6X,7HACTIVE 7X,12X,5HSTART,13X,7X,6HFINISH,7X,
     1 6X,8HCONTINUE)
       PRINT 82
   82  FORMAT(1H0,50H        TASK     I     J        TASK     I     J    OBLGTN,
     1             40H        TASK     I     J        TASK     I     J/)
       DO  85 I=1,M
       IF(CS(I,J).EQ.0)GO TO 852
       NM1=ITN(I)
       I1=IXA(I)
       J1=JXA(I)
C...INITIALIZE PRINT LINE
C...
       NM2=KBL
       I2 =0
       J2 =0
C...
       CIJAY=0.0
       NM3=KBL
       I3=0
       J3=0
C...
       NM4=KBL
       I4=0
       J4=0
C...
       IF(  AC(I,J)*CS(I,J).EQ.0.0)GO TO 841
       NM2= ITN(I)
       I2= IXA(I)
       J2= JXA(I)
       CIJAY= CIJ(I)
  841  CONTINUE
       IF(  BC(I,J)*CS(I,J).EQ.0.0)GO TO 842
       NM3=ITN(I)
       I3= IXA(I)
       J3= JXA(I)
  842  CONTINUE
       IF( (AC(I,J).NE.0.0).OR.(CS(I,J).EQ.0.0) ) GO TO 84
```

```
      NM4=ITN(I)
      I4 =IXA(I)
      J4 =JXA(I)
   84 CONTINUE
      IF(I1.EQ.0) GO TO 85
      PRINT 851,NM1,I1,J1,NM2,I2,J2,CIJAY,NM3,I3,J3,NM4,I4,J4
  851 FORMAT(1H0,2(A10,2I5),F10.2,2(A10,2I5))
      GO TO 85
  852 IF(  BC(I,J)*CIJ( I).EQ.0.0)GO TO 85
      NM1 = KBL
      I1 = 0
      J1 = 0
      NM2 = KBL
      I2 = 0
      J2 = 0
      CIJAY = 0.0
      NM3 = ITN(I)
      I3 = IXA(I)
      J3 = JXA(I)
      NM4 = KBL
      I4 = 0
      J4 = 0
      PRINT 851,NM1,I1,J1,NM2,I2,J2,CIJAY,NM3,I3,J3,NM4,I4,J4
   85 CONTINUE
   97 CONTINUE
C...TABLE  9 - CRITICAL TASK
      TABLE = 9.
C......PRINT 20, TABLE
      PRINT 19
      PRINT 91
   91 FORMAT(1H0,20H TASK NAME     I     J/)
      DO 93 I=1,M
      IF(  ABS( TS(I) ).GT.RP(7)   ) GO TO 93
      PRINT 92, ITN(I), IXA(I), JXA(I)
   92 FORMAT(1H0,A10,2I5)
   93 CONTINUE
C...TABLE  10 - SENSITIVITY ANALYSIS I
      TABLE = 10.
C......PRINT 20, TABLE
      PRINT 19
      PRINT 101
  101 FORMAT(1H0,23H  NODE NO.  SENSITIVITY/)
      DO 103 I=1,M
      IF(ISN(I).LT.1)GO TO 103
      PRINT 102, ISN(I), SN(I)
  102 FORMAT(1H0, I5, 5X, F8.2 )
  103 CONTINUE
C...TABLE 11 - SENSITIVITY ANALYSIS II
      TABLE = 11.
```

```
C......PRINT 20, TABLE
      PRINT 19
      PRINT 111
  111 FORMAT(1H0,20H    SAVING     DELAY/)
      RIT=ITER + 0.0
      IT1 = ITER - 1
      PLMX = PLM(ITER-1) + (RLM(IT1)-TM) * (  (PLM(ITER)-PLM(ITER-1))/
     1         ( (RLM(IT1)-TM) - (RLM(ITER)-TM)  ) )
      DO 113 I=1,IT1
      RIT= RLM(I)- TM
      PIT = PLMX - PLM(I)
      PRINT 112, PIT, RIT
  112 FORMAT(1H0,2F10.2)
  113 CONTINUE
      RETURN
      END
```

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